The characteristic variety of an elliptic algebra

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This is based on joint work with Alex Chirvasitu and S. Paul Smith [2, 3, 4].

There have been several attempts to define the space associated to a noncommutative ring. For a graded algebra over a field $\Bbbk$, one established approach is to look at $\operatorname{QGr} A$, the category of graded A-modules modulo the full subcategory consisting of torsion modules. When the algebra A is commutative and finitely generated in degree one, the category $\operatorname{QGr} A$ is equivalent to the category of quasi-coherent sheaves on $\operatorname{Proj} A$. Thus, for a noncommutative algebra A, we may consider $\operatorname{QGr} A$ as the category of "quasi-coherent sheaves" on the associated "noncommutative projective scheme".

To understand $\operatorname{QGr} A$, the first things one should look at are objects coming from point modules:

Definition 1. Let A be a nonnegatively graded $\Bbbk$-algebra that is finitely generated in degree one. A graded A-module M is called a *point module* if it is cyclic and satisfies

$$\dim_{\Bbbk} M_i = \begin{cases} 1 & \text{if } i \geq 0, \\ 0 & \text{if } i < 0. \end{cases}$$

Artin-Tate-Van den Bergh [1] showed that the point modules are parametrized by a space called the *point scheme*, which is defined as an inverse limit of schemes. Each point module defines a simple object in $\operatorname{QGr} A$. Point modules have played a crucial role in the study of Artin-Schelter regular algebras.

In 1989, Feigin and Odesskii introduced a family of algebras $Q_{n,k}(E,\tau)$ parametrized by an elliptic curve E over $\mathbb{C}$, a closed point $\tau \in E$, and coprime integers $n > k \geq 1$. This is a huge generalization of higher dimensional Sklyanin algebras, and provides flat deformations of polynomial algebras when τ varies.

The aim of this talk is to describe the major component of the point scheme of the elliptic algebra $Q_{n,k}(E,\tau)$, which we call the *characteristic variety*. For a higher dimensional Sklyanin algebra, the characteristic variety is the elliptic curve E and it is the only non-discrete irreducible component of the point scheme. For other elliptic algebras, the characteristic variety depends on the negative continued fraction of the rational number n/k and is realized as the quotient of a product of copies of E by a finite group.

References

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