

The Extension Dimension of Abelian Categories

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Let \mathcal{A} be an abelian category having enough projective objects and enough injective objects. We prove that if \mathcal{A} admits an additive generating object, then the extension dimension and the weak resolution dimension of \mathcal{A} are identical, and they are at most the representation dimension of \mathcal{A} minus two. By using it, for a right Morita ring Λ , we establish the relation between the extension dimension of the category $\text{mod } \Lambda$ of finitely generated right Λ -modules and the representation dimension as well as the global dimension of Λ . In particular, we give an upper bound for the extension dimension of $\text{mod } \Lambda$ in terms of the projective dimension of certain class of simple right Λ -modules and the radical layer length of Λ . It is a joint work with Junling Zheng and Xin Ma.

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