THE FIFTH CHINA-JAPAN-KOREA
INTERNATIONAL SYMPOSIUM ON RING THEORY
(September 10 (Mon.) - 15 (Sat.), 2007)

ABSTRACT

National Olympic Memorial Youth Center
3-1. Yoyogi Kamizono-cho, Shibuya-ku, Tokyo 151-0052, JAPAN

CONTENTS

[Page No.] Names (Alphabetic order of Sir name) and Title

50 C.M.Ringel
The work of Tachikawa on finite-dimensional algebras and their representations
51 S. Tariq Rizvi
On structure of rings of quotients
52 Cosmin S. Roman
ON DIRECT SUMS OF EXTENDING AND BAER MODULES
53 Mohammad Shadab Khan
ON DERIVATIONS IN NEAR RINGS
54 Liang Shen
Characterizations of QF Rings
55 Hongbo Shi
A Note on The Finitistic Dimension Conjecture
56 M. Siles Molina
Algebras of quotients of Leavitt path algebras
57 Patrick F. Smith
Ascending Chain Conditions in Groups, Rings and Modules
58 Ryo Takahashi
On contravariantly finite subcategories of finitely generated modules
59 Akira UEDA
Finitely generated modules over non-commutative valuation rings
60 Masahiko Uhara, Yoshihisa Nagatomi, Kiyoichi Oshiro
skew-matrix ring and applications to QF-ring
61 Lia Vaš
Extending Ring Derivations to Rings and Modules of Quotients
62 Yanhua Wang
Construct bi-Frobenius algebras via quivers
63 Zhixi Wang
The Fundamental Theorems of Entwined Modules
64 Quanshui Wu
Derived Categories over Auslander-Gorenstein Rings
65 Kunio Yamagata
On lifting of idempotents in the endomorphism ring of a module
66 Angelina Yan Mui Chin
CLEAN RINGS AND RELATED CLASSES OF RINGS
67 Hailou Yao
On Simple Connectedness of Minimal Representation-Infinite Algebras
68 Hiroshi Yoshimura
LOCAL QF RINGS WITH RADICAL CUBED ZERO II
69 Jun Zhang
Pushout Artin-Schelter Regular algebras of Global Dimension Four
70 Yingbo Zhang
Professor Liu - Live and Work
71 Moharram A. Khan
A study on gamma nearrings with derivations
72 Shouchuan Zhang
Classification of Element Systems over Finite Commutative Groups
73 Tokuji Araya
Auslander-Reiten conjecture on Gorenstein rings
74 Yoshitomo Baba
On Colocal Pairs
75 Vlastimil Dlab
Approximations of algebras

Derived equivalences and Serre duality for Gorenstein algebras

Hiroki Abe and Mitsuo Hoshino

Let R be a commutative noetherian ring and A a Noether R-algebra, i.e., A is a ring endowed with a ring homomorphism $R \to A$ whose image is contained in the center of A and A is finitely generated as an R-module. Let $c \geq 0$ be an integer. Assume $\mathrm{Ext}^i_R(A, R) = 0$ for $i \neq c$ and set $\Omega = \mathrm{Ext}^c_R(A, R)$. We call A a Gorenstein R-algebra of codimension c if $R_\mathfrak{p}$ is Gorenstein for all $\mathfrak{p} \in \mathrm{Supp}_R(A)$ and Ω is a projective generator for right A-modules. If A is a Gorenstein R-algebra of codimension c, then we see that Ω lies in the center of $\mathrm{Pic}(A)$, the Picard group of A, that Ω is a dualizing complex for A if $\sup\{\dim R_\mathfrak{p} \mid \mathfrak{p} \in \mathrm{Supp}_R(A)\} < \infty$, and that $\mathrm{Ann}_R(A)$ contains an R-regular sequence $x_1, \cdots, x_c$ and A is a Gorenstein S-algebra of codimension 0, where S is the residue ring of R over the ideal generated by $x_1, \cdots, x_c$. Also, we see that our Gorenstein algebras are Gorenstein in the sense of [3]. In particular, commutative Gorenstein algebras are Gorenstein rings.

Our main aim is to demonstrate that Serre duality theory plays an essential role in the theory of derived equivalences for Gorenstein algebras. We extend Serre duality theory (cf. [2]) to Noether algebras. We see that for an arbitrary Noether R-algebra A there exists a bifunctorial isomorphism in Mod-R

$$\mathrm{Hom}_{\mathcal{D}(\text{Mod-}A)}(Y^\bullet, X^\bullet \otimes^{\mathbf{L}}_A V^\bullet) \cong \mathbf{R}\mathrm{Hom}^\bullet_A(X^\bullet, Y^\bullet)^*$$

for $X^\bullet \in \mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{fpd}}$ and $Y^\bullet \in \mathcal{D}(\text{Mod-}A)$, where $V^\bullet = \mathrm{Hom}^\bullet_R(A, I^\bullet)$ with $I^\bullet$ a minimal injective resolution of R and $(-)^* = \mathrm{Hom}_{\mathcal{D}(\text{Mod-}R)}(-, R)$. On the other hand, we know from [1, Theorem 4.7] that if $V^\bullet$ is a dualizing complex for A and if inj dim ${}_AA$ = inj dim $A_A < \infty$ then $- \otimes^{\mathbf{L}}_A V^\bullet$ induces a self-equivalence of $\mathcal{D}^{\mathrm{b}}(\text{mod-}A)$.

Assume A is a Gorenstein R-algebra of codimension c. Let $P^\bullet \in \mathcal{K}^{\mathrm{b}}(\mathcal{P}_A)$ be a tilting complex and $B = \mathrm{End}_{\mathcal{K}(\text{Mod-}A)}(P^\bullet)$. We ask when B is also a Gorenstein R-algebra of codimension c. Set $\nu = - \otimes^{\mathbf{L}}_A \Omega$. Then by Serre duality theory we have an isomorphism of B-bimodules

$$\mathrm{Hom}_{\mathcal{D}(\text{Mod-}A)}(P^\bullet, \nu P^\bullet[i]) \cong \mathrm{Ext}^{i+c}_R(B, R)$$

for all $i \in \mathbb{Z}$. On the other hand, denoting by $\mathcal{S}$ the full subcategory of $\mathcal{D}^-(\text{Mod-}A)$ consisting of complexes $X^\bullet$ with $\mathrm{Hom}_{\mathcal{D}(\text{Mod-}A)}(P^\bullet, X^\bullet[i]) = 0$ for $i \neq 0$, we have an equivalence $\mathrm{Hom}_{\mathcal{D}(\text{Mod-}A)}(P^\bullet, -) : \mathcal{S} \to \text{Mod-}B$ (see [4, Section 4]). Thus B is a Gorenstein R-algebra of codimension c if and only if $\mathrm{add}(\nu P^\bullet) = \mathrm{add}(P^\bullet)$. Unfortunately, this is not the case in general. However, B is a Gorenstein R-algebra of codimension c with $\mathrm{Ext}^c_R(B, R) \cong B$ as B-bimodules if and only if A is a Gorenstein R-algebra of codimension c with $\Omega \cong A$ as A-bimodules.

References

1. H. Abe, Noetherian algebras of finite selfinjective dimension, Comm. Algebra, to appear.
2. A. I. Bondal and M. M. Kapranov, Representable functors, Serre functors, and reconstructions, Izv. Akad. Nauk SSSR Ser. Mat. **53** (1989), no. 6, 1183-1205, 1337.
3. S. Goto and K. Nishida, Towards a theory of Bass numbers with application to Gorenstein algebras, Colloquium Math. **91** (2002), 191–253.
4. J. Rickard, Morita theory for derived categories, J. London Math. Soc. (2) **39** (1989), no. 3, 436–456.

Institute of Mathematics, University of Tsukuba,
Ibaraki, 305-8571, Japan
Email: abeh@math.tsukuba.ac.jp hoshino@math.tsukuba.ac.jp

On generalized Jordan left derivations in certain classes of rings

SHAKIR ALI
Department of Mathematics
Aligarh Muslim University
Aligarh-202002 (India)
E-mail:shakir50@rediffmail.com, shakirali_50@yahoo.com

Let R be an associative ring. An additive mapping $d : R \longrightarrow R$ is called a derivation (resp. Jordan derivation) if $d(xy) = d(x)y + xd(y)$(resp. $d(x^2) = d(x)x + xd(x)$), holds for all $x, y \in R$. An additive mapping $\delta : R \longrightarrow R$ is said to be a left derivation (resp. Jordan left derivation) if $\delta(xy) = x\delta(y) + y\delta(x)$ (resp. $\delta(x^2) = 2x\delta(x)$) holds for all $x, y \in R$. An additive mapping $G : R \longrightarrow R$ is called a *generalized left derivation (resp. generalized Jordan left derivation)* if there exists a Jordan left derivation $\delta : R \longrightarrow R$ such that $G(xy) = xG(y) + y\delta(x)$ (resp. $G(x^2) = xG(x) + x\delta(x)$) holds for all $x, y \in R$.

A mapping $f : R \longrightarrow R$ is said to be commuting on R if $f(x)x = xf(x)$, holds for all $x \in R$. Comparing Jordan left derivation with commuting mapping on a ring R, it turns out that the notion of Jordan left derivations is in a close connection with the commuting mappings on R. There has been considerable interest for commuting mappings on prime rings. The fundamental result in this direction is due to Posner [Proc. Amer. Math. Soc. $8(1957), 1093 - 1100$] which states that *if a prime ring R admits a nonzero derivation that is commuting on R, then R is commutative.* Using rather weaker hypotheses, Brešar and Vukman [Proc. Amer. Math. Soc. 110 (1990), 7-16] obtained a result which shows that the existence of a nonzero Jordan left derivation on a prime ring R of char$R \neq 2, 3$ forces R to be commutative. It was also remarked by Brešar and Vukman that the assumption R is 3-torsion free in the hypotheses of the above result may be avoided.

In the present paper, our objective is to generalize the above mentioned result for generalized Jordan left derivation. Some related results are also discussed.

THE REALIZATION PROBLEM FOR VON NEUMANN REGULAR RINGS

Pere Ara

Universitat Autònoma de Barcelona

Email: para@mat.uab.cat

For a ring R, denote by $\mathcal{V}(R)$ the abelian monoid of isomorphism classes of finitely generated projective right R-modules. This is always a *conical* monoid, meaning that, for $x, y \in \mathcal{V}(R)$, the relation $0 = x+y$ implies $x = y = 0$. If moreover the ring R is von Neumann regular (or just an exchange ring), then $M := \mathcal{V}(R)$ is a *refinement monoid*: whenever $a + b = c + d$ in M there exist $x, y, z, t \in M$ such that $a = x + y$, $b = z + t$, $c = x + z$ and $d = y + t$.

By results of Bergman [5, Theorems 6.2 and 6.4] and Bergman and Dicks [6, page 315], any conical monoid with an order-unit appears as $\mathcal{V}(R)$ for some unital hereditary ring R. We will discuss aspects of the following problem:

Realization Problem for von Neumann Regular Rings: Is every conical refinement monoid *representable*, that is, isomorphic to $\mathcal{V}(R)$, for some von Neumann regular ring R?

By results of Wehrung [7], there are conical refinement cancellative monoids of size $\geq \aleph_2$ which are not representable, but the question is still open for monoids of size $\leq \aleph_1$.

In the paper [2], it is shown that the above problem has a positive answer for a large class of conical refinement monoids, associated to directed graphs. This class of monoids was introduced and studied in [3]. We will state its characterization within finitely generated antisymmetric refinement monoids, obtained recently in [4]. Moreover we will describe some methods to represent classes of monoids which are beyond the class of graph monoids [1].

References

1. P. Ara, Realizing primitive monoids by von Neumann regular rings, *in preparation*.
2. P. Ara and M. Brustenga, The regular algebra of a quiver, *J. Algebra*, **309** (2007), 207–235.
3. P. Ara, M. A. Moreno, and E. Pardo, Nonstable K-theory for graph algebras, *A*lgebr. Represent. Theory, **10** (2007), 157–178.
4. P. Ara, F. Perera, and F. Wehrung, Finitely generated antisymmetric graph monoids, *P*reprint.
5. G. M. Bergman, Coproducts and some universal ring constructions, *T*rans. Amer. Math. Soc., **200** (1974), 33–88.
6. G. M. Bergman, W. Dicks, Universal derivations and universal ring constructions, *P*acific J. Math., **79** (1978), 293–337.
7. F. Wehrung, Non-measurability properties of interpolation vector spaces, *Israel J. Math.*, **103** (1998), 177–206.

LEAVITT PATH ALGEBRAS FOR COUNTABLE GRAPHS

Gene Abrams & Gonzalo Aranda Pino[1]

Department of Mathematics,University of Colorado, Colorado Springs,CO 80933, U.S.A.

Centre de Recerca Matemàtica, Apartat 50, E-08193, Bellaterra (Barcelona), Spain.

Email: abrams@math.uccs.edu & garanda@crm.es

We extend the notion of the Leavitt path algebra of a graph to include all directed countable graphs. We show how various ring-theoretic properties of these more general structures relate to the corresponding properties of Leavitt path algebras of row-finite graphs. Specifically, we identify those graphs for which the corresponding Leavitt path algebra is simple; purely infinite simple; exchange; and semiprime. In our final result, we show that all Leavitt path algebras have zero Jacobson radical.

References

1. G. Abrams, G. Aranda Pino, The Leavitt path algebra of a graph, *J. Algebra* **293 (2)** (2005), 319–334.
2. P. Ara, M.A. Moreno, E. Pardo, Nonstable K-Theory for graph algebras, *Algebra Represent. Theory* (to appear).
3. G. Aranda Pino, E. Pardo, M. Siles Molina, Exchange Leavitt path algebras and stable rank, *J. Algebra* **305 (2)** (2006), 912–936.
4. T. Bates, J. H. Hong, I. Raeburn, W. Szymański, The ideal structure of the C*-algebras of infinite graphs, *Illinois J. Math.* **46 (4)** (2002), 1159–1176.
5. D. Drinen, M. Tomforde, The C*-algebras of arbitrary graphs, *Rocky Mountain J. Math* **35 (1)** (2005), 105–135.
6. I. Raeburn, *Graph algebras.* CBMS Regional Conference Series in Mathematics, **103**, Amer. Math. Soc., Providence, (2005).

[1]Speaker

On Lie Ideals and Generalized (θ, ϕ)-dervations in Rings

Mohammad Ashraf
Department of Mathematics
A.M.U., Aligarh 202002 India
E-mail:mashraf80@hotmail.com

Let R be a ring and U a Lie ideal of R such that $u^2 \in U$. Let θ, ϕ be endomorphisms of R and M be a 2-torsion free R-bimodule such that $mRx = \{0\}$ with $m \in M, x \in R$ implies that either $m = 0$ or $x = 0$. An additive mapping $F : R \longrightarrow M$ is called a generalized (θ, ϕ) - derivation (resp. generalized Jordon (θ, ϕ)-derivation) on U if there exists a (θ, ϕ)-derivation

$d : R \longrightarrow M$ such that $F(uv) = F(u)\theta(v) + \phi(u)d(v)$ (resp. $F(u^2) = F(u)\theta(u) + \phi(u)d(u)$), holds for all $u, v \in U$. It is obvious to see that every generalized (θ, ϕ) - derivation $F : R \longrightarrow M$ is a generalized Jordan (θ, ϕ) - derivation. However, the converse need not be true in general. Corresponding results for derivation was first obtaned by Herstein [Proc. Amer. Math. Soci. 8(1957), 1104-10] which states that every Jordan derivation on a prime ring R is a derivation. This result was further extended in many directions by several authors. In the present paper, we shall discuss such types of results in the setting of generalized derivation. In fact, in the present paper we prove that if θ is one-one and onto, then every generalized Jordon (θ, ϕ)-derivation on U is a generalized (θ, ϕ)-derivation on U. Further it is shown that the above result is also true in the case if the underlying module M is arbitrary and the Lie ideal U of R has a commutator which is free from zero divisor. Finally, a related result for centralizing and commuting generalized derivation on a ring R has been obtained.

GENERALIZED REVERSIBLE RINGS AND THEIR EXTENSIONS

Muhittin Başer

Kocatepe University

Emails: mbaser@aku.edu.tr

Throughout this paper, R denotes an associative ring with identity and α denotes a nonzero non identity endomorphism of a given ring, unless specified otherwise. Recall that a ring is *reduced* if it has no nonzero nilpotent elements. It is well-known that if R is a reduced ring, then the following condition is held: $ab = 0$ implies $ba = 0$ for $a, b \in R$. Cohn [1] called a ring R *reversible* if it holds this condition. Historically, some of the earliest results known to us about *reversible* rings (although not so called at the time) was due to Habeb [2]. Another generalization of a reduced ring is an Armendariz ring. Rege and Chhawchharia [7] called a ring R *Armendariz* if whenever any polynomials $f(x) = a_0 + a_1x + \cdots + a_mx^m$, $g(x) = b_0 + b_1x + \cdots + b_nx^n \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_ib_j = 0$ for each i and j. Recently, the Armendariz property of a ring was extended to skew polynomial rings but with skewed scalar multiplication in [4,3]: For an endomorphism α of a ring R, R is called *α-Armendariz* (resp. *α-skew Armendariz*) if for $p = \sum_{i=0}^{m} a_ix^i$ and $q = \sum_{j=0}^{n} b_jx^j$ in $R[x;\alpha]$, $pq = 0$ implies $a_ib_j = 0$ (resp. $a_i\alpha^i(b_j) = 0$) for all $0 \le i \le m$ and $0 \le j \le n$. On the other hand, an endomorphism α of a ring R is called *rigid* [6] if $a\alpha(a) = 0$ implies $a = 0$ for $a \in R$, and R is an *α-rigid* ring [3] if there exists a rigid endomorphism α of R.

Motivated by the above, for an endomorphism α of a ring R, we call right reversible if whenever $ab = 0$ for $a, b \in R$, then $b\alpha(a) = 0$. A ring R is called *right α-reversible* if there exists a right reversible endomorphism α of R. The notation of an α-reversible ring is a generalization of α-rigid rings as well as an extension of reversible rings. We study characterizations of α-reversible rings and their related properties. And the relationship between α-reversible rings and generalized Armendariz rings is also studied, and so several known results relating to α-rigid and reduced rings can be obtained as corollaries of our results.

References

1. P.M. Cohn, *Reversible rings*, Bull. London Math. Soc. **31** (1999), 641–648.
2. J.M. Habeb, *A note on zero commutative and duo rings*, Math. J. Okayama Univ. **32** (1990), 73–76.
3. C.Y. Hong, N.K. Kim and T.K. Kwak, *On Skew Armendariz Rings*, Comm. Algebra **31 (1)** (2003), 103–122.
4. C.Y. Hong, T.K. Kwak and S.T. Rizvi, *Extensions of generalized Armendariz rings*, Algebra Colloq. **13(2)** (2006), 253–266.
5. N.K. Kim and Y. Lee, *Extensions of reversible rings*, J. Pure and Appl. Algebra **185** (2003), 207–223.
6. J. Krempa, *Some examples of reduced rings*, Algebra Colloq. **3 (4)** (1996), 289–300.
7. M.B. Rege and S. Chhawchharia, *Armendariz rings*, Proc. Japan Acad. Ser. A Math. Sci. **73** (1997), 14–17.

