AS-regularity of geometric algebras of plane cubic curves

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A geometric algebra $A = \mathcal{A}(E, \sigma)$ introduced by Mori [4] is a quadratic algebra which determines and is determined by the pair (E, σ) where E is a projective scheme and $\sigma \in \operatorname{Aut} E$. In noncommutative algebraic geometry, AS-regular algebras are the most important class of algebras to study, and Artin-Tate-Van den Bergh [1] showed that every 3-dimensional quadratic AS-regular algebra is a geometric algebra where E is $\mathbb{P}^2$ or a cubic curve in $\mathbb{P}^2$. In this talk, we study its converse.

Suppose that E is a cubic curve in $\mathbb{P}^2$. If E is singular, then a geometric algebra $A = \mathcal{A}(E, \sigma)$ is AS-regular for almost all $\sigma \in \operatorname{Aut} E$ by [3]. This is not the case if E is smooth. If E is smooth, then we choose a suitable $\tau \in \operatorname{Aut} E$ of finite order as in [2] so that every $\sigma \in \operatorname{Aut} E$ can be written as $\sigma = \sigma_p \tau^i$ where $p \in E$, $i \in \mathbb{Z}_{|\tau|}$ and $|\tau|$ is the order of τ. One of the main results of [3] is that we characterize AS-regularity of a geometric algebra $A = \mathcal{A}(E, \sigma_p \tau^i)$ in terms of the pair (p, i). It turns out that if $i = 0$, then A is always an AS-regular algebra (called a Sklyanin algebra). On the other hand, if $i \neq 0$, then A is hardly ever AS-regular.

If time permits, we will explain how to check AS-regularity using a twist of a superpotential in the sense of [5].

References

1. M. Artin, J. Tate and M. Van den Bergh, *Some algebras associated to automorphisms of elliptic curves*, The Grothendieck Festschrift, vol. 1, Progress in Mathematics vol. 86 (Brikhäuser, Basel, 1990) 33–85.
2. A. Itaba and M. Matsuno, *Defining relations of* 3*-dimensional quadratic AS-regular algebras*, to appear in Mathematical Journal of Okayama Univ., (arXiv:1806.04940).
3. A. Itaba and M. Matsuno, *Twisted superpotentials of* 3*-dimensional quadratic AS-regular algebras*, arXiv:1905.02502.
4. I. Mori, *Non commutative projective schemes and point schemes*, Algebras, Rings and Their Representations, World Sci. Hackensack, N. J., (2006), 215–239.
5. I. Mori and S. P. Smith, *m-Koszul Artin-Schelter regular algebras*, J. Algebra. **446** (2016), 373–399.

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