Efficient generation of ideals in core subalgebras of the polynomial ring $k[t]$ over a field k

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This is a joint work [1] with Naoki Endo, Shiro Goto, and Yuki Yamamoto.

In this talk, we find efficient systems of generators for ideals in certain subalgebras R of the polynomial ring $S = k[t]$ with one indeterminate t over a field k. The class of subalgebras which we explore in this talk naturally includes the semigroup rings $k[H]$ of numerical semigroups H.

Let R be a k-subalgebra of S. We say that R is a core of S, if $t^{c_0}S \subseteq R$ for some integer $c_0 > 0$. If R is a core of S, then

$$k[t^{c_0}, t^{c_0+1}, \ldots, t^{2c_0-1}] \subseteq R \subseteq S,$$

and a given k-subalgebra R of S is a core of S if and only if $R \supseteq k[H]$ for some numerical semigroup H. Therefore, once R is a core of S, R is a finitely generated k-algebra of dimension one, and S is a birational module-finite extension of R with $t^{c_0}S \subseteq R : S$. Typical examples of cores are, of course, the semigroup rings $k[H]$ of numerical semigroups H. However, cores of S do not necessarily arise as semigroup rings for some numerical semigroups.

Let R be a core k-subalgebra of S. Take $f \in R$ such that $f(0) = 1$. We consider the ideal $I = fS \cap R$.

Problem 1. (1) Determine the minimal number of generators of I.

(2) Find a system of generators of I.

The problem (1) is already known by classical results given by O. Forster [2] and R. G. Swan [3]. In this talk, we give a method to solve the problem (2) and we recover the known results on the problem (1) in our situation.

References

1. N. Endo, S. Goto, N. Matsuoka, and Y. Yamamoto, *Efficient generation of ideals in core subalgebras of the polynomial ring $k[t]$ over a field k*, arXiv:1904.11708.
2. O. Forster, *Über die Anzahl der Erzeugenden eines Ideals in einem Noetherschen Ring*, Math. Z., **84** (1964), 80–87.
3. R. G. Swan, *The number of generators of a module*, Math. Z., **102** (1967), 318–322.

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