Noetherian-like properties in polynomial and power series rings

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There are several Noetherian-like properties, e.g., Noetherian spectrum, Laskerian, strong-finite-type (SFT), piecewise Noetherian property. We investigate the stability of such properties under polynomial and power series extensions. In particular, we show that for a nonzero SFT prime ideal \( P \) of a Prüfer domain \( D \), the following statements are equivalent: (1) \( D[[X]]_{P[[X]]} \) is Noetherian; (2) \( \text{ht} \ P = 1 \) and \( k[[X]] = D[[X]]_{D[[X]](0)} \), where \( D = D/P \) and \( k \) is the quotient field of \( D \); (3) \( D[[X]]_{P[[X]]} \) is a valuation domain. As a corollary, we also show that for a Prüfer domain \( D \), \( D[[X]] \) is piecewise Noetherian if and only if \( D \) is Noetherian.

REFERENCES

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