Some Progress on Clean Rings

Jianlong Chen
Department of Mathematics
Southeast University
Nanjing, 210096, P.R.China

Abstract

Clean ring is introduced by Nicholson in 1977. Since then, it have attracted many ring theory experts to do further researches. During the study of clean rings, some related clean ring classes and many challengeable open questions are arisen, such as five open problems about strongly clean rings asked by Nicholson. Now the study of clean rings has been a hotspot in the ring theory. In this talk, We will give some recent progress about clean rings. It contains the following contents.
(1) Clean ring (group ring, g(x)-clean ring, 2-clean ring).
(2) Strongly clean ring (matrix ring, triangular matrix ring and power series ring).
(3) Uniquely clean group ring
(4) Uniquely strongly clean ring (example, characterization and relation).
(5) Some questions about clean rings.

A Class of Near-Rings from Rings

2000 Mathematics Subject Classification: 16Y30

Key Words: near-rings, (e, t)-near-rings, right permutable, LSD, RSD, prime ideals

Yong Uk Cho

Department of Mathematics Education, SillaUniversity,

Pusan 617-736, Korea

e-mail: yuchosilla.ac.kr

ABSRTRACT.

In this paper, from a given ring $(R, +, \cdot)$, we will construct that a new kind of near-ring. To construct a new class of near-rings from the base ring $(R, +, \cdot)$, the procedure is to begin with a ring $(R, +, \cdot)$ and a new multiplication denoted by $*$ on $(R, +)$ so as to obtain a new near-ring $(R, +, *)$. The new multiplication is expressed in terms of the original addition and multiplication by defining $a * b$ to be a polynomial in a and b with fixed (central) orthogonal idempotents e and t in R, for all a, b in R. We will also consider some relations of substructures of base rings and those of new kinds of near-rings, for example, subnear-rings, R-subgroups, invariant substructures and ideals, in particular, prime like ideals of a new near-rings and investigate some of their properties.

J. Clay [1] classified almost all abelian near-rings on given finite abelian groups of order ≤ 7. Our purpose is to introduce a special method for constructing a new class of abelian near-rings from arbitrary rings with (central) orthogonal idempotents.

One-sided ideals in simple rings

John Clark

Abstract

One-sided ideals in simple ringsabstract: This talk is on joint work with Dinh Van Huynh. Using a variation on the concept of a CS module, we describe exactly when a simple ring is isomorphic to a ring of matrices over a Bezout domain. Our techniques are then applied to characterise simple rings which are right and left Goldie, right and left semihereditary.

On Divisible and Torsionfree Modules

Nanqing Ding
Department of Mathematics, Nanjing University, Nanjing 210093, China
E-mail: nqding@nju.edu.cn

Abstract

This talk is a report on joint work with Lixin Mao.

Let R be a ring. A left R-module M is said to be *divisible* (or *P-injective*) if $\mathrm{Ext}^1(R/Ra, M) = 0$ for all $a \in R$. A right R-module N is called *torsionfree* if $\mathrm{Tor}_1(N, R/Ra) = 0$ for all $a \in R$. The definitions of divisible and torsionfree modules coincide with the classical ones in case R is a commutative domain. It is clear that a right R-module N is torsionfree if and only if the character module N^+ is divisible by the standard isomorphism $\mathrm{Ext}^1(R/Ra, N^+) \cong \mathrm{Tor}_1(N, R/Ra)^+$ for every $a \in R$.

In this paper, a ring R is called left P-coherent in case each principal left ideal of R is finitely presented. A left R-module M (resp. right R-module N) is called D-injective (resp. D-flat) if $\mathrm{Ext}^1(G, M) = 0$ (resp. $\mathrm{Tor}_1(N, G) = 0$) for every divisible left R-module G. It is shown that every left R-module over a left P-coherent ring R has a divisible cover; a left R-module M is D-injective if and only if M is the kernel of a divisible precover $A \to B$ with A injective; a finitely presented right R-module L over a left P-coherent ring R is D-flat if and only if L is the cokernel of a torsionfree preenvelope $K \to F$ with F flat. We also study the divisible and torsionfree dimensions of modules and rings. As applications, some new characterizations of von Neumann regular rings and PP rings are given.

Key Words: P-coherent ring; divisible module; torsionfree module; D-injective module; D-flat module; Warfield cotorsion module; (pre)cover; (pre)envelope.

2000 Mathematics Subject Classification: 16D50; 16D40; 16E10; 18G10.

A TILED ORDER OF FINITE GLOBAL DIMENSION WITH NO NEAT PRIMITIVE IDEMPOTENT

Hisaaki Fujita and Akira Oshima

Let R be a discrete valuation ring with a unique maximal ideal πR and a quotient field K, and let $F = R/\pi R$ be the residue class field. Let $n \geq 2$ be an integer and $\{\lambda_{ij} \mid 1 \leq i, j \leq n\}$ a set of n^2 integers satisfying

$$\lambda_{ii} = 0, \quad \lambda_{ik} + \lambda_{kj} \geq \lambda_{ij}, \quad \lambda_{ij} + \lambda_{ji} > 0 \quad (\text{if } \ i \neq j)$$

for all $1 \leq i, j, k \leq n$. Then $\Lambda = (\pi^{\lambda_{ij}} R)$ is a basic semiperfect Noetherian R-subalgebra of the full $n \times n$ matrix algebra $\mathrm{M}_n(K)$. We call such Λ a *tiled R-order* in $\mathrm{M}_n(K)$.

Let S be a semiperfect Noetherian ring and e a primitive idempotent of S. Following Ágoston, Dlab and Wakamatsu [1], we call e a *neat* primitive idempotent if $\mathrm{Ext}^i_S(V, V) = 0$ for all $i \geq 1$, where V is a simple right S-module with $Ve \neq 0$.

It is shown by Jategaonkar [4] that for a fixed $n \geq 2$, there are only finitely many tiled R-orders of finite global dimension in $\mathrm{M}_n(K)$, up to isomorphisms. It is not known, however, what is the maximum finite global dimension among them, while some examples of tiled R-orders having large global dimension are studied by some authors. In such examples, neat primitive idempotents play an essential role. Then in [3] a question was posed, that is, "Does any tiled R-order of finite global dimension have a neat primitive idempotent?", which can be considered as an improved version of Jategaonkar's conjecture disproved by Kirkman and Kuzmanovich [5] and [2] for all $n \geq 6$.

In this talk, for an arbitrary prime p, we construct a tiled R-order Λ in $\mathrm{M}_n(K)$ such that global dimension $\mathrm{gld}\Lambda = 5$ if characteristic $\mathrm{char}F \neq p$, and $\mathrm{gld}\Lambda = \infty$ if $\mathrm{char}F = p$, where $n = 4p + 5$. In the case of $\mathrm{char}F \neq p$, Λ is a tiled R-order of finite global dimension which has no neat primitive idempotent, so that the above question is solved.

In order to compute global dimension of Λ, we use the theory of Rump [6]. Namely, we compute projective resolutions in the category of finite dimensional Ω-representations over F, where Ω is the infinite poset of indecomposable Λ-projectives in a fixed simple $\mathrm{M}_n(K)$-module.

References

1. I. Ágoston, V. Dlab, T. Wakamatsu, *Neat algebras*, Comm. Algebra 19 (2) (1991), 433-442.
2. H. Fujita, *Tiled orders of finite global dimension*, Trans. Amer. Math. Soc. 322 (1990), 329-341.
3. H. Fujita, *Neat idempotents and tiled orders having large global dimension*, J. Algebra 256 (2002), 194-210.
4. V.A. Jategaonkar, *Global dimension of tiled orders over a discrete valuation ring*, Trans. Amer. Math. Soc. 196 (1974), 313-330.
5. E.Kirkman, J. Kuzmanovich, *Global dimension of a class of tiled orders*, J. Algebra 127 (1989), 57-72.
6. W. Rump, *Discrete posets, cell complexes, and the global dimension of tiled orders*, Comm. Algebra 24 (1996), 55-107.

Institute of Mathematics
University of Tsukuba
Tsukuba Ibaraki, 305-8571 JAPAN
E-mail: fujita@math.tsukuba.ac.jp

Zero-Divisor Semigroups of Some Simple Graphs [2]

Tang Gaohua[3], Su Huadong and Ren Beishang
Department of Mathematics, Guangxi Teachers' College,
Nanning 530001, P. R. China

The zero-divisor graph of a commutative semigroup with zero is a graph whose vertices are the nonzero zero-divisors of the semigroup, with two distinct vertices joined by an edge in case their product in the semigroup is zero. In this paper, we give formulas to calculate the numbers of non-isomorphic zero-divisor semigroups corresponding to the graph $K_n - e$ (the complete graph K_n deleted an edge), star graphs $K_{1,n}$, two-star graphs $T_{m,n}$ and windmill graphs respectively.

1. Main Results

Theorem 1. *The number of non-isomorphic zero-divisor semigroups corresponding to the graph $K_n - e$(the complete graph K_n deleted an edge) is*

$$H(n) = 4\sum_{i=0}^{n-2} K(i) + 3\sum_{i=0}^{n-4}(n-3-i)K(i) + \frac{1}{2}\sum_{i=0}^{n-5}(n-4-i)(n-3-i)K(i).$$

Where $K(n) = \sum_{k=0}^{n} P(k)$ is the number of non-isomorphic zero-divisor semigroups corresponding to the complete graph K_n and $P(n)$ denotes the number of partitions of n.

Theorem 2. *Let $n \geq 2$. Then the number of non-isomorphic zero-divisor semigroups corresponding to the star graph $K_{1,n}$ is*

$$S(n) = n + 2 + 2f(n-1) + 2f(n).$$

Where $f(n)$ denotes the number of non-isomorphic commutative semigroups with n elements.

Theorem 3. *Let m and n be positive integers. Let $T(m,n)$ denote the number of non-isomorphic zero-divisor semigroups corresponding to the two-star graph $T_{m,n}$. Then*
(1) $T(m,n) = f(m) + f(n)$, if $m \neq n$;
(2) $T(m,n) = f(n)$, if $m = n$.
Where $f(n)$ denotes the number of non-isomorphic commutative semigroups with n elements.

Theorem 4. *Let $n \geq 2$, $m \geq 0$ and $F_{n,m}$ denotes the windmill graphs, a special refinement of the star graph $K_{1,2n+m}$. Then there are $\frac{1}{2}(m+1)(n+1)(n+2)$ non-isomorphic zero-divisor semigroups corresponding to the graph $F_{m,n}$.*

[2]Supported by Guangxi Natural Sciences Foundation(0575052, 0640070), Innovation Project of Guangxi Graduate Education(2006106030701M05) and Scientific Research Foundation of Guangxi Educational Committee

[3]E-mail: huadongsu@sohu.com

Duals of Modules and Quasitriangular Hopf Algebras

Guilong Liu
School of Information Sciences,
Beijing Language and Culture University, Beijing 100083, China
Email address: liuguilong@blcu.edu.cn

Abstract

Let C be a coalgebra and M a right C-comodule then the dual space M^* of M is a right C^*-module. If we begin with a right A-module M, however, difficulties arise. In general, M^* is not a comodule. This paper studies the finite dual comodule.

Let A be an algebra and M be a right A-module. We show that $M^0 = \{f|f \in M^*, f(MI) = 0$, for some ideal I of A such that $dimA/I < \infty\}$ is a right A^0-comodule. We also show that the finite dual comodule of Yetter-Drinfeld module, H-module coalgebra and Hopf module are Yetter-Drinfeld module, H -comodule algebra and Hopf module respectively. Let (H, R) be a quasitriangular Hopf algebra with $R = \sum a_i \otimes b_i \in H \otimes H$. On the set H we define a new coproduct $\Delta' = R\Delta h = \sum a_i h_1 \otimes b_i h_2$. We show that $(H, \Delta', \varepsilon)$ is a coalgebra. If we denote this coalgrbra $(H, \Delta', \varepsilon)$ by H' then we show that the map $u : H' \to H$ by $u(h) = h$ is a convolution invertible. Applying this new coproduct to the Drinfeld double $D(H)$, we show $(D(H))' \cong (M_n(k))^*$ as coalgebra. Here $n = \dim H$, and $M_n(k)$ is the algebra of all $n \times n$ square matrices over field k.

SYMMETRY IN NONCOMMUTATIVE NOETHERIAN RINGS

C. R. Hajarnavis
University of Warwick
Email: crh@maths.warwick.ac.uk

The Jacobson radical of a ring is left-right symmetrical. It is the intersection of maximal right ideals as well as maximal left ideals. A similar symmetry holds for semisimple Artinian rings. This two-sidedness plays a crucial role in the use of these concepts. Here we recall that such a symmetry also exists for invertible ideals and describe applications which are dependent on this property.

Let R be a prime Noetherian ring and Q the quotient ring of R . Define $I^* = \{q \in Q : qI \subseteq R\}$ and $I^+ = \{q \in Q : Iq \subseteq R\}$. Then we say that I is *right invertible* if $II^+ = R$, *left invertible* if $I^*I = R$ and *invertible* if both hold. We have shown [3] that in a Noetherian (semi)prime polynomial identity (PI) ring a maximal ideal is right invertible if and only if it is left invertible. In particular, such an ideal is projective. This result was extended to a general ideal I in [2]. Some applications of this result are as follows [1,2]:

Theorem 1. *(with Chatters and Lissaman): Let R be a Noetherian PI ring. Let M be a non-idempotent maximal ideal of R such that M_R is projective. Then M has the left Artin-Rees property and M contains a right regular element of R .*

Recall that a module M_R is called a *generator* if R_R is a direct summand of $M \oplus ... \oplus M$. (finite direct sum of copies of M). We can show:

Theorem 2. *(with Braun): Let R be a Noetherian PI ring. Let I be a two sided ideal of R which is a right generator. Then I is right projective and satisfies the left Artin-Rees property.*

Thus a generator ideal is a progenerator.

Extending these methods we obtain information on the structure of the ideal class group of the centre of certain rings of finite global dimension. This class includes the enveloping algebra of a finite dimensional Lie algebra over a field of characteristic p .

References

1. A. W. Chatters, C. R. Hajarnavis and R. M. Lissaman, *Projective prime ideals and localisation in PI rings*, J. London Math. Soc. **(2) 64** (2001), 1-12.
2. A. Braun and C. R. Hajarnavis, *Generator ideals in Noetherian PI rings*, J. Algebra **247** (2002), 134-152.
3. C. R. Hajarnavis, *One-sided invertibility and localisation II*, Glasgow J. Math. **39** (1995), 15-19.

GROUP ACTIONS IN A UNIT-REGULAR RING, II

Juncheol Han
Department of Mathematics Education
Pusan National University
Pusan, 609-735, Korea
email jchanpusan.ac.kr

Abstract

Let R be a ring with unity, X the set of all nonzero, nonunits of R and G the group of all units of R. We will consider some group actions on X by G, the left (resp. right) regular action and the conjugate action. In this paper, by investigating these group actions we can have some results as follows: First, if $E(R)$, the set of all nonzero nontrivial idempotents of a unit-regular ring R, is commuting, then $o_\ell(x) = o_r(x)$, $o_c(x) = \{x\}$ for all $x \in X$ where $o_\ell(x)$ (resp. $o_r(x)$, $o_c(x)$) is the orbit of x under the left regular (resp. right regular, conjugate) action on X by G and R is abelian regular. Secondly, if R is a unit-regular ring with unity 1 such that G is a finitely generated abelian group and $2 = 1 + 1 \in G$, then G is a finite abelian group. Finally, if R is an abelian regular ring such that G is an abelian group, then R is a commutative ring.

References

1. K. R. Goodearl, *von Neumann Regualr Rings*, Pitman Publishing Limited, London,1979
2. J. Han, *Regular action in a ring with a finite number of orbits*, Comm. in Algebra 25(7) 1997, 2227 – 2236
3. J. Han, *Group actions in a unit-regular ring*, Comm. in Algebra 27(7) 1999,3353 – 3361
4. J. Han, *The structure of a semiperfect ring*, Journal of Korean Math. Soc., to be appeared 2007

On Hochschild cohomology ring of an order of a quaternion algebra

Takao Hayami

Let R be a commutative ring and Λ an R-algebra which is a finitely generated projective R-module. If M is a Λ-bimodule (i.e., a $\Lambda^{\mathrm{e}} = \Lambda \otimes_R \Lambda^{\mathrm{op}}$-module), then the nth Hochschild cohomology of Λ with coefficients in M is defined by $H^n(\Lambda, M) := \mathrm{Ext}^n_{\Lambda^{\mathrm{e}}}(\Lambda, M)$. We set $HH^n(\Lambda) = H^n(\Lambda, \Lambda)$. The Yoneda product gives $HH^*(\Lambda) := \bigoplus_{n\geq 0} HH^n(\Lambda)$ a graded ring structure with $1 \in Z\Lambda \simeq HH^0(\Lambda)$ where $Z\Lambda$ denotes the center of Λ. $HH^*(\Lambda)$ is called the Hochschild cohomology ring of Λ.

Let G denote the generalized quaternion 2-group of order 2^{r+2} for $r \geq 1$:

$$Q_{2^r} = \langle x, y \mid x^{2^{r+1}} = 1, x^{2^r} = y^2, yxy^{-1} = x^{-1} \rangle.$$

We set $e = (1 - x^{2^r})/2 \in \mathbb{Q}G$ and denote xe by ζ, a primitive 2^{r+1}-th root of e. Then e is a central idempotent of $\mathbb{Q}G$ and $\mathbb{Q}Ge$ is the quaternion algebra over the field $K := \mathbb{Q}(\zeta + \zeta^{-1})$ with identity e, that is, $\mathbb{Q}Ge = K \oplus Ki \oplus Kj \oplus Kij$ where we set $i = x^{2^{r-1}}e$ and $j = ye$ (see [1, (7.40)]). Note that $i^2 = j^2 = -e, ij = -ji$ hold. In the following we set $R = \mathbb{Z}[\zeta + \zeta^{-1}]$, the ring of integers of K, and we set $\Gamma = \mathbb{Z}Ge = R \oplus R\zeta \oplus Rj \oplus R\zeta j$. Note that R is a commuting parameter ring, because y commutes with $x + x^{-1}$. Then Γ is an R-order of $\mathbb{Q}Ge$. In particular if $r = 1$, $\Gamma = \mathbb{Z}e \oplus \mathbb{Z}i \oplus \mathbb{Z}j \oplus \mathbb{Z}ij$ is just the quaternion algebra over $\mathbb{Z}$.

We give an efficient bimodule projective resolution of Γ, and we determine the ring structure of the Hochschild cohomology $HH^*(\Gamma)$ by calculating the Yoneda products using this bimodule projective resolution. We have the following theorem (see [2]):

Theorem. (1) *If $r = 1$, then the Hochschild cohomology ring $HH^*(\Gamma)$ is isomorphic to*

$$\mathbb{Z}[A, B, C]/(2A, 2B, 2C, A^2 + B^2 + C^2),$$

where $\deg A = \deg B = \deg C = 1$.

