TRIANGULARIZATION OF MATRICES AND POLYNOMIAL MAPS

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In this talk, we present conditions for a set of matrices satisfying a permutation identity to be simultaneously triangularizable. A set S of $n \times n$ matrices over an algebraically closed field is called σ-permutable if S satisfies a permutation identity:

$$A_1 A_2 \cdots A_r = A_{\sigma(1)} A_{\sigma(2)} \cdots A_{\sigma(r)}, \text{ for all } A_1, A_2, \ldots, A_r \in S,$$

for some nonidentity permutation $\sigma \in S_r$. We prove that S is triangularizable if S is σ-permutable for some nonidentity permutation $\sigma \in S_r$ with $\Delta(\sigma) = 1$, where $\Delta(\sigma) = \gcd\{|\sigma(i) - i| \mid 1 \leq i \leq r\}$. As applications, we generalize the Radjavi's result on triangularization of matrices with permutable trace and results of Yan and Tang on linear triangularization of polynomial maps. Joint work with Yueyue Li and Yan Tian.

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