

# CHARACTERIZATIONS OF RADICALS IN SKEW POLYNOMIAL AND SKEW LAURENT POLYNOMIAL RINGS

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Throughout this paper, all rings are assumed to be associative with 1. We let  $R$  denote an arbitrary ring, and let  $\sigma$  be any automorphism of  $R$ . By  $R[x; \sigma]$  ( $R[x, x^{-1}; \sigma]$ ) we mean the skew (Laurent) polynomial ring over  $R$ , subject to the (left) skewing condition  $xr = \sigma(r)x$  ( $x^{-1}r = \sigma^{-1}(r)x^{-1}$ ) for each  $r \in R$ .

Pearson and Stephenson [4] characterized the prime radical of a skew polynomial ring as  $P(R[x; \sigma]) = (P(R) \cap P_\sigma(R)) + P_\sigma(R)xR[x; \sigma]$  where  $P_\sigma(R)$  is the intersection of all strongly  $\sigma$ -prime ideals of  $R$ , which is the so-called  $\sigma$ -prime radical of  $R$ . The prime radical of a skew Laurent polynomial ring was investigated by Cheon et al. [1]. They showed that  $P(R[x, x^{-1}; \sigma]) = P_{(\sigma, \sigma^{-1})}(R)[x, x^{-1}; \sigma]$ , where  $P_{(\sigma, \sigma^{-1})}(R)$  denotes the intersection of all  $(\sigma, \sigma^{-1})$ -prime ideals of  $R$ .

On the other hand, Ferrero [2] characterized the generalized nilradical of skew polynomial and skew Laurent polynomial rings as  $N(R[x; \sigma]) = N(R) + N_\sigma(R)xR[x; \sigma]$  and  $N(R[x, x^{-1}; \sigma]) = N_\sigma(R)[x, x^{-1}; \sigma]$ , where  $N_\sigma(R)$  is the intersection of all the  $\sigma$ -ideals of  $R$  which are also completely prime ideals. Ferrero also showed that  $s(R[x; \sigma]) = s(R) + s_\sigma(R)xR[x; \sigma]$ , where  $s(R)$  ( $s_\sigma(R)$ ) denotes the  $(\sigma)$ -strongly prime radical of  $R$  in [3].

For the continuation of the study of radicals of skew (Laurent) polynomial rings, in this paper, we conduct the study showing that radicals between the prime radical and the generalized nilradical have similar forms to the above formulas.

In particular, we first give a complete description of the Levitzki radical of a skew (Laurent) polynomial ring through the prime ideals and skewed prime ideals in the base ring. We next provide formulas similar to the above expression of radicals for the strongly prime radical and the uniformly strongly prime radical of these rings.

## REFERENCES

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