(2) *If $r \geq 2$, then the Hochschild cohomology ring $HH^*(\Gamma)$ is isomorphic to*

$$R[A, B, C, D]/((\zeta + \zeta^{-1})A, (\zeta + \zeta^{-1})B, (\zeta + \zeta^{-1})C, 2^r D, A^2 + B^2 + C^2, BC - 2^{r-1}\eta D),$$

where $\eta = 2e/(\zeta + \zeta^{-1})$ $(\in R)$, $\deg A = \deg B = \deg C = 1$ *and* $\deg D = 2$.

In the case $r = 1$, this Hochschild cohomology ring is already known by [3, Section 3.4].

References

1. C.W. Curtis and I. Reiner, *Methods of representation theory. Vol. I. With applications to finite groups and orders*, Wiley-Interscience, New York, 1981.
2. T. Hayami, *Hochschild cohomology ring of an order of a simple component of the rational group ring of the generalized quaternion group*, Comm. Algebra (to appear).
3. K. Sanada, *On the Hochschild cohomology of crossed products*, Comm. Algebra **21** (1993), 2727–2748.

Department of Mathematics
Faculty of Science
Tokyo University of Science
Wakamiya-cho 26, Shinjuku-ku, Tokyo 162-0827, JAPAN
Email: hayami@rs.kagu.tus.ac.jp

Auslander-type Conditions

Zhaoyong Huang
Department of Mathematics, Nanjing University, Nanjing 210093, P. R. China
E-mail: huangzy@nju.edu.cn

Let R be a two-sided noetherian ring and mod R the category of finitely generated left R-modules. For a positive integer n, R is called a *n-Gorenstein ring* if the flat dimension of the i-th term in a minimal injective resolution of R_R is at most $i-1$ for any $1 \leq i \leq n$. R is said to satisfy the *Auslander condition* if it is k-Gorenstein for all n. For a module M in mod R (resp mod R^{op}) and a non-negative integer i, we denote grade$M \geq i$ if $\mathrm{Ext}^j_R(M, R) = 0$ for any $0 \leq j < i$, and denote s.grade$M \geq i$ if grade$N \geq i$ for any submodule N of M. It was showed by Auslander in 1975 that R is k-Gorenstein if and only if s.grade$\mathrm{Ext}^i_R(M, R) \geq i$ for any $M \in$mod R and $1 \leq i \leq n$, and in particular, the notion of n-Gorenstein rings (and hence that of the Auslander condition) is left-right symmetric.

Let $n, k \geq 0$. We say that R is $G_n(k)$ if s.grade$\mathrm{Ext}^{i+k}_R(M, R) \geq i$ holds for any $M \in$ mod R and $1 \leq i \leq n$. Then R is $G_n(k)$ if and only if the flat dimension of the i-th term in a minimal injective resolution of R_R is at most $i + k - 1$ for any $1 \leq i \leq n$. Thus $G_n(0)$ (resp. $G_\infty(0)$) is just the n-Gorenstein ring (resp. the Auslander condition). Similarly, we say that R is $g_n(k)$ if grade$\mathrm{Ext}^{i+k}_R(M, R) \geq i$ holds for any $M \in$ mod R and $1 \leq i \leq n$.

Noetherian rings satisfying 'Auslander-type conditions' on self-injective resolutions can be regarded as certain non-commutative analogs of commutative Gorenstein rings. Such conditions, especially dominant dimension and the n-Gorenstein ring, play a crucial role in representation theory and non-commutative algebraic geometry. They are also interesting from the viewpoint of some homological conjectures.

In this talk, I summarize some developments on the study of Auslander-type conditions.

References

1. M. Auslander and I. Reiten, *Syzygy modules for Noetherian rings*, J. Algebra **183**(1996), 167–185.
2. M. Hoshino and K. Nishida, *A generalization of the Auslander formula*, in: Representations of Algebras and Related Topics, Fields Institute Communications **45**, Amer. Math. Soc., Providence, Rhode Island, 2005, pp.175–186.
3. Z.Y. Huang and O. Iyama, *Auslander-type condition and cotorsion theory*, Preprint.
4. Y. Iwanaga and H. Sato, *On Auslander's n-Gorenstein rings*, J. Pure Appl. Algebra **106**(1996), 61–76.

Characterizations of elements in prime radicals of skew polynomial rings and skew Laurent polynomial rings

Chan Huh

Pusan National University

Abstract

We show that the θ-prime radical of a ring R is the set of all strongly θ-nilpotent elements, where θ is an automorphism of R. Also we observe some conditions under which the θ-prime radical of R coincides with the prime radical. Moreover we characterize elements in the prime radical of skew Laurent polynomial ring, by studying ':θ-(semi)primeness.

Applications of Harada rings and Kupisch series for Harada rings

Ken-ichi Iwase

In this talk, we show several results as applications of left H-rings and discuss Kupisch series for left H-rings. Among others, we show the following results:

Theorem 1. If R is a left perfect ring whose factor rings are right QF-3, then R is a Nakayama ring.

Theorem 2. If R is a semiprimary QF-2 ring with ACC or DCC for right annihilator ideals, then R is QF-3.

Theorem 3. If R is a basic indecomposable left H-ring with a simple projective right R-module, then R can be represented as a factor ring of an upper triangular matrix ring over a division ring.

Theorem 4. For a left H-ring R, the following conditions are equivalent:
(1) R is a Nakayama ring.
(2) $R/S_i(R_R)$ is a Nakayama QF-ring for $i = 1, 2$.
(3) $R/S_i(R_R)$ is a Nakayama QF-ring for $i = 0, 1, 2, \cdots$.
(4) R is right Kasch and $R/S(R_R)$ is right QF-2.
(5) $R/S_i(R_R)$ is QF for $i = 1, 2$.
(6) $R/S_i(R_R)$ is QF for $i = 0, 1, 2, \cdots$.

Tennouji senior high school
attached to Osaka kyouiku University
4-88, Minami-kawahori,Tennouji, Osaka , 543-0054 JAPAN
Email: iwase@cc.osaka-kyoiku.ac.jp

Koszul differential graded algebras and BGG correspondence

J.-W. He Department of Mathematics, Shaoxing College of Arts and Science,

Shaoxing 312000, China

Email: hejw@fudan.edu.cn

Institute of Mathematics, Fudan University, Shanghai 200433, China

Email: qswu@fudan.edu.cn

abstract

The concept of Koszul differential graded algebra (Koszul DG algebra) is introduced. Koszul DG algebras exist extensively. It is shown that any finite dimensional local algebra over an algebraically closed field is the Ext-algebra of some Koszul DG algebra, and quasi-Koszul and strongly quasi-Koszul algebras can also be realized by Koszul DG algebras. Koszul DG algebras have nice properties similar to the classic Koszul algebras: The Yoneda algebra of the Ext-algebra of a Koszul DG algebra A is isomorphic to the cohomology algebra $H(A)$; There is an equivalence (and a duality) between some subcategories of the derived category of A and of the derived category of its Ext-algebra. When the Koszul DG algebra A is AS-regular it is proved that the Ext-algebra E of A is Frobenius. In this case, similar to the classical BGG correspondence, there is a correspondence between the stable category of E and some sub quotient category of the derived category of A.

Valuation rings in the quotient ring of a skew polynomial ring

John S. Kauta
Department of Mathematics, Faculty of Science, Universiti Brunei Darussalam, Bandar Seri Begawan BE1410, BRUNEI.
e-mail: kauta@fos.ubd.edu.bn

Let K be a field, and let σ be an automorphism of K of finite order, say n. One can form a skew polynomial ring $K[X, \sigma]$ over K with the usual rules of multiplication defined by the commutation rule: $Xa = \sigma(a)X \ \forall \ a \in K$. Let $K(X, \sigma)$ denote the skew field of quotients of $K[X, \sigma]$. If F is the fixed field of σ and $a \in F \setminus \{0\}$, then one can also form the cyclic algebra $(K/F, \sigma, a)$. The aim of this talk is to describe the connection between the noncommutative valuation theories of $K(X, \sigma)$ and $(K/F, \sigma, a)$. In particular, we will discuss semihereditary orders and Dubrovin valuation rings of the two central simple algebras.

References

1. H. H. Brungs and M. Schröder, *Valuation Rings in Ore Extensions*, J. Algebra **235** (2001), 665-680.
2. G. Xie, S. Kobayashi, H. Marubayashi, N. Popescu, and C. Vraciu, *Noncommutative Valuation Rings of the Quotient Artinian Ring of a Skew Polynomial Ring*, Algebras and Representation Theory **8** (2005), 57-68.

ON NON-δ-M-COSINGULAR COMPLETELY $\oplus$-δ_M-SUPPLEMENTED MODULES

Derya Keskin Tütüncü, Fatma Kaynarca and M. Tamer KOŞAN

Hacettepe University and Afyonkarahisar Kocatepe University

Emails: keskin@hacettepe.edu.tr and fatmakaynarca@hotmail.com and mtkosan@aku.edu.tr

In this note all modules will be unital right R-modules.

A module N in $\sigma[M]$ is called *M-singular* if $N \cong L/K$ for an $L \in \sigma[M]$ and K is essential in L. Let $A \leq N \in \sigma[M]$. A is called a *δ_M-small submodule* of N if whenever $N = A + X$ and N/X is M-singular for $X \leq N$ we have $N = X$ and we denote it by $A \ll_{\delta_M} N$. Note that δ_M-small submodules are the generalization of δ-small submodules in the category of $Mod - R$ defined by Zhou in [3].

Let $A \leq B \leq N \in \sigma[M]$. A is called a *δ_M-coessential submodule* of B in N if $B/A \ll_{\delta_M} N/A$ and we denote it by "$A \subseteq_{ce}^{\delta_M} B$ in N". A submodule A of N is said to be *δ_M-coclosed* in N if it has no proper δ_M-coessential submodule in N and we denote it by "$A \subseteq_{cc}^{\delta_M} N$". Let $A \leq B \leq N \in \sigma[M]$. If $A \subseteq_{ce}^{\delta_M} B$ in N and $A \subseteq_{cc}^{\delta_M} N$, then we say that A is a *δ_M-coclosure* of B in N. By [1], A is a δ_M-coclosure of B in N if and only if A is a minimal δ_M-coessential submodule of B in N (because the class of M-singular modules is closed under homomorphic images).

Inspired by the definitions and characterizations of the class $\overline{Z}_M(N)$ given in [2], one may define the submodule $\overline{Z}_{\delta_M}(N) = \bigcap\{Kerg \mid g : N \longrightarrow T, T \in \mathcal{DM}\}$ where $\mathcal{DM}$ is the class of δ-M-small modules. Any module $N \in \sigma[M]$ is called a *δ-M-cosingular* (*non-δ-M-cosingular*)*module* if $\overline{Z}_{\delta_M}(N) = 0$ ($\overline{Z}_{\delta_M}(N) = N$).

Let $N \in \sigma[M]$ and $X \leq N$. A *δ_M-supplement* of X in N is a submodule K of N with $N = K + X$ and $X \cap K \ll_{\delta_M} K$. If every submodule of N has a direct summand δ_M-supplement in N, then N is called *$\oplus$-δ_M-supplemented*. We call any module N in $\sigma[M]$ *completely $\oplus$-δ_M-supplemented* if every direct summand of N is $\oplus$-δ_M-supplemented.

Some of our main results can be stated as follows:

THEOREM 1: Let $N \in \sigma[M]$ be a non-δ-M-cosingular $\oplus$-δ_M-supplemented module. Then N is (D_3) if and only if N has the summand intersection property.

THEOREM 2: Let $N \in \sigma[M]$ such that $\overline{Z}_{\delta_M}(N)$ has a coclosure (or δ_M-coclosure) in N. Then N is (completely) $\oplus$-δ_M-supplemented if and only if $N = \overline{Z}^2_{\delta_M}(N) \oplus K$ for some submodule K of N such that K and $\overline{Z}^2_{\delta_M}(N)$ are (completely) $\oplus$-δ_M-supplemented.

THEOREM 3: Let $N \in \sigma[M]$. Then N is lifting if and only if $N = \overline{Z}^2_{\delta_M}(N) \oplus K$ for some submodule K of N such that K and $\overline{Z}^2_{\delta_M}(N)$ are lifting and K and $\overline{Z}^2_{\delta_M}(N)$ are relatively projective.

References

1. D.Keskin, *On lifting Modules*, Comm Alg. **28(7)** (2000) 3427-3440.
2. Y. Talebi and N. Vanaja, *The Torsion Theory Cogenerated by M-small Modules*, Comm. Alg.**30(3)** (2002) 1449-1460.
3. Y. Zhou, *Generalizations of Perfect, Semiperfect and Semiregular Rings*, Algebra Colloquim **7(3)** (2000) 305-318.

ON THE RELATIVE (QUASI-) DISCRETENESS OF MODULES

Derya Keskin Tütüncü and Nil Orhan Ertaş

Hacettepe University and Süleyman Demirel University

Emails: keskin@hacettepe.edu.tr and orhannil@yahoo.com

In recent years, several authors (c.f. [1]-[5]) have studied discrete modules and other related concepts as interesting generalizations of the concept of projectivity and lifting property of modules. Although relative projectivity has been studied in detail, it appears that not much work has yet been done to study the discreteness of an arbitrary module M with respect to another module N, namely the N-discreteness of M. In [3], Oshiro generalized the concepts of lifting, quasi-discrete and discrete modules only for submodules taken from a given family u . This was done by considering the properties

$\mathsf{u}-(D_1)$ For all $A \in \mathsf{u}$ there exists a direct summand N of M such that $A/N \ll M/N$.

$\mathsf{u}-(D_2)$ For any $A \in \mathsf{u}$ with $A \leq_d M$, any sequence $M \longrightarrow M/A \longrightarrow 0$ splits.

$\mathsf{u}-(D_3)$ For any $A \in \mathsf{u}$ and $B \leq M$ with $A, B \leq_d M$ and $M = A+B$, $A \cap B \leq_d M$.

Throughout this paper u will be a family of submodules of any unital right R-module M which is closed under coessential submodules (namely, if $A \in \mathsf{u}$ and $A/B \ll M/B$, then $B \in \mathsf{u}$) and (α):

(α) For $A \in \mathsf{u}$ and $B \leq M$, $M/A \cong M/B$ implies that $B \in \mathsf{u}$.

Oshiro called the module M u -*lifting*, u -*quasi-semiperfect* or u -*semiperfect*, if M satisfies $\mathsf{u}-(D_1)$, $\mathsf{u}-(D_1)$ and $\mathsf{u}-(D_3)$, or $\mathsf{u}-(D_1)$ and $\mathsf{u}-(D_2)$, respectively. In this paper we use the terms of *quasi-discrete* and *discrete* instead of quasi-semiperfect and semiperfect, respectively. When u consists of all submodules of M then M is u -lifting, u -quasi-discrete or u -discrete, respectively, iff M is lifting, quasi-discrete or discrete.

Let M and N be modules. Consider the class

$\mathcal{B}(M,N) = \{A \leq M \mid \exists X \leq N, \exists f \in Hom(M,N/X), Kerf/A \ll M/A\}$. M is said to be N-*lifting*, N-*quasi-discrete* or N-*discrete* when M is u -lifting, u -quasi-discrete or u -discrete for $\mathsf{u} = \mathcal{B}(M,N)$, respectively. In this paper we prove the following:

Theorem 1. *Let* $0 \longrightarrow N' \xrightarrow{f} N \xrightarrow{g} N'' \longrightarrow 0$ *be an exact sequence. If* M *is* N*-lifting((quasi)-discrete), then it is both* N'*- and* N''*-lifting((quasi)-discrete).*

Theorem 2. *Let* $0 \longrightarrow N' \xrightarrow{f} N \xrightarrow{g} N'' \longrightarrow 0$ *be an exact sequence and let* M *be an* N*-amply supplemented module. Assume that* $\mathcal{B}(M/T,N)$ *is closed under supplement submodules for every factor module* M/T *of* M*. Then* M *is* N*-lifting(quasi-discrete) iff it is both* N'*-lifting(quasi-discrete) and* N''*-lifting(quasi-discrete).*

References

1. L.Ganesan and N.Vanaja, *Strongly Discrete Modules*, Comm. in Algebra **35** (2007) 897-913.
2. D.Keskin and A.Harmanci, *A Relative Version of the Lifting Property of Modules*, Algebra Colloquium **11(3)** (2004) 361-370.
3. K.Oshiro, *Semiperfect Modules and Quasi-semiperfect Modules*, Osaka J. Math. **20** (1983) 337-372.
4. R.Wisbauer, *Foundations of Module and Ring Theory*, Gordon and Breach, Philadelphia (1991).
5. S.H.Mohamed and B.J.Müller, *Continuous and Discrete Modules*, London Math. Soc. LNS 147 Cambridge Univ. Press, Cambridge (1990).

ON IRREDUCIBLE FACTORS OF THE POLYNOMIAL $\mathbf{f(x) - g(y)}$

Sudesh K. Khanduja
Department of Mathematics,
Panjab University
Chandigarh-160014, India.
E-mail: skhand@pu.ac.in

Abstract. A well known result of Ehrenfeucht states that a polynomial $f_1(x_1)+...+f_n(x_n)$ with complex coefficients is irreducible over the field of complex numbers provided the degrees of $f_1(x_1),...,f_n(x_n)$ have the greatest common divisor one. In 1964, Tverberg extended this result by showing that when K is a field of characteristic zero, then each polynomial $f_1(x_1)+...+f_n(x_n)$ over K in $n \geq 3$ variables is irreducible. He also proved that a polynomial $f_1(x_1)+f_2(x_2)$ with degrees of f_1 and f_2 coprime, is always irreducible over a field of arbitrary characteristic. In 2001, Bhatia and Khanduja generalized the above result by proving that if r is the g.c.d. of the degrees of the polynomials $f(x)$ and $g(y)$ in the variables x and y having leading coefficients c and d, then $f(x) - g(y)$ is irreducible over a field K provided $x^r - \frac{c}{d}$ is K-irreducible. Indeed the above result was proved for Generalized Difference polynomials. In this lecture, all irreducible factors of polynomials of the form $f(x) - g(y)$ over an arbitrary field are described. It will also be proved that the number of irreducible factors of $f(x) - g(y)$ over a field K (counting multiplicities) does not exceed the greatest common divisor of the degrees of $f(x), g(y)$. It will be deduced that if $f(x)$ and $g(y)$ are non-constant polynomials with coefficients in the field $\mathbb{Q}$ of rational numbers and deg$f(x)$ is a prime number, then $f(x) - g(y)$ is a product of at most two irreducible polynomials over $\mathbb{Q}$. This contributes to a problem raised by J. W. S. Cassels which asks for what polynomials f, is the polynomial $\frac{f(x)-f(y)}{x-y}$ reducible. These results have jointly been proved with Prof. A. J. Engler, State University of Campinas, Brazil.

The generalized finite intersection property

Hong Kee Kim

Gyeongsang National University

Abstract

A generalization of Baer ring and Noetherian ring is the generalized finite intersection property that is defined by the following; A ring R is said to have the generalized left finite intersection property if there exists a finite subset X' of R such that the left annihilator of any subset X of R is the left annihilator of X'. In this talking, there exists a ring having the generalized left finite intersection property but neither Baer ring nor Noetherian ring. Also, we find some results about a ring having the generalized left finite intersection property.

