

On balanced Auslander–Dlab–Ringel algebras

Takahide Adachi, Aaron Chan and Mayu Tsukamoto

Osaka Prefecture University, Nagoya University and Yamaguchi University

Email: adachi@mi.s.osakafu-u.ac.jp, aaron.kychan@gmail.com, tsukamot@yamaguchi-u.ac.jp

Many algebras that appear in representation theory and algebraic geometry often fall into the following two classes at the same time. One is the class of Koszul algebras introduced by Priddy [6], and the other is the class of quasi-hereditary algebras introduced by Cline, Parshall, and Scott [3]. These two classes of algebras exhibit two different forms of dualities - the Koszul duality [2] and the Ringel duality [7]. One sufficient condition for the two dualities of a Koszul quasi-hereditary algebra to commute is given by Mazorchuk [5], and an algebra that satisfies such a condition is called a *balanced algebra* nowadays.

In ring theory, one interesting class of quasi-hereditary algebras is given by the *Auslander–Dlab–Ringel algebras*. The class was first studied by Auslander in [1] and subsequently by Dlab and Ringel in [4] to give a construction of algebras with finite global dimensions.

In this talk, we will explain in slightly more details about the aforementioned classes of algebras, and will give a sufficient condition for Auslander–Dlab–Ringel algebras to be balanced.

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