Ideal Approximation Theory

Xianhui Fu

Northeast Normal University

Email: fuxh476@nenu.edu.cn

In the general setting of an exact category, the idea of the classical theory of approximations is to select a suitable subcategory to approximate arbitrary objects by the ones from this subcategory. While the idea of ideal approximation theory is to give morphisms and ideals of categories equal status as objects and subcategories.

Ideal Approximation Theory for exact categories is devoted to the study of precovering ideals, and the dual notion of preenveloping ideals, with emphasis on the notion of a special precovering (respectively, special preenveloping) ideal. A main technical tool to develop this theory is the mono-epi exact structure on the category of morphisms over an exact category which is introduced and analyzed in [5]. Along the way, several important results, such as ideal versions of Salce's Lemma [4, 5], Wakamatsu's Lemma [5], Eklof's Lemma [3] and Bongartz's Lemma [6] which are fundamental tools in classical theory, and an analogy of Ghost's Lemma [5] in triangulated categories, have been derived. The ideal approximation theory has been used to the study of ring and representation theory. For examples, (1) it is used to give an affirmative answer to an question asked by Benson and Gnacadja concerning sharp upper bounds for the phantom number of a finite group [5]; and (2) it is used to prove a partial dual of a result of Xu [10]: if R is a right coherent ring, and the class of pure projective right R-modules is closed under extensions, then every FP-projective module is pure projective [3].

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