On quasi continuous rings

Jin Yong Kim*
Department of Mathematics and Institute of Natural Sciences
Kyung Hee University
Suwon 449–701, Korea
E-mail: jykim@khu.ac.kr
Jae Kyung Doh and Jae Keol Park[a]
Department of Mathematics
Busan National University
Busan 609–735, Korea
E-mail[a]: jkpark@pusan.ac.kr

In this paper, two-sided quasi-continuous rings or left CS rings with certain chain conditions are QF. For example, it is shown that two-sided quasi-continuous left perfect rings with DCC on left annihilators are QF. Also it is proved that left CS left GP-injective rings with ACC on left annihilators are QF.

Morita duality and recent development

Kazutoshi Koike

About 50 years ago, Morita and Azumaya published epoch-making papers [6] and [1] independently, which are origins of the theory of Morita duality. Morita [6] studied equivalences (covariant category equivalences) between the full categories of modules over two rings and dualities (contravariant category equivalence) between resonably large subcategories of right and left modules over two rings. Azumaya [1] studied dualities between the category of finitely generated right and left modules over two artinian rings, as a generalization of dualities induced by quasi-Frobenius rings. Now the equivalence and duality of Morita are called Morita equivalence and Morita duality, respectively, and play very important roles of the ring theory. The duality of Azumaya is interpretated as an essential case of Morita duality.

Following these works, many ring-theorists had investigated Morita duality. Particularly, they had been interested in (Morita) self-duality. There are few classes of rings with self-duality. Finite dimensional algebras over fields, quasi-Frobenius rings and (artinian) serial rings are such classes of rings. Azumaya [2] conjectured that every artinian exact ring has a self-duality. The class of artinian exact rings contains serial rings and commutative rings, which are known as rings with self-duality. However the conjecture still remains open.

In this talk, we shall present fundamental results about Morita duality (self-duality) and recent development including the authors results of [3, 4, 5] for self-duality of Harada rings (certain QF-3 artinian rings, which can be regarded as a generalization of quiasi-Frobenius rings and serial rings) and Azumaya's conjecture.

References

1. G. Azumaya. A duality theory for injective modules. *Amer. J. Math.*, 81:249–278, 1959.
2. G. Azumaya. Exact and serial rings. *J. Algebra*, 85(2):477–489, 1983.
3. K. Koike. Examples of QF rings without Nakayama automorphism and H-rings without self-duality. *J. Algebra*, 241(2):731–744, 2001.
4. K. Koike. Almost self-duality and Harada rings. *J. Algebra*, 254(2):336–361, 2002.
5. K. Koike. Self-duality of quasi-Harada rings and locally distributive rings. *J. Algebra*, 302(2):613–645, 2006.
6. K. Morita. Duality for modules and its applications to the theory of rings with minimum condition. *Sci. Rep. Tokyo Kyoiku Daigaku Sect. A*, 6:83–142, 1958.

Okinawa National College of Technology
905 Henoko, Nago City
Okinawa 905-2192, JAPAN
Email: koike@okinawa-ct.ac.jp

Representation theory from a certain point of view

Shigeo Koshitani (Chiba University)

Abstract

We will be discussing representation theory, first of all, of finite dimensional algebras and also of finite groups over a sufficiently large field. We will be trying to explain what is going on in the area in terms of several conjectures and so on.

Reference

[1] Charles W. Curtis, Pioneers of Representation Theory: Frobenius, Burnside, Schur, and Brauer, Amer. Math. Soc., London Math. Soc., History of Mathematics, Vol.15, 1999.

Compartible algebra structures of Lie algebras

Fujio Kubo

Hiroshima University

Email: remakubo@amath.hiroshima-u.ac.jp

1. Introduction

The compartible algebra products $*$ of a finite-dimensional semisimple Lie algebra $\mathfrak{g}$ with a Lie bracket $[-,-]$ is defined to be satisfying the Leibniz law:

$$[x, y * z] = [x, y] * z + y * [x, z].$$

If $\mathfrak{g}$ is simple of all the types except A_n of $n \geq 2$, then the compartible algebra products must be the scalar multiples of the Lie bracket $[-,-]$. In case that $\mathfrak{g}$ is simple of type A_n of $n \geq 2$, such a product is a sum of a scalar multiple of $[-,-]$ and a deformed one of the ordinal associative products on the full $(n+1) \times (n+1)$ matix algebra. Then we give an alternative proof to the triviality of the compartible associative algebra structures of a semisimple Lie algebra $\mathfrak{g}$ which was shown in Kubo [3].

2. Explicit forms

We can consider such a product as an element of $\mathrm{Hom}_{\mathfrak{g}}(\mathfrak{g} \otimes \mathfrak{g}, \mathfrak{g})$, and give the explicit form of it.

Theorem 1. *Let g be a simple Lie algebra over k.*
(1) *If g is not of type A_n of $n \geq 2$ then* $\mathrm{Hom}_g(g \otimes g, g) = kL$.
(2) *If g is of type A_n of $n \geq 2$ then* $\mathrm{Hom}_g(g \otimes g, g) = kL \oplus kD$.
Here L and D are of the forms:

$$L(x \otimes y) := [x, y], \quad D(x \otimes y) = xy - \frac{1}{n+1}\mathrm{Tr}(xy)E.$$

We will give an elementary proof based on the basic knowldge found in the text books [1, 2].

As for the compartible *associative* algebra structures, we have the following interesting result.

Corollary 2 (Kubo[3])**.** *Let $\mathfrak{g}$ be a semisimple Lie algebra with a Lie bracket $[-,-]$. If $\phi \in \mathrm{Hom}_{\mathfrak{g}}(\mathfrak{g} \otimes \mathfrak{g}, \mathfrak{g})$ satisfies the associative law: $\phi(\phi(x \otimes y) \otimes z) = \phi(x \otimes \phi(y \otimes z))$ for $x, y, z \in \mathfrak{g}$, then $\phi = 0$.*

References

1. J.E. Humphreys, *Introduction to Lie algebras and Representation Theory*, Springer Verlag, New York, 1972.
2. N. Jacobson, *Lie Algebras*, Interscience, New York, 1962.
3. F. Kubo, *Finire-dimensional non-commutative Poisson algebras*, J. Pure Appl. Algebra **113** (1996), 307–314.

On Broué's abelian defect group conjecture in representation theory of finite groups

Naoko Kunugi

Tokyo University of Science

In modular representation theory of finite groups, it is important to consider relations between representations of a given finite group and representations of its p-local subgroups. In fact, there are several conjectures based on such a concept. I will talk about, in particular, Broué's abelian defect group conjecture.

Let G be a finite group. Let p be a prime and $(K, \mathcal{O}, k)$ a p-modular system, that is, $\mathcal{O}$ is a complete discrete valuation ring, K is a quotient field of $\mathcal{O}$ of characteristic 0 and k is a residue field of $\mathcal{O}$ of characteristic p.

For $R \in \{\mathcal{O}, k\}$, we have an indecomposable decompositon

$$RG = A_0 \oplus \cdots \oplus A_n$$

of RG as algebras. We call each A_i a block of RG. For an indecomplsable (right) RG-module U, there is a unique block A such that $U \cdot A \neq 0$. In this case, we say that U belongs to A. We call a block to which the trivial RG-module belongs the principal block of RG.

Let A be a block of RG. A defect group of A is a minimal p-subgroup D of G such that the multiplication map $A \otimes_{RD} A \to A$ splits as an (A, A)-bimodule homomorphism.

The following theorem is very important.

Theorem 1 (Brauer's first main theorem)**.** *Let D be a p-subgroup of G. Then there is a one to one correspondence between the set of blocks of RG with defect group D and the set of blocks of $RN_G(D)$ with defect group D.*

The theorem does not say anything about relations between representations of corresponding blocks. On the other hand, Broué conjectures the following.

Abelian Defect Group Conjecture (Broué [1, 2]). Let A be a block of RG with defect group D and B the Brauer correspondent block in $N_G(D)$ with defect group D. If D is abelian, then two blocks A and B would be derived equivalent.

References

1. M. Broué, Isométries parfaites, types de blocs, catégories dérivées, *Astérisque* **181-182** (1990), 61–92.
2. M. Broué, Equivalences of blocks of group algebras, in Finite Dimensional Algebras and Related Topics, (edited by V. Dlab and L.L. Scott) Kluwer Acad. Pub., Dordrecht, 1994, pp.1–26.

Extended Armendariz rings and rigid rings

Tai Keun Kwak
Department of Mathematics
Daejin University
Pocheon 487–711, Korea
Email: tkkwak@daejin.ac.kr

A ring R is called Armendariz if whenever the product of any two polynomials in $R[x]$ over R is zero, then so is the product of any pair of coefficients from the two polynomials. Such rings have been extensively studied in literature. For a ring endomorphism α, we introduce the notions of α-Armendariz rings and α-skew Armendariz rings which are generalizations of α-rigid rings and extensions of Armendariz rings, by considering the polynomials in the skew polynomial ring $R[x;\alpha]$ in place of the ring $R[x]$. We investigate their properties and study on the relationship between the Baerness and p.p.-property of a ring R and these of the skew polynomial ring $R[x;\alpha]$ in case R is extended Armendariz. In particular, we show that there is a strong connection among extended Armendariz rings and rigid rings. Several known results follow as consequences of our results.

On strongly NI rings

Chang Ik Lee, Yang Lee and Sung Ju Ryu

Department of Mathematics
Pusan National University

Email: cilee@pusan.ac.kr, ylee@pusan.ac.kr and sjryu@pusan.ac.kr

Kim and Lee [5] called a ring strongly 2-primal if every factor rings of itself are 2-primal. The study of strongly 2-primal rings was initiated by Birkenmeier at el. [1]. Commutative rings are more similar to such rings than to 2-primal rings.
In this talk, we define the concept of strongly NI rings which is a generalization of both strongly 2-primal rings and NI rings. Thus, it is related to NIness, 2-primalness and strong 2-primalness. Using some examples, each of them is not equivalent to strong NIness.

First, we show that a ring is strongly NI if and only if every strongly prime ideals are completely prime.
Next we also study the basic structure of strongly NI rings. We see whether new rings of old ones preserve strong NIness. Moreover we see some examples and counter examples suitable for questions raised naturally.
Finally, we also see some classes of rings under which NI rings and strongly NI rings coincide.

References

1. BHL G.F. Birkenmeier, H.E. Heatherly, E.K. Lee, *Completely prime ideals and associated radicals*, in: S.K. Jain, S.T. Rizvi (Eds.), Proceedings of the Biennial Ohio State-Denison Conference, 1992, World Scientific, Singapore, 1993, 102–129.
2. BKP G.F. Birkenmeier, J.Y. Kim, J.K. Park, *Regularity conditions and the simplicity of prime factor rings*, J. Pure Appl. Algebra 115 (1997), 213-230.
3. HK C.Y. Hong, T.K. Kwak, *On minimal strongly prime ideals*, Comm. Algebra 28 (2000), 4867–4878.
4. HJL S.U. Hwang, Y.C. Jeon, Y. Lee, *Structure and topological conditions of NI rings*, J. Algebra 302 (2006), 186–199.
5. KL N.K. Kim, Y. Lee, *On rings whose prime ideals are completely prime*, J. Pure Appl. Algebra 170 (2002), 255–265.
6. LHK Y. Lee, C. Huh, H.K. Kim, *Questions on 2-primal rings*, Comm. Algebra 26 (1998), 595–600.
7. Ma G. Marks, *On 2-primal Ore extensions*, Comm. Algebra 29 (2001), 2113–2123.
8. Ro L.H. Rowen, *Ring Theory*, Academic Press, Inc., San Diego, 1991.
9. Sm A. Smoktunowicz, *Polynomial rings over nil rings need not be nil*, J. Algebra 233 (2000), 427–436.

On the Behrens radical of matrix rings and polynomial rings

P.-H. Lee and E. R. Puczyłowski

keywords: Brown-McCoy radical,Behrens radical, polynomial ring

It is shown that the Behrens radical of a polynomial ring, in either commuting or non-commuting indeterminates, has the form of "polynomials over an ideal" and, in the case of non-commuting indeterminates, for a given coefficient ring, the ideal does not depend on the cardinality of sets of indeterminates. However, in contrast to the Brown-McCoy radical, it is not true that if the polynomial ring in an infinite set of commuting indeterminates over a ring R is Behrens radical, then the polynomial ring in an infinite set of non-commuting indeterminates over R is Behrens radical. This is connected with the fact that matrix rings over Behrens radical rings need not be Behrens radical. The class of Behrens radical rings, which is closed under taking matrix rings, is described.

ORE EXTENSIONS WHICH ARE GPI-RINGS

Chen-Lian Chuang and Tsiu-Kwen Lee

Department of Mathematics, National Taiwan University
Taipei 106, Taiwan
chuang@math.ntu.edu.tw and tklee@math.ntu.edu.tw

Abstract. Let R be a prime ring and δ a σ-derivation of R, where σ is an automorphism of R. It is proved that the skew polynomial ring $R[t;\sigma,\delta]$ is a GPI-ring (PI-ring resp.) if and only if R is a GPI-ring (PI-ring resp.), δ is quasi-algebraic, and σ is quasi-inner. If $R[t;\sigma,\delta]$ is a GPI-ring then $\mathrm{soc}(Q[t;\sigma,\delta]\tilde{C}) = \Big(\mathrm{soc}(Q)[t;\sigma,\delta]\Big)\tilde{C}$, where Q is the symmetric Martindale quotient ring of R and where $\tilde{C}$ denotes the extended centroid of $Q[t;\sigma,\delta]$. If $R[t;\sigma,\delta]$ is a PI-ring, its PI-degree is determined as follows:

(1) $\text{PI-deg}(R[t;\sigma,\delta]) = \text{PI-deg}(R) \times \text{Out-deg}(\delta)$ if δ is X-outer, and
(2) $\text{PI-deg}(R[t;\sigma,\delta]) = \text{PI-deg}(R) \times \text{Out-deg}(\sigma)$ if δ is X-inner.

† *2000 Mathematics Subject Classification. 16W20, 16W25, 16A05, 16R50.*
‡ *Key words and phrases. Prime ring, PI-degree, skew polynomial ring, GPI, quasi-algebraic, quasi-inner.*
Members of Mathematics Division, NCTS (Taipei Office).
Corresponding author: Tsiu-Kwen Lee.

On Property (A)

Chan Yong Hong
Department of Mathematics and Research Institute for Basic Sciences
Kyung Hee University, Seoul 131–701, Korea
Nam Kyun Kim
College of Liberal Arts, Hanbat National University
Daejeon 305-719, Korea
Yang Lee, Sung Ju Ryu, and Dong Eun Yeo
Department of Mathematics, Pusan National University
Pusan 609-735,Korea

E-mail addresses: hcy@khu.ac.kr (C.Y. Hong), nkkim@hanbat.ac.kr (N.K. Kim), ylee@pusan.ac.kr (Y. Lee), sung1530@dreamwiz.com (S.J. Ryu), yde1229@nate.com (D.E. Yeo)

One of important properties for commutative Noetherian rings is that the annihilator of an ideal I consists entirely of zero-divisors is nonzero [6, Theorem 82]. However this result fails for some non-Noetherian rings, even if the ideal I is finitely generated.

Huckaba and Keller [5] introduced the following: a commutative ring R has *Property* (A) if every finitely generated ideal of R consisting entirely of zero-divisors has a nonzero annihilator.

Property (A) was originally studied by Quentel [9]. Quentel used the term Condition (C) for Property (A). The class of rings with Property (A) contains many important kinds of commutative rings, e.g., Noetherian rings, rings whose prime ideals are maximal, the polynomial rings and rings whose classical ring of quotients are von Neumann regular. Using Property (A), Hinkle and Huckaba [3] extend the concept of Kronecker function rings from integral domains to rings with zero-divisors. Many useful results for commutative rings with Property (A) can be found in [1, 2, 4, 5, 7, 8, 9, etc.], and they contain several results which are useful studying commutative rings with zero-divisors.

Property (A) is closely connected with the following annihilator condition. Lucas [7] said that a commutative ring R has the *annihilator condition* (briefly, *a.c.*) provided that given a finitely generated ideal I of R, there exists an element $b \in R$ such that the annihilator of I is equal to the annihilator of b. Property (a.c.) was originally introduced by Henriksen and Jerison [2] on reduced rings. Noet that polynomial rings over reduced rings, Bezout rings (finitely generated ideals are principal), and many other important kinds of commutative rings have Property (a.c.). Property (A) and (a.c.) are equivalent conditions on a reduced ring whose space of minimal prime ideals is compact. However, these two conditions are not equivalent in general [7]. Recently, Lucas [8] also studied the zero-divisor graph of rings with Property (A).

We continue the study of rings with Property (A) on noncommutative rings. We first extend the concept of Property (A) to noncommutative rings, and observe structures of that and related rings.

We will say that a ring (possibly noncommutative) R has *right* (*left*) *Property* (A) if for every finitely generated ideal I consists entirely of left (right) zero-divisors, there exists nonzero $a \in R$ ($b \in R$) such that $Ia = 0$ ($bI = 0$). A ring R is called to have *Property* (A) if R has right and left Property (A).

It is shown that biregular rings have Property (A).

We next study several extensions of rings with Property (A) including matrix rings, polynomial rings, and classical quotient rings. It is proved that the Property (A) can go up to matrix rings. Adding to those, we also study when the space of minimal prime ideals of rings with Property (A) is compact.

Furthermore we will observe the Property (A) in other kinds of extensions and related concepts with some conditions (if necessary).

References

1. F. Azarpanah, O.A.S. Karamzadeh, A. Rezai Aliabad, On ideals consisting entirely of zero divisors , Comm. Algebra vol. 28 , 2000 , pp. 1061–1073
2. M. Henriksen, M. Jerison , The space of minimal prime ideals of a commutative ring , Trans. Amer. Math. Soc. , 1965vol. 115, pp. 110–130
3. G. Hinkle and J.A. Huckaba , The generalized Kronecker function ring and the ring $R(X)$, *J. reine angew. Math.* , 1977 vol. 292 , pp. 25–36
4. J.A. Huckaba , Commutative rings with zero divisors , Marcel-Dekker Inc., New York, 1988
5. J.A. Huckaba, J.M. Keller , Annihilation of ideals in commutative rings , Pacific J. Math. vol. 83 , 1979 , pp. 375–379
6. I. Kaplansky, Commutative rings , Allyn and Bacon, Boston, 1970
7. T.G. Lucas, Two annihilator conditions: property (A) and (a.c.) , Comm. Algebra vol. 14 , 1986 , pp. 557–580
8. T.G. Lucas,The diameter of a zero divisor graph , J. Algebra, vol. 301 , 2006 , pp. 174–193
9. Y. Quentel , Sur la compacité du spectre minimal d'un anneau , Bull. Soc. Math. France , 1971 vol. 99 , pp. 265–272

Weak Global Dimension of Coherent Rings

Lixin Mao
Department of Mathematics, Nanjing University, Nanjing 210093, China
Institute of Mathematics, Nanjing Institute of Technology, Nanjing 211167, China
E-mail: maolx2 hotmail.com

Nanqing Ding
Department of Mathematics, Nanjing University, Nanjing 210093, China
E-mail: nqding@nju.edu.cn

Abstract

We study the weak global dimension of coherent rings in terms of the left FP-injective resolutions of modules. Let R be a left coherent ring and $\mathcal{FI}$ the class of all FP-injective left R-modules. It is shown that $wD(R) \leq n$ ($n \geq 1$) if and only if every nth $\mathcal{FI}$-syzygy of a left R-module is FP-injective; and $wD(R) \leq n$ ($n \geq 2$) if and only if every $(n-2)$th $\mathcal{FI}$-syzygy in a minimal $\mathcal{FI}$-resolution of a left R-module has an FP-injective cover with the unique mapping property. Some results for the weak global dimension of commutative coherent rings are also given.

