Unique Factorization property of non-UFDs

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A unique factorization domain (UFD) is an integral domain in which each nonzero nonunit can be written uniquely as a finite product of irreducible elements, and in this case, each irreducible element is a prime element. Let D be an integral domain and t be the so-called t-operation on D. As in [1], D is called a *weakly factorial domain* (WFD) if each nonzero nonunit of D can be written as a finite product of primary elements. Two primary elements a, b of D will be said to be *distinct* if $\sqrt{aD} \neq \sqrt{bD}$. Let D be a WFD, and note that if

$$x = x_1 \cdots x_n = a_1 \cdots a_m$$

are two finite products of distinct primary elements of D, then $n = m$ and $x_i D = a_i D$ for $i = 1, \ldots, n$ by reordering if necessary. Hence, each nonzero nonunit of a WFD can be written uniquely as a finite product of distinct primary elements.

Following [5], we say that a nonzero nonunit $x \in D$ is *homogeneous* if x is contained in a unique maximal t-ideal of D. Then, in this talk, we will say that D is a *homogeneous factorization domain* (HoFD) if each nonzero nonunit of D can be written as a finite product of pairwise t-comaximal homogeneous elements. The notion of HoFDs was first introduced in [2], where the authors called an HoFD a t-pure domain. Clearly, primary elements are homogeneous. Thus, the notion of HoFDs is a natural generalization of WFDs, and we have the following implications:

$$\text{UFD} \Rightarrow \text{Weakly factorial GCD-domain} \Rightarrow \text{WFD} \Rightarrow \text{HoFD}.$$

In this talk, we first show that the expression of an element of an HoFD is unique as in the case of WFDs. Then, among other things, we show that (1) a PvMD D is an HoFD if and only if $D[X]$, the polynomial ring over D, is an HoFD and (2) D is a weakly Matlis GCD-domain if and only if $D[X]$ is an HoFD with t-Spec$(D[X])$ treed. We also study the HoFD property of $A + XB[X]$ constructions, pullbacks, and semigroup rings. This talk is based on [3, 4].

References

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