Key Words: weak global dimension; FP-injective dimension; syzygy; (pre)cover.
2000 Mathematics Subject Classification: 16E10; 16E05; 16D50.

On τ-coherent rings

Kanzo Masaike

Let R be a ring with identity and τ the Lambek torsion theory, which is cogenerated by the injective hull of the left (or right) R-module R. In the following we shall say that R is left τ-*coherent*, if every finitely generated left ideal is τ-finitely presented. An R-homomorphism $f : M \to N$ is called τ-*epimorphism*, if *coker* f is τ-*torsion*.

On the other hand, R is said to be a *left (resp. right)* TF *ring*, if every finitely generated τ-torsion free left (resp. right) R-model is embedded into a free module. Furthemore, we shall say that a submodule N of a left R-module M is *rationally closed*, if M/N is τ-torsion free. We give a necessary and sufficient condition of left TF ring R for which R becomes a right TF ring.

Theorem 1. *Let* Q *be a maximal left quotient ring of* R. *Then* R *is a left* TF *ring, if and only if* Q *is a left* TF *ring and for every* $q \in Q$, $q^{-1}R$ *is a* τ-*finitely generated dense right ideal of* R, *where* $q^{-1}R = \{r \in R | qr \in Q\}$.

Theorem 2. *Let R be a left TF ring. Then, the following conditions are equivalent.*

(1) *R is a right TF ring.*

(2) *R is left* τ *-coherent, and for every inverse system of* τ-*epimorphism* $\{f_\lambda : R \to M_\lambda\}_{\lambda \in \Lambda}$ *with each* M_λ *a torsionless left R-module,* $\varprojlim f_\lambda$ *is a* τ *-epimorphism* .

Proposition 3. *Let* Q *be a maximal left quotient ring of* R *such that every finitely generated left* Q-*submodule of* $\oplus_{i=1}^{n} Q$, *a finite direct sum of copies of the left* Q-*module* Q, *is a rationally closed. Then, the following conditions are equivalent.*

(1) *R is left* τ *-coherent.*

(2) *Q is flat as a right R-module.*

(3) *Q is* Π*-flat as a right R-module.*

Department of Mathematics
Tokyo Gakugei University
Koganei, Tokyo, JAPAN
Email: masaike@u-gakugei.ac.jp

Totally Projective Modules And The Extentions Of Bounded $QTAG$-Modules

A. Mehdi
Department of Mathematics
Aligarh Muslim University, Aligarh-202 002, INDIA
email: alveera_mehdi@rediffmail.com

Abstract:A $QTAG$-module M is a unital module over an associative ring with unity such that every finitely generated submodule of any homomorphic image of M is a direct sum of uniserial modules.For a uniform element $x \in M$, height of x is $\sup\{d(U/xR)\}$ where U runs through all the uniserial modules containing x. $H_k(M)$ is the submodule of M generated by the elements of height atleast k and $M^1 = \bigcap_{k=0}^{\infty} H_k(M)$ or $H_\omega M$. M is k-bounded if $H(x) \leq k$ for all $x \in M$. A submodule N of M is h-pure in M if $H_k(N) = H_k(M) \cap N$ for all $k \in Z^+$ and it is nice if for all ordinals σ, $H_\sigma(M/N) = (H_\sigma(M) + N)/N$. M is σ-projective if $H_\sigma(\mathrm{Ext}(M, C) = 0$ for all $QTAG$-modules C and it is totally projective if it is σ-projective for all ordinals σ. For a submodule $N \subset M$ and ordinal α, α^{th} Ulm-Kaplansky invariant of M with respect to M is defined as $f_\alpha(M, N)$ which is the cardinality of the minimal generating set of $(\mathrm{Soc}(H_\alpha(M))/(\mathrm{Soc}(H_\alpha(M)) \cap (\mathrm{Soc}(H_{\alpha+1}(M) + M)))$. If $N = 0$, $f_\alpha(M, N) = f_M(\alpha)$.

In this paper we study totally projective $QTAG$-modules and the extensions of bounded $QTAG$-modules.In the first section we study totally projective modules M/N and M'/N' where N,N' are isomorphic nice submodules of$M$ andM' respectively.In fact the height preserving isomorphism between nice submodules is extented to the isomorphism from$M$ ontoM' with the help of Ulm-Kaplansky invariants.In the second section extensions of the bounded $QTAG$-modules are studied.Here the invariants are automorphism classes of bounded submodules of the extending module together with the cardinality of the minimal generating set of maximal summand of the extension module. The equivalence of epimorphisms is the main tool in this study.

An Introduction to Noncommutative Algebraic Geometry

Izuru Mori

Shizuoka University

Email: simouri@ipc.shizuoka.ac.jp

Since classification of low dimensional projective schemes has been active and successful in algebraic geometry for many years, one of the major projects in noncommutative algebraic geometry is to classify low dimensional noncommutative projective schemes defined by Artin and Zhang [4] (1994). In this talk, we will survey this project. Classification of noncommutative projective curves were completed by Artin and Stafford [2] (1995). For classification of noncommutative projective surfaces, we have the following conjecture due to Artin [1] (1997); every noncommutative projective surface is birationally equivalent to either (1) a quantum projective plane, (2) a quantum ruled surface, or (3) a surface finite over its center. Classification of quantum projective planes were completed by Artin, Tate and Van den Bergh [3] (1990), however, classification of other types of surfaces together with the above conjecture are still open.

References

1. M. Artin, *Some Problems on Three-dimensional Graded Domains*, Representation theory and algebraic geometry, London Math. Soc. Lecture Note Ser. **238** Cambridge Univ. Press (1997), 1-19.
2. M. Artin and J.T. Stafford, *Noncommutatuve Graded Domains with Quadratic Growth*, Invent. Math. **122** (1995), 231-276.
3. M. Artin, J. Tate and M. Van den Bergh, *Some Algebras Associated to Automorphisms of Elliptic Curves*, in The Grothendieck Festschrift Vol. **1** Birkhauser, Basel (1990), 33-85.
4. M. Artin and J.J. Zhang, *Noncommutative Projective Schemes*, Adv. Math. **109** (1994), 228-287.
5. J.T. Stafford and M. Van den Bergh, *Noncommutative Curves and Noncommutative Surfaces*, Bull. Amer. Math. Soc. **38** (2001), 171-216.

Some congruences concerning finite groups

Kaoru Motose

In this talk, we present a lemma about orders of normal subgroups in a transitive group of prime degree. This lemma has an application to prove simplicity of the alternative group A_5 of degree 5, and 4-transitive Mathieu groups $M_{11}, M_{12}, M_{23}, M_{24}$. Please use this lemma for your lecture to your students about group theory or Galois theory.

In the remaining time, if it is possible, I would like to propagandize Feit-Thompson conjecture because it is not so popular, to mathematician, even to finite group theorists and number theorists.

References

1. Apostol, T. M., *The Resultant of the Cyclotomic Polynomials $F_m(ax)$ and $F_n(bx)$* , Math. Comput. **29** (1975), 1-6.
2. Feit, W. and Thompson, J. G., *A Solvability Criterion for Finite Groups and Some Consequences* , Proc. Nat. Acad. Sci. USA **48** (1962), 968-970.
3. Huppert, B., *Endliche Gruppen I*, Grundlehren, **134** (1967), Springer.
4. Huppert, B. and Blackburn, N., *Finite Groups III*, Grundlehren, **243** (1982), Springer.
5. Passman, D.S., *Permutation Groups* , 1968, Benjamin.
6. Stephens, N. M., *On the Feit-Thompson Conjecture*, Math. Comput. **25** (1971), 625.

Emeritus Professor of Hirosaki University
Toriage 5-13-5, Hirosaki 036-8171 JAPAN
Email: moka.mocha_no_kaori@snow.ocn.ne.jp

Hochschild Cohomology of Algebras with Stratifying Ideals

Hiroshi Nagase

This is joint work with Steffen Koenig.

1. Long exact sequences

When studying Hochschild cohomology it is natural to try relating cohomology H(B) of a finite dimesional algebra B to H(A) of an 'easier' or 'smaller' algebra A. In the case that B is a one-point extension of A, by Happel [?], two long exact sequences with terms of H(B) and H(A) have been studied . Happel's long exact sequences have been generalized to triangular matrix algebras, for example by Michelena and Platzeck [5].

We would like to try generalizing these results. We consider the case that B has an ideal I which gives a fully faithful functor from derived category of B/I to that of B. Such an ideal is called a *stratifying* ideal by Cline, Parshall and Scott[2]. Heredity ideals are examples of stratifying ideals and any triangular matrix algebra has a stratifying ideal.

It is known that stratifying ideals are idempotent ideals, namely any stratifying ideal of B has the form of BeB for some idempotent e in B. By using of this idempotent e , we get the following two long exact sequences;

$$\cdots \to \mathrm{Ext}^{\mathrm{n}}_{\mathrm{B^e}}(\mathrm{B},\mathrm{BeB}) \to \mathrm{H^n(B)} \to \mathrm{H^n(A)} \to \mathrm{Ext}^{\mathrm{n+1}}_{\mathrm{B^e}}(\mathrm{B},\mathrm{BeB}) \to \cdots, \text{ and}$$

$$\cdots \to \mathrm{Ext}^{\mathrm{n}}_{\mathrm{B^e}}(\mathrm{A},\mathrm{BeB}) \to \mathrm{H^n(B)} \to \mathrm{H^n(A)} \oplus \mathrm{H^n(eBe)} \to \mathrm{Ext}^{\mathrm{n+1}}_{\mathrm{B^e}}(\mathrm{A},\mathrm{BeB}) \to \cdots,$$

where $A = B/BeB$ and $B^e = B \otimes B^{op}$.

2. Nakayama algebras

We apply the second long exact sequence above to Nakayama algebras. For any Nakayama algebra B, if B is not self-injective algebra, then B has a stratifying ideal BeB with B/BeB a ground field. Since eBe is also Nakayama algebra, if eBe is not self-injective, then eBe also has a stratifying ideal. So we can continue this reduction until the algebra is reduced into a self-injective algebra. Hochschild cohomology of self-injective Nakayama algebras have been studied by Erdmann and Holm [3], and also by Bardzell, Locateli and Marcos [1]. By using their results and the second long exact sequence above, we will compare the Hochschild cohomology of the Nakayama algebra B and that of a self-injective Nakayama algebra.

References

1. M. J. Bardzell, A. C. Locateli and E. N. Marcos: *On the Hochschild cohomology of truncated algebras*, Comm. in Algebra, **28(3)** (2000), 1615-1639.
2. E.Cline, B.Parshall and L.Scott, *Stratifying endomorphism algebras*, Memoir AMS **591** (1996), i-vi, 1-119.
3. K. Erdmann and T. Holm: *Twisted bimodules and Hochschild cohomology for selfinjective algebras of class A_n*, Forum Math., **11** (1999), 177-201.
4. D. Happe:NAGAEl, *Hochschild cohomology of finite dimensional algebras* , Lecture Note in Math. **1404** (1989), 108–112.
5. S.Michelena and M.I.Platzeck: *Hochschild cohomology of triangular matrix algebras*, J. Algebra, **233** (2000), 502-525.

Nara National College of Technology
22 Yata-cho, Yamatokoriyama,
Nara 639-1080 JAPAN
Email: nagase@libe.nara-k.ac.jp

PRIMITIVITY OF GROUP RINGS OF EXTENSIONS OF FREE GROUPS

Tsunekazu Nishinaka

Okayama Shoka University

Email: nishinak@po.osu.ac.jp

1. Abstract

Let G be a group, and let $\varphi : G \longrightarrow G$ be a monomorphism. The ascending HNN extension corresponding to φ is the group $G_\varphi = \langle G, t | t^{-1}gt = \varphi(g) \rangle$. Let n be a positive integer and $G_m^n = \langle x_1, \cdots, x_m \mid x_i^n, (x_i x_j)^n, 1 \leq i, j \leq m \rangle$. A ring is (right) primitive if it has a faithful irreducible (right) module. Let F be a free group and K a field. Our purpose of this lecture is the study of primitivity of group rings KF_φ and KG_m^n.

If $G \neq 1$ is a finite group or an abelian group, then the group ring KG can never be primitive. The first nontrivial example of primitive group ring was offered by Formanek and Snider [3] in 1972. After that, several examples which include a result for primitivity of group rings of free products of groups [2] were constructed. However, there is at present no viable conjecture as to when KG is primitive for arbitrary groups.

The ascending HNN extension G_φ of a group G is a well-studied class of groups. Recently, Borisov and Sapir, in their paper [1], have shown that the ascending HNN extension F_φ of a finitely generated free group F is residually finite (cf.[4]). Then by reduction method based on residual properties and on series in groups, we can see that KF_φ is semiprimitive if the characteristic of K is zero. One might therefore hope that KF_φ is semiprimitive for any field K. We showed that KF_φ is semiprimitive for all K even if the rank of F is countably infinite. In fact, KF_φ is primitive except the case with nontrivial center.

Now, let R_i $(i = 0, 1, 2, \cdots)$ be the set of positive reduced words of length i in the free group $\langle x_1, \cdots, x_m \rangle$, and let $B(m, n, i) = \langle x_1, \cdots, x_m \mid w^n,\ w \in \cup_{k=0}^{i} R_k \rangle$. Then $B(m, n, 0)$ is simply a free group of rank m, and $B(m, n, 1)$ a free product of cyclic groups of order n. It is known that these groups are residually finite, and that $B(m, n, \infty)$, which is called the m-generator free Burnside group of exponent n, is not residually finite for any sufficiently large n. It is also known that group rings $KB(m, n, 0)$ and $KB(m, n, 1)$ are primitive if $m > 1$ and $n > 2$. We can show that $G_m^n = B(m, n, 2)$ is an extension of a free group if $n > 2$ and $n \geq 2m - 3 > 0$. As a concequence, in this case, it can be seen that G_m^n is residually finite, and that KG_m^n is primitive for any K.

References

1. A. B. Borisov and M. Sapir, Polynomial maps over finite fields and residual finiteness of mapping tori of group endomorphisms, Invent. Math., 160(2)(2005), 341-356
2. E. Formanek, Group rings of free products are primitive, J. Algebra, 26(1973), 508-511
3. E. Formanek and R. L. Snider, Primitive group rings, Proc. Amer. Math. Soc., 36(1972), 357-360
4. R. Geoghegan, M. L. Mihalik, M. Sapir, and T. Wise, Ascending HNN extensions of finitely generated free groups are Hopfian, Bull. London Math. Soc., 33(3)(2001), 292-298

Partial Skew Polynomial Rings and Jacobson Rings

Wagner de Oliveira Cortes

Pure and Applied Mathematics
Federal University of Rio Grande do Sul (UFRGS)

We consider rings R with a partial action α of a cyclic infinite group G on R, $R[x;\alpha]$ the partial skew polynomial rings and $R < x;\alpha >$ the the partial skew Laurent of polynomial. We generalized the well known results about Jacobson rings and strongly Jacobson rings in skew polynomial rings and skew Laurent of polynomial rings to partial skew polynomial rings and partial skew Laurent of polynomial rings.

Computing the maximal algebra of quotients of a Lie algebra

M. Brešar, F. Perera, J. Sánchez Ortega*, M. Siles Molina

Department of Mathematics, University of Maribor, FNM, Koroška 160, 2000 Maribor, Slovenia.

Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain.

Departamento de Álgebra, Geometría y Topología, Universidad de Málaga, 29071 Málaga, Spain.

Email: bresar@uni-mb.si, perera@mat.uab.cat, jsanchez@agt.cie.uma.es, mercedes@agt.cie.uma.es

The theory of algebras of quotients of associative algebras has a rich history and is still an active research area. The notion of "being an algebra of quotients" has found recently parallels for Jordan systems (see e.g. [4, 1, 3]).

In [6] M. Siles Molina initiated the study of algebras of quotients of Lie algebras. She introduced the notion of a general (abstract) algebra of quotients of a Lie algebra, and also, as a special concrete example, the notion of the maximal algebra of quotients $Q_m(L)$ of a semiprime Lie algebra L. In order to foresee the importance of this concept in the non-associative setting, F. Perera and M. Siles Molina undertook in [5] a study of the relationship between Lie and associative quotients. M. Cabrera and J. Sánchez Ortega have proved in [2] that similar results hold for the skew case.

Our main goal is to compute $Q_m(L)$ for some Lie algebras L. Specifically, we are interested in Lie algebras of the form $L = A^-/Z$, where A^- is the Lie algebra associated to a prime associative algebra A and Z is the center of A, and in Lie algebras of the form $L = K/Z_K$, where K is the Lie algebra of skew elements of a prime associative algebra with involution and Z_K is its center. We also study natural questions, namely, whether $Q_m(I)$, where I is an essential ideal of a Lie algebra L, is equal to $Q_m(L)$ and whether taking the maximal algebra of quotients is a closure operation, that is, $Q_m(Q_m(L)) = Q_m(L)$. We will be able to give a positive answer to the first question provided that L satisfies a certain condition. On the other hand, we will show that in some special situations the answer to the second question is positive, namely, if L is a simple Lie algebra or if $L = A^-/Z$, where A is either a simple algebra (satisfying a minor technical assumption) or an affine PI prime algebra (i.e. a finitely generated prime algebra which satisfies a polynomial identity). In general, however, it is not true.

References

1. Anquela, J. A., García, E., Gómez Lozano, M., *Maximal algebras of Martindale-like quotients of strongly prime linear Jordan algebras*, J. Algebra **280** (2004), 367-383.
2. Cabrera, M., Sánchez Ortega, J., *Lie quotients for skew Lie algebras.* (Preprint).
3. García, E., Gómez Lozano, M., *Jordan System of Martindale-like Quotients.* J. Pure Appl. Algebra **194** (2004), no. 1-2, 127-145.
4. Martínez, C., *The ring of fractions of a Jordan algebra*, J. Algebra **237** (2001), 798-812.
5. Perera, F., Siles Molina, M., *Associative and Lie algebras of quotients.* Pub. Math. (To appear).
6. Siles Molina, M., *Algebras of quotients of Lie algebras.* J. Pure Appl. Algebra **188** (2004), 175-188.

Professor Harada — Person and Work

Kiyoichi Oshiro

Yamaguchi University

Email: oshiro@yamaguchi-u.ac.jp

I introduce "Person and Work" of Professor Manabu Harada by talking following:

Person

(1) University days at Osaka City University as a student of Professor Keizo Asano.

(2) Professor days for almost 40s at Osaka City University.

(3) After retired from Osaka City University.

Research field

(1) Hereditary order, hereditaruy noetherian prime rings.

(2) Krull-Remak-Schmidt-Azumaya's Theorem.

(3) Non-small modules and non-cosmall modules.

(4) Lifting property and extending property for modules.

(5) Simple injective modules and mini-injective modules.

(6) New artinian rings (H-rings), and its sense and applications.

δ-M-SMALL and δ-HARADA MODULES

M. Tamer KOŞAN [1] and A. Çiğdem ÖZCAN [2]

[1] Kocatepe University, Department of Mathematics,ANS Campus, Afyon, Turkey
[2] Hacettepe University, Department of Mathematics, 06800 Beytepe Ankara, Turkey

Email: mtkosan@aku.edu.tr, ozcan@hacettepe.edu.tr

In the early 1980s, Harada found a new class of Artinian rings, and Oshiro in [1] calls these rings *Harada rings* (briefly H-ring). Generalizing the concept of H-rings to module theory, Harada modules were introduced by Jayaraman and Vanaja in [2] by using the category $\sigma[M]$.

In this article, as a generalization of Harada modules, δ-Harada modules are defined by using the concept of "δ–small submodules" of Zhou in [3]. Some properties of δ-Harada modules are investigated and the following characterization of H-rings is also obtained.

Theorem. The following are equivalent for a ring R.

1. R is a right H-ring.
2. R is right Noetherian and every non-δ-small right R-module contains a non-zero injective submodule.
3. R is right Artinian and every non-δ-small right R-module contains a non-zero injective submodule.
4. R is right perfect and for any exact sequence of right R-modules $P \xrightarrow{f} N \longrightarrow 0$ where N is injective and $Ker(f)$ is δ-small in P, P is a direct sum of an injective module and a semisimple projective module.

References

1. K. Oshiro, *Lifting modules, extending modules and their applications to QF-rings*, Hokkaido Math. J., **13** (1984) 310-338.
2. M. Jayaraman, N. Vanaja, *Harada modules*, Comm.Algebra, **28** (2000) 3703-3726.
3. Y. Zhou, *Generalizations of perfect, semiperfect, and semiregular rings*, Alg. Coll., **7(3)** (2000) 305-318.

Quasi-Baer Ring Hulls and Applications to C^*-Algebras

Jae Keol Park
Department of Mathematics
Busan National Univesity
Busan 609-735, South Korea
E-mail: jkpark@pusan.ac.kr

Recall that a ring with identity is called *quasi-Baer* if the right annihilator of each ideal is generated by an idempotent as a right ideal. In this talk, we discuss the existence of quasi-Baer ring hulls of semiprime rings and their applications to C^*-algebras. We consider connections between the concepts of boundedly centrally closed and quasi-Baer for the class of C^*-algebras. In particular, we characterize a C^*-algebra whose local multiplier algebra is a direct product of prime C^*-algebras. As a corollary, we characterize those C^*-algebras which satisfy a PI and have only finitely many minimal prime ideals (This is a joint work with Gary F. Birkenmeier and S. Tariq Rizvi).

GENERALIZED DERIVATIONS IN PRIME RINGS

Nadeem-ur-Rehman

Department of Mathematics
Aligarh Muslim University
Aligarh 202002 (INDIA)

Email: rehman100@gmail.com

Let R be an associative ring with center $Z(R)$. For each $x, y \in R$ denote the commutator $xy - yx$ by $[x, y]$ and the anti-commutator $xy + yx$ by $x \circ y$. Recall that a ring R is prime if for any $a, b \in R$, $aRb = \{0\}$ implies that $a = 0$ or $b = 0$. For any $a, b \in R$ we set $[a, b]_{\alpha,\beta} = a\alpha(b) - \beta(b)a$. An additive function $F : R \longrightarrow R$ is called a generalized inner derivation if $F(x) = ax + xb$ for fixed $a, b \in R$. For such a mapping F, it is easy to see that

$$F(xy) = F(x)y + x[y, b] = F(x)y + xI_b(y) \text{ for all } x, y \in R.$$

This observation leads to the following definition, given in [Comm. Algebra 26(1998), 1149-1166]: an additive mapping $F : R \longrightarrow R$ is called a generalized derivation with associated derivation d if

$$F(xy) = F(x)y + xd(y) \text{ for all } x, y \in R.$$

Familiar examples of generalized derivations are derivations and generalized inner derivations. Since the sum of two generalized derivations is a generalized derivation, every map of the form $F(x) = cx + d(x)$, where c is fixed element of R and d is a derivation, is a generalized derivation; and if R has 1, all generalized derivations have this form.

Let α and β be the endomorphisms of R. An additive map $d : R \longrightarrow R$ is called an (α, β)-derivation if $d(xy) = d(x)\alpha(y) + \beta(x)d(y)$ holds for all $x, y \in R$. An $(1, 1)$-derivation is called simply a derivation, where 1 is the identity map. For fixed a, the map $d_a : R \longrightarrow R$ given by $d_a(x) = [a, x]_{\alpha,\beta}$ for all $x \in R$ is an (α, β)-drivation which is said to be an (α, β)-inner derivation. An additive mapping $F : R \longrightarrow R$ will be called a generalized (α, β)-inner derivation if $F(x) = a\alpha(x) + \beta(x)b$ for some fixed $a, b \in R$ and all $x \in R$. A simple computation yields that if F is a generalized (α, β)-inner derivation, then we have for all $x, y \in R$, $F(xy) = F(x)\alpha(y) + \beta(x)d_{-b}(y)$, where d_{-b} is an (α, β)-inner deivation. In this viewpoint, an additive map $F : R \longrightarrow R$ will be called generalized (α, β)-derivation associated with d if there exists an (α, β)-derivation $d : R \longrightarrow R$ such that

$$F(xy) = F(x)\alpha(y) + \beta(x)d(y) \text{ for all } x, y \in R.$$

An $(1, 1)$-genaralized derivation is called simply a generalized derivation, where 1 is an identity map. In the present paper, we shall discuss the commutativity of prime rings admitting a generalized (α, β)-deviation F satisfying any one of the properties: (i) $[F(x), x]_{\alpha,\beta} = 0$, (ii) $F([u, v]) = 0$, (iii) $F(u \circ v) = 0$, (iv) $F([x, y]) = [x, y]_{\alpha,\beta}$, (v) $F(xy) = \alpha(xy)$, (vi) $F(x \circ y) = (x \circ y)_{\alpha,\beta}$, (vii) $F(xy) - \alpha(xy) \in Z(R)$ and $(viii)$ $F(x)F(y) - \alpha(xy) \in Z(R)$, for all x, y in some appropriate subset of R.

The relevance and the ubiquity of Prüfer modules

C.M.Ringel

Let R be a ring. An R-module M will be said to be a Prüfer module provided there exists a locally nilpotent, surjective endomorphism of M which has kernel of finite length. In the lecture we want to outline the relevance, but also the ubiquity of Prüfer modules. Any Prüfer module which is not of finite type gives rise to a generic module, thus to infinite families of indecomposable modules with fixed endo-length (here we are in the setting of the second Brauer-Thrall conjecture). The aim of the lecture is to exhibit a construction procedure for Prüfer modules which yields a wealth of such modules.

The work of Tachikawa on finite-dimensional algebras and their representations

C.M.Ringel

The aim of the lecture is to outline some of the main contributions of Tachikawa to the representation theory of finite-dimensional algebras. We mainly will concentrate on the early years: on results concerning the classification of indecomposable modules, the importance of the dominant dimension and on the structure of selfinjective aLgebras and their generalisations (such as QF-1 and QF-3 algebras). Key topics will be the double centralizer condition (and Schur-Weyl duality), the repetitive algebras and trivial extensions as well as the various homological conjectures. Tachikawa's considerations had a lasting effect on the modern development of the representation theory of finite-dimensional algebras and we want to sketch in which way these results have been used, extended and transformed in recent years.

On structure of rings of quotients

S. Tariq Rizvi
Department of Mathematics
Ohio State University
Lima, OH 45804-3576
E-mail: rizvi.1osu.edu

Abstract. We introduce the notion of Johnson dimension (jdim) of an $S-R$ bimodule M where S and R are any rings. We use our methods to provide a characterization of an arbitrary ring R for which $Q(R)$, the maximal right ring of quotients of R, is a direct product of prime rings. This removes the condition of nonsingularity from a result of of Jain, Lam and Leroy and generalizes it. As a consequence, we obtain a structure theorem for the idempotent closure $RB(Q(R))$, the subring of Q generated by R and the central idempotents of $Q(R)$. Some related results on dimensions of rings will be presented. (This is a joint work with Gary F. Birkenmeier and Jae K. Park.)

ON DIRECT SUMS OF EXTENDING AND BAER MODULES

Cosmin S. Roman

The Ohio State University, Lima

Email: cosmin@math.ohio-state.edu

Let R be a ring with unit, not necessarily commutative; all modules are considered to be right R-modules.

An open question, that has only received partial answers till to date, is to find necessary and sufficient conditions for a direct sum of extending modules to be extending. An extending R-module is one for which every submodule is essential in a direct summand. The concept generalizes those of injective and (quasi-) continuous modules. A closely related concept is that of a Baer Module. We define an R-module M to be Baer if, denoting by $S = End_R(M)$ the ring of endomorphisms of M, the left annihilator in S of any submodule of M is a left ideal of S generated by an idempotent; equivalently, the right annihilator in M of any left ideal of S is a direct summand of M.

Using this concept, we will present some newer developments which may provide an answer to the above question in some situations under the assumption of nonsingularity (and even under some weaker form of nonsingularity). Some applications of our results will be discussed (joint work with S.Tariq Rizvi).

References

1. Chase, S.U., Direct Products Of Modules, Trans. Am. Math. Soc. 1960, *97*, no. 3, 457–473
2. Cohn, P.M., *Free Rings and Their Relations*, Academic Press: London & New York, 1971
3. Dung, N.V.; Huynh, D.V.; Smith, P.F.; Wisbauer, R., *Extending Modules*, Pitman Research Notes in Mathematics Series, 313; Longman Scientific & Technical: Harlow, 1994
4. Faith, C., Embedding Torsionless Modules in Projectives, Publ. Mat. 1990, *34*, no. 2, 379–387
5. Mohamed, S.H.; Mller, B.J., *Continuous and Discrete Modules*, London Mathematical Society Lecture Note Series, 147, Cambridge University Press, Cambridge, 1990
6. Rizvi, S.T.; Roman, C.S., Baer And Quasi-Baer Modules, Comm. Alg. **2004**, *32(1)*, 103–123
7. Rizvi, S.T.; Roman, C.S., Baer Property of Modules and Applications, *Advances in Ring Theory, (Proceedings of the 4th China-Japan-Korea International Conference)* ((eds.)), publisher, city, **2005**, 225-241
8. Tsukerman, G.M., Rings of Endomorphisms of Free Modules, Sibirsk. Mat. Ž. 7 **1966**, *7*, 1161–1167
9. Stephenson, V.; Tsukerman, G.M., Rings of Endomorphisms of Projective Modules, Sibirsk. Mat. Ž. **1970**, *11*, 228–232

ON DERIVATIONS IN NEAR RINGS

Mohammad Shadab Khan
Department of Commerce
Aligarh Muslim University
Aligarh-202002 (India)
E-mail:km_shadab@rediffmail.com

Let N be a left near-ring. An additive mapping $d : N \longrightarrow N$ is said to be a derivation on N if $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$. An additive mapping $d : N \longrightarrow N$ is called a (σ, τ)-derivation if there exist endomorphisms $\sigma, \tau : N \longrightarrow N$ such that $d(xy) = \sigma(x)d(y) + d(x)\tau(y)$ for all $x, y \in N$.

There is an increasing body of evidence that prime near-rings with derivations have ring like behavior, indeed, there are several results, see for example; Bell[Kluwer Academic Publishers Netherlands (1997), 191-197], Beidar et.al[Comm. Algebra 24(5)(1996), 1581-1589], Wang[Proc. Amer. Math. Soc. 121 (1994), 361-366] and Ashraf et. al[Arch. Math. 40(2004), 281-286] asserting that the existence of a suitably-constrained derivation on a prime near-ring forces the near-ring to be ring.

In this paper, our aim is to generalizes some results due to Wang [Proc. Amer. Math. Soc. 121 (1994), 361-366] on near-rings admitting a special type of derivation namely (σ, τ)-derivation, where σ and τ are endomorphisms of the near-ring.

Characterizations of QF Rings

Liang Shen
A joint work with Jianlong Chen
Department of Mathematics, Southeast University, Nanjing, China
Email: lshen@seu.edu.cn

Let R be a ring. A right ideal I of R is called small if for any proper right ideal K of R, $I+K \neq R$. R is called right small injective if every homomorphism from a small right ideal of R to R_R can be extended to a homomorphism from R_R to R_R. R is called right $\aleph_0$-injective if every homomorphism from a countably generated right ideal of R to R_R can be extended to a homomorphism from R_R to R_R.

Some characterizations of QF rings are given by small injectivity, mininjectivity, simple injectivity and $\aleph_0$-injectivity. It is proved that:
(1) If R is left and right mininjective, with ACC on right annihilators such that $S_r \subseteq^{ess} R_R$, then R is QF.
(2) If R is right simple injective, with ACC on right annihilators such that $S_r \subseteq^{ess} R_R$, then R is QF.
(3) If R is right small injective, with ACC on right annihilators such that $S_r \subseteq^{ess} R_R$, then R is QF.
(4) If R is left perfect, left and right small injective, then R is QF.
(5) If R is semilocal, right small injective and with ACC on right annihilators, then R is QF.
(6) If R is right noetherian and left $\aleph_0$-injective, then R is QF.

A Note on The Finitistic Dimension Conjecture

Hongbo Shi

Nanjing University of Finance and Economics

Email: dshi782000@yahoo.com

For a large class of rings R, we first observe some relationships between the R-projective and respectively eRe-projective resolutions, where e is certain well-chosen idempotent of R, and then use the observations to draw some relations between the finitistic dimensions of the two rings. The results are then applied to relations algebras in an effort to offer some insight into the nature of the finitistic dimension conjecture for this special and important class of algebras.

References

1. K. Erdmann, T. Holm, O. Iyama and J. Schröer, *Radical Embedding and Representation Dimension*, Advances in Mathematics, **185**(2004), 159-177.
2. C.D. Feustel, E.L. Green, Ellen Kirkman, and James Kuzmanovich, *Constructing Projective Resolutions*, Comm. Algebra, **21**(1993), 1869-1887.
3. E.L. Green, E. Kirkman and J. Kuzmanovich, *Finitistic dimensions of finite-dimensional monomial algebras*, J. Algebra **136**(1991), 37-51.
4. K. Igusa and D. Zacharia, *Syzygy pairs in a monomial algebra*, Proc. Amer. Math. Soc. **108**(1990), 601-604.
5. Changchang Xi, *On The Finitistic Dimension Conjecture I: Related to Representation-finite Algebras*, J. Pure Appl. Algebra 193(2004)287-305.
6. B. Zimmermann-Huisgen, *Predicting Syzygies over Monomial Relations Algebras*, Manuscripta Math. **70**(1991), 157 - 182.
7. B. Zimmermann-Huisgen, *Homological domino effects and the first Finitistic Dimension Conjecture*, Invent. math. **108**(1992), 369 - 383.

Algebras of quotients of Leavitt path algebras

M. Siles Molina
Departamento de Álgebra, Geometría y Topología, Universidad de Málaga, 29071 Málaga, Spain
Email:mercedes@agt.cie.uma.es

2000 Mathematics Subject Classification: Primary 16D70
*Key Words:*Leavitt path algebra,socle, algebra of quotients, Fountain-Gould right order, Toeplitz algebra

Leavitt path algebras of row-finite graphs have attracted a good deal of attention recently. Although their origin can be traced back to the pioneering work of Leavitt in his quest for finding universal rings failing to satisfy the IBN property (see [**?**]:SILES), they have only come to life as such in recent years (see, e.g. [1], and [4]). Speaking informally, a Leavitt path algebra is a graph K-algebra over a field K together with two naturally occurring relations, known as the *Cuntz-Krieger relations*. This terminology evolves from the construction due to Cuntz and Krieger of algebras defined out of finite $\{0,1\}$-matrices towards the so-called graph C^*-algebras (see [9]).

As with their analytic relatives, Leavitt path algebras provide a source of examples of rings whose algebraic structure is determined by highly visual properties of the underlying graph (see [7]). For example, conditions on the graph allow us to decide when the corresponding Leavitt path algebra is simple [1], purely infinite simple [2], exchange [6], finite-dimensional [3]. Some of the graph conditions parallel the corresponding structural properties that one encounters in C^*-algebras (notably simplicity and the exchange property), but the routes towards the proofs are in general quite different.

A whole range of examples of algebras arise as Leavitt path algebras. Besides the (now already) classical examples investigated by Leavitt, we also find algebraic analogues of the Toeplitz algebra (which will be described here), arbitrary matrices over the Laurent polynomial ring $K[x,x^{-1}]$ over a field K, and, up to Morita equivalence, all K-ultramatricial algebras (i.e. countable direct limits of K-matricial algebras).

In this talk we will start by showing that the Leavitt path algebra of a (row-finite) graph is an algebra of quotients of the corresponding path algebra. The path algebra is semiprime if and only if whenever there is a path connecting two vertices, there is another one in the opposite direction. Semiprimeness is studied because, for acyclic graphs, the Leavitt path algebra is a Fountain-Gould algebra of right quotients of any semiprime subalgebra containing the path algebra (and a Moore-Penrose algebra of right quotients of any subalgebra with involution that contains the path algebra). The maximal algebras of quotients of Leavitt path algebras with essential socle (equivalently the associated graph satisfies that every vertex connects to a line point) can be obtained.

References

1. G. Abrams, G. Aranda Pino, The Leavitt path algebra of a graph, *J. Algebra* **293 (2)** (2005), 319–334.
2. G. Abrams, G. Aranda Pino, Purely infinite simple Leavitt path algebras, *J. Pure Appl. Algebra* **207 (3)** (2006), 553–563.
3. G. Abrams, G. Aranda Pino, M. Siles Molina, Finite dimensional Leavitt path algebras, *J. Pure Appl. Algebra* (to appear) DOI: 10.1016/j.jpaa.2006.07.013
4. P. Ara, M.A. Moreno, E. Pardo, Nonstable K-Theory for graph algebras, *Algebra Represent. Theory* (to appear).
5. G. Aranda Pino, E. Pardo, M. Siles Molina, Exchange Leavitt path algebras and stable rank, *J. Algebra* **305 (2)** (2006), 912–936.
6. G. Aranda Pino, E. Pardo, M. Siles Molina, Exchange Leavitt path algebras and stable rank, *J. Algebra* **305 (2)** (2006), 912–936.
7. G. Aranda Pino, F. Perera, M. Siles Molina, eds., *Graph algebras: bridging the gap between analysis and algebra*, ISBN: 978-84-9747-177-0, University of Málaga Press, Málaga, Spain (2007).
8. W.G. Leavitt, The module type of a ring, *Trans. A.M.S.* **103** (1962), 113–130.
9. I. Raeburn, *Graph algebras*, CBMS Regional Conference Series in Mathematics, **103**, Amer. Math. Soc., Providence, (2005).

Ascending Chain Conditions in Groups, Rings and Modules

Patrick F. Smith

Department of Mathematics
University of Glasgow
Glasgow G12 8QW, Scotland UK

Email: pfs@maths.gla.ac.uk

The chain conditions in question are of the following type: given a positive integer n, an algebraic object satisfies the ascending chain condition on n-generated subobjects. For example, every free group satisfies the ascending chain condition on n-generated subgroups, for every positive integer n. If R is a right Noetherian ring then sometimes every free right R-module satisfies the ascending chain condition on n-generated submodules, for every positive integer n, but sometimes this does not happen.

On contravariantly finite subcategories of finitely generated modules

Ryo Takahashi

Shinshu University

Email: takahasi@math.shinshu-u.ac.jp

The notion of a contravariantly finite subcategory, which is also called a (pre)covering class, was first introduced over artin algebras by Auslander and Smalø [4] in connection with studying the problem of which subcategories admit almost split sequences. The notion of a resolving subcategory was introduced by Auslander and Bridger [1] in the study of modules of G-dimension zero, which are now also called totally reflexive modules. Auslander and Reiten [3] proved that the notion of a contravariantly finite resolving subcategory is closely related to tilting theory. There is also an application of contravariantly finite resolving subcategories to the study of the finitistic dimension conjecture.

In this talk, we will study contravariantly finite resolving subcategories over commutative rings. Let R be a commutative noetherian henselian (e.g. complete) local ring. (Note that any commutative artinian local ring is complete, hence henselian.) We denote by $\operatorname{mod} R$ the category of finitely generated R-modules, by $\mathcal{F}(R)$ the subcategory of free R-modules, by $\mathcal{C}(R)$ the subcategory of maximal Cohen-Macaulay R-modules, and by $\mathcal{G}(R)$ the subcategory of totally reflexive R-modules. The subcategory $\mathcal{F}(R)$ is always a contravariantly finite resolving subcategory, and so is $\mathcal{C}(R)$ provided that R is Cohen-Macaulay. The latter fact is known as the Cohen-Macaulay approximation theorem, which was proved by Auslander and Buchweitz [**?**]. The subcategory $\mathcal{G}(R)$ is always resolving, and coincides with $\mathcal{C}(R)$ if R is Gorenstein.

The main result of this talk is the following.

Theorem 1. *If R is Gorenstein, then all the contravariantly finite resolving subcategories of* $\operatorname{mod} R$ *are $\mathcal{F}(R)$, $\mathcal{C}(R)$ and* $\operatorname{mod} R$.

Actually we can prove several more general results than the above theorem. One of them yields another proof of the following theorem, which is the main result of [5].

Theorem 2. *Suppose that there is a nonfree R-module in $\mathcal{G}(R)$. If $\mathcal{G}(R)$ is contravariantly finite in* $\operatorname{mod} R$*, then R is Gorenstein.*

References

1. M. Auslander; M. Bridger, Stable module theory, Memoirs of the American Mathematical Society, No. 94, *American Mathematical Society, Providence, R.I.*, 1969.
2. M. Auslander; R.-O. Buchweitz, The homological theory of maximal Cohen-Macaulay approximations, Colloque en l'honneur de Pierre Samuel (Orsay, 1987), *Mém. Soc. Math. France (N.S.)* No. 38, (1989), 5–37.
3. M. Auslander; I. Reiten, Applications of contravariantly finite subcategories, *Adv. Math.* **86** (1991), no. 1, 111–152.
4. M. Auslander; S. O. Smalø, Preprojective modules over Artin algebras, *J. Algebra* **66** (1980), no. 1, 61–122.
5. L. W. Christensen; G. Piepmeyer; J. Striuli; R. Takahashi, Finite Gorenstein representation type implies simple singularity, preprint (2007), `http://arxiv.org/abs/0704.3421`.
6. R. Takahashi, Contravariantly finite resolving subcategories over commutative rings, preprint (2007).

Finitely generated modules over non-commutative valuation rings

Akira UEDA

Shimane University

Email: ueda@riko.shimane-u.ac.jp

Finitely generated modules over commutative valuation rings have been investigated since 1980's (see [1]). We will report some results about finitely generated modules over non-commutative valuation rings.

A subring R of a division ring D is said to be an *total valuation ring* if, for any non-zero element d of D, we have $d \in R$ or $d^{-1} \in R$. Furthermore, if $dRd^{-1} = R$ holds for any non-zero element d of D, then R is called an *invariant valuation ring.*

An R-submodule N of a left R-module M is said to be *relatively divisible* (an *RD-module* for short) if $aN = N \cap aM$ for any $a \in M$.

Then we have the following results:

Proposition 1. *Let R be an invariant valuation ring and let M be a finitely generated left R-module. Then there exists a sequence*

$$0 = M_0 \subset M_1 \subset \cdots \subset M_n = M$$

of R-submodules of M such that

(1) *each M_i is an RD-submodule of M, and*
(2) *M_i/M_{i-1} is cyclic ($i = 1, 2, \cdots, n$).*

The sequence in Proposition 1 is called an *RD-composition series* of M.

Proposition 2. *Let R be an invariant valuation ring. Then any two RD-composition series of a finitely generated left R-module M is isomorphic and the length of RD-composition series of M is equal to the number of minimal generators of M.*

References

1. L. Fuchs and L. Salce, *Modules over non-Noetherian domains*, AMS, 2001.
2. H. Marubayashi, H. Miyamaoto and A. Ueda, *Non-commutative valuation rings and semi-hereditary orders*, Kluwer Academic Publishers, 1997.

skew-matrix ring and applications to QF-ring

Masahiko Uhara, Yoshihisa Nagatomi, Kiyoichi Oshiro

In this talk, we discuss a skew-matrix ring which is introduced by Kupisch and Oshiro through the study on Nakayama rings. As applications of these rings, we show several results on QF-rings as follows:

(1) Let R be a skew-matrix ring over a local QF-ring Q;

$$R \simeq \begin{pmatrix} Q & \cdots & Q \\ & \cdots & \\ Q & \cdots & Q \end{pmatrix}_{\sigma,c,n}$$

with $c \in \mathrm{J}(Q)$,$\sigma \in \mathrm{Aut}(Q)$.Then R is a QF-ring with non-identity cyclic Nakayama permutation.

(2) Let R be a basic indecomposable artinian ring R is said to be a strongly QF-ring if and only if eRe is QF for any $e = e^2 \in R$. Followring are equivalent:

(a) R is a strongly QF-ring

(b) R is a QF-ring with identity Nakayama permutation or R is represented as a skew-matrix ring $(Q)_{\sigma,c,n}$ over a local QF-ring with $c \in \mathrm{J}(Q)$.

(3) For a given permutation $\xi = \begin{pmatrix} 1 & 2 & \cdots & n \\ p_1 & p_2 & \cdots & p_n \end{pmatrix}$, there is a basic indecomposable QF-ring $R = e_1R \oplus \cdots \oplus e_nR$ with a Nakayama permutation $\begin{pmatrix} e_1 & e_2 & \cdots & e_n \\ e_{p_1} & e_{p_2} & \cdots & e_{p_n} \end{pmatrix}$ where $\{e_1, \ldots, e_n\}$ is a complete set of orthogonal primitive idempotents.

References

1. M. Hoshino, *Strongly quasi-Frobenius rings*, Comm. Algebra, (2000), 3585–3599.
2. H. Kupisch, *Uber ein Klasse von Ringen mit Minimalbedingung* II, Arch. Math. 26 (1975), 23-35.
3. K. Oshiro, *Theories of Harada in Artinian rings*, International Sympojium on Ring Theory, (Kyongju, 1999), Brikhauser Boston, Boston (2001), 279–328.
4. Y. Yukimoto, *On decomposition of strongly quasi-Frobenius rings*, Comm. Algebra 28 (2000), 1111–1113.

E-mail:yu_uhara@yahoo.co.jp

Extending Ring Derivations to Rings and Modules of Quotients first line

Lia Vaš

Department of Mathematics, Physics and Statistics, University of the Sciences in Philadelphia, 600 S. 43rd St., Philadelphia, PA 19104

Email: l.vas@usip.edu

Recall that a *derivation* on a ring R is an additive mapping $\delta : R \to R$ with $\delta(rs) = \delta(r)s + r\delta(s)$ for all $r, s \in R$. An additive mapping $d : M \to M$ on a right R-module M is a δ-*derivation* if $d(xr) = d(x)r + x\delta(r)$ for all $x \in M$ and $r \in R$. As these concepts are not intrinsically ring theoretic notions, it is of interest to study how they agree with the concepts that are intrinsically ring theoretic. Specifically, continuing the study from [1], [2], and [3], we study how derivations agree with hereditary torsion theories. In [1], a torsion theory is said to be *differential* if $d(\mathcal{T}(M))$ is in the torsion submodule $\mathcal{T}(M)$ for every module M and every derivation d. Equivalently, a torsion theory is differential if every derivation on a module extends uniquely to a derivation on its module of quotients.

In this talk, we review some results from [3] and continue the study of extensions of derivations to rings and modules of quotients. Firstly, we demonstrate that every perfect torsion theory for a ring R is differential. Perfect torsion theories are those in which the module of quotients $M_{\mathfrak{F}}$ of every module M is isomorphic to the tensor product $M \otimes_R R_{\mathfrak{F}}$ where $R_{\mathfrak{F}}$ is the right ring of quotients. Thus, every module of quotients is determined solely by the right ring of quotients. Since the classical torsion theory of a right Ore ring has this property, a perfect torsion theory is a way to generalize the classical torsion theory in cases when the ring might not be right Ore. We prove that a perfect torsion theory is differential and give an explicit construction of the extension of a derivation from a right R-module M to the module of quotients $M_{\mathfrak{F}}$. Secondly, we prove that the Lambek and Goldie torsion theories for any ring are differential. Lastly, we address the following question: assuming that a ring derivation can be extended to a ring of quotients Q_1 contained in another ring of quotients Q_2, when can a derivation on Q_1 be extended to Q_2? In the cases when it can, we say that the extensions *agree.* More generally, we consider the following question: when does a derivation on module of quotients extends to the module of quotients with respect to a larger torsion theory? We prove that such extension is possible if both torsion theories are differential or if the smaller is differential and the module of quotients with respect to the smaller torsion theory is torsion-free in the larger torsion theory. We derive some corollaries of this results. In particular, we prove that the extensions on the maximal right ring of quotients, the classical right ring of quotients, the ring of quotients with respect to the Goldie torsion theory, and the total right ring of quotients (the largest perfect right ring of quotients) all agree.

References

1. P. E. Bland, *Differential torsion theory*, Journal of Pure and Applied Algebra **204** (2006) 1 – 8.
2. J. S. Golan, *Extensions of derivations to modules of quotients*, Comm. algebra **9** (1981) no. 3, 275 – 281.
3. L. Vaš, *Differentiability of Torsion Theories,* Journal of Pure and Applied Algebra **210** (2007) no. 3, 847 – 853.

Construct bi-Frobenius algebras via quivers

Yanhua Wang (joint with Xiaowu Chen and Pu Zhang)
Department of Applied Mathematics,
Shanghai University of Finance and Economics,
Shanghai 200433, P.R.China
E-mail: yhw@mail.shufe.edu.cn
Supported by the Chinese NSF (Grant No. 10271113)

Using quiver technique, we construct a class of (non)graded bi-Frobenius algebras. In particular, this kind of bi-Frobenius algebras are not Hopf algebras. We also classify a class of graded bi-Frobenius algebras via certain equations of structure coefficients.

1. Introduction

Typical Frobenius algebras are finite group algebras. In general, a finite-dimensional Hopf algebra is a Frobenius algebra (see Larson and Sweedler [LS], or 2.1.3 in Montgomery [M]). Relations of the Frobenius algebras with the Yang-Baxter equations and with the topological quantum field theory can be founded in [Kad] and [A], respectively. As a natural generalization of finite-dimensional Hopf algebras, the concept of a bi-Frobenius algebra was introduced by Doi and Takeuchi [DT] (see also [Kop]). Roughly speaking, this is a Frobenius algebra as well as a Frobenius coalgebra together with an antipode. Except for an example given in 2.5 in [DT], there are few explicit constructions of bi-Frobenius algebras which are not finite-dimensional Hopf algebras. The aim of this note is to provide such an explicit construction via quivers.

Motivations of our construction is the quiver method in the representation theory of algebras, see Ringel [R], coalgebra structure on quivers considered by Chin and Montgomery [CM], and constructing Hopf quiver and quiver quantum groups by Cibils and Rosso [C], [CR], and E. Green and Solberg [GS], etc.

We start from the algebra $\mathbb{K}Z_n/J^d$, where $\mathbb{K}Z_n$ is the path algebra of the basic cycle with n vertices and J is the ideal generated by arrows with $d \geq 2$ an integer. This is an augmented Frobenius algebra, and it is a symmetric if and only if $d \equiv 1 (\mathrm{mod}\ n)$ (see Theorem 2.3 below). Endowed with a suitable Frobenius coalgebra structure, this Frobenius algebra becomes a bi-Frobenius algebra, which is not a Hopf algebra (see Theorem 3.3).

The authors thank the referee for pointing out the reference [D]. By Lemma 1.2 in [D], one can prove the both Frobeniusness of $\mathbb{K}Z_n/J^d$ by showing that it has a bijective bi-Frobenius antipode. So we leave the proof of the Frobeniusness of algebra (see Lemma 2.1 (ii)) and of coalgebra (see Lemma 3.2) to Theorem 3.3.

Throughout let $\mathbb{K}$ be a field. All algebras and coalgebras are over $\mathbb{K}$. The notations Hom and $\otimes$ are over $\mathbb{K}$.

References

[A] L. Abrams, Two-dimensional topological quantum field theories and Frobenius algebras, J. Knot Theory and Its Ramifications 5(1996), 569-587.

[CM] W. Chin and S. Montgomery, Basic coalgebras, AMS/IP Stud. Adv. Math. 4, AMS 1997.

[C] C. Cibils, A quiver quantum group, Commun. Math. Phys. 157(1993), 459-477.

[CR] C. Cibils and M. Rosso, Hopf quivers, J. Algebra 254(2002), 241-251.

[D] Y. Doi, Substructures of bi-Frobenius algebras, J. Algebra, 256(2002), 568-582.

[DT] Y. Doi and M. Takeuchi, Bi-Frobenius algebras, Contemp. Math. 267, AMS 2000, 67-97.

[GS] E. L. Green and O. Solberg, Basic Hopf algebras and quantum groups, Math. Z. 229(1998), 45-76.

[Kad] L. Kadison, New examples of Frobenius extensions, University Lecture Series, Vol. 14, AMS. 1999.

[Kop] M. Koppinen, On algebras with two multiplications, including Hopf algebras and Bose-Mesner algebras, J. Algebra 182(1996), 256-273.

[LS] R. G. Larson and M. Sweedler, An associative orthogonal bilinear form for Hopf algebras, Amer. J. Math. 91(1969), 75-93.

[M] S. Montgomery, Hopf algebras and their actions on rings, CBMS Lectures 82, AMS. Providence. RI, 1993.

[R] C. M. Ringel, Tame algebras and integral quadratic forms, Lecture Notes in Math. 1099, Springer 1984.

The Fundamental Theorems of Entwined Modules

Wang Zhixi (joint with Hou Bo)

Department of Mathematics, Capital Normal University, Beijing 100037, P.R.China

Email: wangzhx@mail.cnu.edu.cn

Let $\langle C, \Delta, \varepsilon \rangle$ be a coalgebra, $\langle A, \mu, \eta \rangle$ an algebra and a k–linear map $\psi : C \otimes A \to A \otimes C$ such that $\psi(C \otimes \mu) = (\mu \otimes C)(A \otimes \psi)(\psi \otimes A)$, $(A \otimes \Delta)\psi = (\psi \otimes C)(C \otimes \psi)(\Delta \otimes A)$, $\psi(C \otimes \eta) = \eta \otimes C$, $(A \otimes \varepsilon)\psi = \varepsilon \otimes A$. Then C and A are said to be entwined by ψ and the triple (A, C, ψ) is called an entwining structure, denoted by $(A, C)_\psi$. Let $\mathbf{M}_A^C(\psi)$ be the category of right entwined modules such that $A \in \mathbf{M}_A^C(\psi)$ with the action $a \triangleleft b = ab$ and the coaction $\rho^A(a) = a_0 \otimes a_1$. For any $M \in \mathbf{M}_A^C(\psi)$, set $M^{\mathrm{co}C} = \{m \in M \mid \rho^M(m \triangleleft a) = m_0 \triangleleft a_\alpha \otimes m_1^\alpha = m \triangleleft a_0 \otimes a_1, \ \forall a \in A\}$. The main result is the theorem.

Theorem 1. *Let $(A, C)_\psi$ be an entwining structure. If there exists a right C–comodule map $\phi : C \to A$, which is convolution invertible, then for any right entwined module $M \in \mathbf{M}_A^C(\psi)$,*

$$\Theta : M^{\mathrm{co}C} \otimes_B A \to M, \quad m \otimes a \mapsto m \triangleleft a \tag{1}$$

is an isomorphism of entwined modules.

References

1. Sweedler M. E., *Hopf Algebras*, New York: Benjamin, 1969.
2. Doi Y. and Taheuchi M., *Hopf–galois extensions of algebras, the Miyashita–Ulbrich action, and Azamaya algebras*, J. Algebra, **121**(1989), 488–516.
3. Bohm G. and Nill F. and Szlachanyi K., *Weak Hopf algebras 1: Integral theory and C^*–structure*, J. Algebra, **221**(1999), 385–438.
4. Zhang Liangyun and Zhu Shenglin, *Fundamental theorems of weak Doi-Hopf modulees and semisimple weak smash product algebras*, Comm. in Algebra, **32(9)**(2004), 3403–3415.
5. Brzezinski T., *On modules associated to coalgebra–Galois extensions*, J. Algebra, **215**(1999), 290–317.
6. Caenepeel S. and Groot E.De, *Modules over weak entwining structures*, in *New trends in Hopf algbra theory (La Falda, 1999)*, 31–54, Comtemp. Math., **267**, Amer. Math. Soc., Providence, RI, 2000.
7. Brzezinski T. and Hajac P. M., *Coalgebra extensions and algebra coextensions of Galois type*, Comm. in Algebra, **27**(1999), 1347–1367.
8. Brzezinski T. and Majid S., *Coalgebra bundles*, Comm. Math. Phys., **191**(1998), 467–492.
9. Yetter D.N., *Quantum groups and representations of monoidal categories*, Math. Proc. Cambridge Philos. Soc., **108**(1990), 261–290.
10. Doi Y., *Unifying Hopf modules*, J. Algebra, **153**(1992), 373–385.
11. Koppinen M., *Variations on the smash product with application to group–graded rings*, J. Pure Appl. Algebra, **104**(1994), 61–80.
12. Bohm G., *Doi–Hopf modules over weak Hopf algebras*, Comm. in Algebra, **28**(2000), 4687–4698.
13. Doi Y., *On the structure of relative Hopf modules*, Comm. in Algebra, **11(3)**(1983), 243–255.

Derived Categories over Auslander-Gorenstein Rings

Q. -S. Wu

Institute of Math.
Fudan University
Shanghai, 200433, China

Email: qswu@fudan.edu.cn

In late sixties of last century, M. Auslander introduced various homological conditions on noetherian rings, which turned out very important in non-commutative ring theory. Our main object in this talk is Auslander-Gorenstein rings, which can be viewed as a non-commutative analogue of commutative local Gorenstein rings and as a generalization of quasi-Frobenius rings. Besides some well-known examples such as rings of differential operators (Weyl algebras) and universal enveloping algebras of finite dimensional Lie algebras, non-commutative Iwasawa algebras over compact p-adic Lie groups ([5]) and noetherian PI Hopf algebras (quantum groups)([7]) are Auslander-Gorenstein.

Let A-Mod stand for the category of left A-modules, and A-mod for the full subcategory of finitely generated left A-modules. Also, we denote by A^{op}-Mod and A^{op}-mod their right analogues.

If A is a Frobenius algebra, or more generally, a quasi-Frobenius ring, it is well known that the pair of the exact contravariant functors

$$F = \mathrm{Hom}_A(-, {}_AA) : A\text{-mod} \to A^{op}\text{-mod} \quad \text{and} \quad G = \mathrm{Hom}_A(-, A_A) : A^{op}\text{-mod} \to A\text{-mod}$$

gives a duality between A-mod and A^{op}-mod.

This duality was generalized to a so called step-duality for an Auslander-Gorenstein ring with injective dimension d in [1]. In the talk, we will define the grade for bounded complexes in $D^b_{fg}(A\text{-Mod})$. Using the generalized Ischebeck spectral sequence, we will give a step-duality between certain subcategories of $D^b_{fg}(A\text{-Mod})$ and $D^b_{fg}(A^{op}\text{-Mod})$ when A is an Auslander-Gorenstein ring. The step-duality in [1] between A-mod and A^{op}-mod can be induced from the one between $D^b_{fg}(A\text{-Mod})$ and $D^b_{fg}(A^{op}\text{-Mod})$ by using the heart of the t-structures of triangulated categories ([3]).

This is a joint work with G. -B. Zhuang.

References

1. K. Ajitabh, S. P. Smith and J. J. Zhang, Auslander-Gorenstein rings, *Comm. in Algebra* **26** (1998).
2. J. -E. Björk, The Auslander condition on noetherian rings, Séminaire Dubreil-Malliavin 1987-88, *Lecture Notes in Math.* 1404, Springer Verlag, 1989, pp. 137-173.
3. S. I. Gelfand and Yu. I. Manin, *Homological Algebra*, Springer-Verlag, 1999.
4. A. Neeman, *Triangulated Categories*, Annals of Mathematics Studies **148**, Princeton University Press, Princeton, NJ, 2001.
5. O. Venjakob, *On the structure theory of the Iwasawa algebra of a p-adic Lie group*, J. Eur. Math. Soc. **4** (2002), 271-311.
6. C. A. Weibel, *An Introduction to Homological Algebra*, Cambridge University Press, 1994.
7. Q.-S. Wu and J. J. Zhang, *Noetherian PI Hopf algebras are Gorenstein*, Trans. Amer. Math. Soc. **355** (2003), 1043-1066.

On lifting of idempotents in the endomorphism ring of a module

Kunio Yamagata

Tokyo University of Agriculture and Technology

Email: yamagata@cc.tuat.ac.jp

Let M be a module over a ring with an identity. In the case when M is quasi-injective, by a classical theorem of Faith – Utusmi, Lambek and Johnson, the endomorphism ring H of M has the following properties:

(1) an idempotent of $H \operatorname{Rad} H$ can be lifted to H,

(2) $H \operatorname{Rad} H$ is a (von Neumann) regular ring.

Moreover, in this case, $\operatorname{Rad} H$ coincides with the ideal $L(H)$ of endomorphisms with large kernel. Thus, in this talk, we consider the problem: do the properties (1)(2) essentially depend on the Jacobson radical or the ideal $L(M)$? As an application, it will be shown a class of modules M with the properties with respect to $L(M)$ but not $\operatorname{Rad} H$.

References

1. C. Faith; Y. Utumi, *Quasi-injective modules and their endomorphism rings*, Arch. Math. **15** (1964), 166–174.

CLEAN RINGS AND RELATED CLASSES OF RINGS

Angelina Y.M. Chin

University of Malaya

Email: acym@um.edu.my

Let R be a ring with identity. An element in R is said to be clean if it is the sum of a unit and an idempotent. If this representation is unique, then the element is said to be uniquely clean. R is said to be clean (uniquely clean) if all its elements are clean (uniquely clean). In this talk, we discuss some characteristics of clean rings and related rings as well as relations between these rings. We also present some results on group rings whose nontrivial homomorphic images are uniquely clean.

On Simple Connectedness of Minimal Representation-Infinite Algebras

Hailou Yao Guoqiang Han

(College of Applied Science, Beijing University of Technology,
100 Pingleyuan, Chaoyang District, Beijing 100022, P.R. China)
email: yaohl@bjut.edu.cn

Abstract: Let A be a connected minimal representaion-infinite algebra over an algebraically closed field k, in this article we investigate the simple connectedness and strongly simple connectedness of the algebra A, we obtained that A is simply connected if and only if its first Hochschild cohomology group $H^1(A)$ is trivial. We also give other equivalent conditions for simple connectedness of A. Finally, we give a condition for strongly simple connectedness of the algebra A.

Our main results are as follows.

Theorem 2.1 Suppose that an algebra A is minimal representation-infinite with a preprojective component, then A is simply connected if and only if $H^1(A) = 0$.

Theorem 2.2 Suppose that A is a minimal representation-infinite connected algebra, e is a primitive idempotent in A, If $H^1(A) = 0$, then $H^1(A/AeA) = 0$.

Theorem 2.3 If A is a minimal representation-infinite connected algebra, then A is simply connected if and only if $H^1(A) = 0$.

Theorem 3.1 Let A be a minimal representation-infinite algebra, then the following conditons are equivalent.

(a) A is simply connected;

(b)A is triangular, moreover, there exists one presentation $A \cong kQ_A/I_A$ such that each irreducible cycle in Q_A is an irreducible contour, and for each irreducible contour (p, q), we have $p, q \notin I_A$ and $p - \lambda q \in I_A$ for some nonzero $\lambda \in k$.

(c) A is triangular, and for any presentation $A \cong kQ_A/I_A$ all irreducible cycles in Q_A are irreducible contours, and, for each irreducible contour (p, q) we have $p, q \notin I_A$ and $p - \lambda q \in I_A$ for some nonzero $\lambda \in k$.

Theorem 4.1 Let A be a minimal representation-infinite simply connected algebra, then A is strongly simply connected if and only if A/Ae_xA is strongly simply connected for each source or sink x in Q_A where (Q_A, I_A) is any presentation of A and e_x is a primitive idempotent corresponding to x.

Keywords Simple connected Algebras, Minimal Representaion-Infinite Algebras, Hochschild Cohomology Groups.

[3]Supported by the Natural Foundation of China (grant No. 10371036), the Natural Science foundation of Beijing (grant No. 1042001),the fund of Beijing Education Committee(KM200610005024) and the Fundamental research Fund of College of Applied Science in Beijing University of Technology

LOCAL QF RINGS WITH RADICAL CUBED ZERO II

Hiroshi Yoshimura

Graduate School of Science and Engineering
Yamaguchi University
Yamaguchi 753-8512 JAPAN

Email: yoshi@yamaguchi-u.ac.jp

This is a joint work with Isao Kikumasa and Kiyoichi Oshiro.

For a given (not necessarily commutative) field k, how can we construct local quasi-Frobenius (QF) rings Λ satisfying the condition

$$(*) \quad \Lambda/Rad\,\Lambda \cong k \quad \text{and} \quad (Rad\,\Lambda)^3 = 0\,?$$

Local QF rings Λ with $(*)$ which are algebras over a commutative field k can be constructed by factor algebras of free algebras over k and the k-isomorphism classes of those algebras can be determined under some condition ([1]), but the situation in case Λ are not algebras over a commutative field is considerably complicated. In this talk, for a given field k, we give a construction of local QF rings Λ with $(*)$ of split type (i.e., $\Lambda \cap Rad\,\Lambda = 0$) and consider the number of isomorphism classes of those local QF rings.

References

1. I. Kikumasa and H. Yoshimura, *Isomorphism classes of algebras with radical cube zero*, Advances in Ring Theory, Proceedings of the 4-th China-Japan-Korea International Conference, pp. 106–117, World Scientific, 2005.

Pushout Artin-Schelter Regular algebras of Global Dimension Four

Jun Zhang

Abstract: in general k algebra pushouts of AS-regular algebras are not regular. We notice that with some restrictions, if two quadratic regular 3 dimensional algebras A and B both contain the same cubic regular 3 dimensional algebras C, then the k algebra pushout of A and B over C is a quadratic regular algebra of global dimension 4. Under our restrictions, we give a description of all such pushout algebras.

Professor Liu Shaoxue — Life and work

Zhang Yingbo

School of Mathematics, Beijing Normal University, 100875 China

Email: zhangyb@bnu.edu.cn

Professor Liu Shaoxue was born on November 6 of 1929 in Liaoyang, a city of Liaoning Province in Northern China. He graduated from the Department of Mathematics of Beijing Normal University in 1951. In September 1953, he was sent to study in the Mechanics and Mathematics Department of Moscow University of the Formal Soviet Union under the guidance of A.G.Kurosh, a prestigious expert in Algebra, and received Ph.D degree in 1956.

Professor Liu Shaoxue's Ph.D thesis 'On Decomposition of Infinite Algebras' proves that an extension Jordan algebra of a locally finite algebra by a locally finite algebra is still locally finite; and an extension Lie algebra of a locally finite algebra by a locally finite algebra is still locally finite in case it is an algebraic Lie algebra. Thus an Jordan algebra has Livitzki radical, and a solvable Lie algebra is locally finite. His another paper 'On algebras in which every subalgebra is an ideal' published in 1964 gives a complete description of the Hamilton algebras. Professor's textbook book 'Rings and Algebras' published in 1983 for the graduated students is wildly used in China.

During a visit to Tsukuba University in 1988, Professor Liu and Professor H.Tachikawa suggested to hold the China-Japan international Symposium on ring theory, and the first Symposium was held in Guangxi of China.

Professor Liu decided to change the research project to the representation theory of algebras for his Ph.D students at the age of 56. Because of his personality of kindness and humorous, he makes a lot of good friends. Many foreign experts, especially Claus Ringel, gave great helps to establish a research group of representation theory of algebras in China. Now most of Liu's students are working in several best Universities of China as Professors. Since the efforts in the education of graduated students, Professor Liu won an award for excellent teaching of Beijing City in 1991.

Professor Liu is a distinguished Mathematician and mathematical Educationist.

A study on gamma nearrings with derivations

Moharram A. Khan
Centre for Interdisciplinary Research in Basic Sciences (CIRBSc) Jamia Millia Islamia, Jamia Nagar New Delhi-110025, India
Email: moharram_a@yahoo.com

Abstract

In this talk, we first discuss how close the gamma nearrings are to gamma rings and then generalize results from gamma rings to gamma nearring with suitable constraints on gamma rings. Secondly, we introduce the notion of Gamma (a,b)-derivation on prime and semi prime gamma rings as well as gamma prime and semi prime nearring and to obtain some related results using derivations and generalized derivations on prime and semi prime gamma nearring. At last, we close our discussion with some open problems.

Classification of Element Systems over Finite Commutative Groups

Shouchuan Zhang [a], Min Wu [a,b], Hengtai Wang [a]
a: Department of Mathematics, Hunan University, Changsha, 410082
b: Department of Mathematics, Tsinghua University, Beijing, 100084

Classification of Hopf algebras was developed and popularized in the last decade of the twentieth century, which would have applications to a number of other areas of mathematics, aside from its intrinsic algebraic interest. In mathematical physics, Drinfeld's and Jambo's work was to provide solutions to quantum Yang-Baxter equation. In conformal field theory, I. Frenkel and Y. Zhu have shown how to assign a Hopf algebra to any conformal field theory model[8]. In topology, quasi-triangular and ribbon Hopf algebras provide many invariants of knots, links, tangles and 3-manifolds[9, 10, 12, 13]. In operator algebras, Hopf algebras can be assigned as an invariant for certain extensions.

Researches on the classification of Hopf algebra is in the ascendant. N. Andruskiewitsch and H. J. Schneider have obtained interesting result in classification of finite-dimensional pointed Hopf algebras with commutative coradical [1, 2, 3, 4]. More recently, they have also researched this problem in case of non-commutative coradical. Pavel Etingof and Shlomo Gelaki gave the complete and explicit classification of finite-dimensional triangular Hopf algebras over an algebraically closed field k of characteristic 0 [7]. The classification of monomial Hopf algebras, which are a class of co-path Hopf algebras, and simple-pointed sub-Hopf algebras of co-path Hopf algebras were recently obtained in [6] and [11], respectively.

Assume that k is an algebraically closed field of characteristic zero with a primitive $|G|$th root of 1 and G is a finite abelian group. Element systems with characters can be applied to classify quiver Hopf algebras, multiple Taft algebras over G and Nichols algebras in ${}^{FG}_{FG}\mathcal{YD}$ (see [15, Theorem 3, Theorem 4]). In this paper, we obtained the formula computing the number of isomorphic classes of element systems with characters over finite commutative group G.

References

1. N. Andruskiewitsch and H. J. Schneider, Hopf algebras of order p^2 and braided Hopf algebras, J. Alg. **199** (1998), 430–454.
2. N. Andruskiewitsch and H. J. Schneider, Lifting of quantum linear spaces and pointed Hopf algebras of order p^3, J. Alg. **209** (1998), 645–691.
3. N. Andruskiewitsch and H. J. Schneider, Pointed Hopf algebras, new directions in Hopf algebras, edited by S. Montgomery and H. J. Schneider. Cambradge University Press, 2002.
4. N. Andruskiewitsch and H. J. Schneider, Finite quantum groups and Cartan matrices, Adv. Math. **154** (2000), 1–45.
5. Charles W. Curtis, Irving Reiner. Representation theory of finite group and associative algebras, New York:John Wiley&Sons, 1962.
6. X. W. Chen, H. L. Huang, Y. Ye and P. Zhang, Monomial Hopf algebras, J. Alg. **275** (2004), 212–232.
7. Pavel Etingof, Shlomo Gelaki, The classification of finite-dimensional triangular Hopf algebras over an algebraically closed field of characteristic 0, eprint arXiv:math/0202258.
8. I. Frenkel and Y. Zhu, Vertex operator algebras associated to representations of affine and Virasoro
algebras, Duke Math. J. , 66 (1992), 123-168.
9. M. A. Hennings, Hopf algebras and regular isotopy invariants for link diagrams. Pro. Cambridge Phil. Soc 109 (1991), 59-77.
10. L. Kauffman, invariants of links and 3-manifolds via Hopf algebras, in Geometry and Physics, Marcel Dekker Lecture Notes in Pure and Appl. Math. vol 184, 1997, 471-479.
11. F. Van Oystaeyen and P. Zhang, Quiver Hopf algebras, J. Alg. **280** (2004), 577–589.
12. D. E. Radford, On Kauffman's knot invariants arising from finite-dimensional Hopf algebras, in Advances in Hopf Algebras. Marcel Dekker Lecture Notes in Pure and Appl. Math vol 158 (1994), 205-266.
13. N. Yu. Reshetikhin and V. G. Turaev, Ribbon graphs and their invariants derived from quantum groups. Commun. Math. Phys. **127** (1990), 1–26.
14. Michio Suzuki, Group Theory I, New York:Springer-Verlag, 1978.
15. Shouchuan Zhang, Yaozhong Zhang, Huixiang Chen, Classification of PM Quiver Hopf Algebras, math. QA/0410150.

Auslander-Reiten conjecture on Gorenstein rings

Tokuji Araya
Nara University of Education
Email: araya@math.okayama-u.ac.jp

Auslander and Reiten conjectured the generalized Nakayama conjecture (GNC) in [1]

(GNC) For an Artin algebra Λ, every indecomposable injective Λ-module appears as a direct summand in the minimal injective resolution of Λ.

They showed that (GNC) is equivalent to the following;

For an Artin algebra Λ, if M is a finitely generated Λ-module and $\mathrm{Ext}^i_\Lambda(M, M \oplus \Lambda) = 0$ $(\forall i > 0)$, then M is projective.

M. Auslander, S. Ding, and Ø. Solberg widened the context to algebras over commutative local rings [2].

(ARC) For a commutative Noetherian local ring R, if M is a finitely generated R-module and $\mathrm{Ext}^i_R(M, M \oplus R) = 0$ $(\forall i > 0)$, then M is projective.

They showed in [2] that if R is a complete intersection, then R satisfies (ARC). The main theorem of this lecture is following;

Theorem 1. *Let R be a Gorenstein ring whose Krull-dimension d is at least 2. If R_p satisfies* (ARC) *for all $p \in \mathrm{Spec}R$ with $\mathrm{ht}\, p \leq 1$, then R satisfies* (ARC).

References

1. M. Auslander and I. Reiten, *On a generalized version of the Nakayama conjecture*, Proc. Amer. Math. Soc. **52** (1975), 69–74.
2. M. Auslander, S. Ding and Ø.Solberg, *Liftings and Weak Liftings of Modules*, J. Algebra **156** (1993), 273–317.
3. Y. Yoshino, *Cohen-Macaulay modules over Cohen-Macaulay rings*, London Mathematical Society Lecture Note Series, vol. 146, Cambridge University Press, Cambridge, 1990.

On Colocal Pairs

Yoshitomo BABA

In [1] K. R. Fuller characterized indecomposable injective projective modules over artinian rings using i-pairs. In [2] the Fuller's theorems were generalized to indecomposable projective quasi-injective modules and indecomposable quasi-projective injective modules over artinian rings. And in [3] the Fuller's theorems were studied minutely. Further in [4], [5] M. Hoshino and T. Sumioka extended these results to perfect rings. In my talk, we studies the results in [2] from the point of view of [3], [4].

References

1. K. R. Fuller, *On indecomposable injectives over artinian rings*, Pacific J. Math. **29** (1968), 343–354.
2. Y. Baba, *Injectivity of quasi-projective modules, projectivity of quasi-injective modules, and projective cover of injective modules*, J. of Algebra **155** (1993), 415–434.
3. Y. Baba and K. Oshiro, *On a theorem of Fuller*, J. of Algebra **154** (1993), 86–94.
4. M. Hoshino and T. Sumioka, *Injective pairs in perfect rings*, Osaka J. Math. **35** (1998), 501–508.
5. M. Hoshino and T. Sumioka, *Colocal pairs in perfect rings*, Osaka J. Math. **36** (1999), 587–603.

Department of Mathematics
Osaka Kyoiku University
Kashiwara, Osaka 582-8582 JAPAN
Email: ybaba@cc.osaka-kyoiku.ac.jp

Approximations of algebras

Vlastimil Dlab (Carleton University, Ottawa, Canada)

Expanding the idea of standardization introduced by Dlab–Ringel earlier, Ágoston–Dlab–Lukács have introduced two operators, Σ and Ω, acting on all finite dimensional associative algebras. These operators define the Δ- and $\bar{\Delta}$-equivalences whose equivalence classes are in bijection with the standardly stratified algebras, and partition all algebras into rooted trees of finite length labelled by properly stratified algebras. The linear representations of the semigroups $athcalS_n = \langle a, b \,|\, a^2 = a,\ b^2 = b,\ (ab)^{n-1}a = (ab)^{n-1} \rangle$ acting via Σ and Ω on the algebras with n irreducible representations can be fully described: There are just $2(2n-1)$ indecomposable representations of $\mathcal{S}_n$ and all are uniserial.