On a cubical generalization of preprojective algebras

Hiroyuki Minamoto

Osaka Prefecture Unversity

Email: minamoto@mi.s.osakafu-u.ac.jp

In this abstract K denotes a field of char $K = 0$ and Q denotes a finite acyclic quiver.

Recall that the preprojective algebra $\Pi(Q) = K\overline{Q}/(\rho)$ is the path algebra $K\overline{Q}$ of the double quiver $\overline{Q}$ of Q with the mesh relation $\rho = \sum_{\alpha \in Q_1} \alpha\alpha^* - \alpha^*\alpha$. It is an important mathematical object having rich representation theory and plenty of applications. In this joint work with M. Herschend, we study a cubical generalization $\Lambda = \Lambda(Q) := K\overline{Q}/([a, \rho] \mid a \in \overline{Q}_1)$ where $[-, +]$ is the commutator. We note that our algebra Λ is a special case of algebras $\Lambda_{\lambda,\mu}$ introduced by Etingof-Rains [4], which is a special case of algebras Λ_Φ introduced by Cachazo-Katz-Vafa [2]. However, our algebra Λ of very special case has intriguing properties, among other things it provides the universal Auslander-Reiten triangle.

We may equip Λ with a grading by setting $\deg \alpha = 0, \deg \alpha^* := 1$ for $\alpha \in Q_1$. Λ_1 We introduce an algebra to be $A = A(Q) := \begin{pmatrix} KQ & \Lambda_1 \\ 0 & KQ \end{pmatrix}$ where Λ_1 is the degree 1-part of Λ.

We note that Etingof-Latour-Rains [3] showed that if Q is a ADE-quiver, then Λ is symmetric. We summarize existing results on the algebras Λ and A.

Theorem 1. *(1) Λ is finite dimensional if and only if Q is an ADE-quiver if and only if A is 2-representation finite algebra. Assume that this is the case. Then Λ is a stably 3-Calabi-Yau symmetric algebra. Moreover we have an isomorphism $\Lambda \cong \bigoplus_{M \in \operatorname{ind} KQ} M \otimes_K M$ of KQ-modules.*

(2) Λ is infinite dimensional if and only if Q is not an ADE-quiver A is 2-representation infinite algebra. Assume this is the case. Then Λ is graded coherent and 3-Calabi-Yau.

(3) In any case, the 2-quasi-Veronese algebra of Λ is isomorphic to the 3-preprojective algebra of A. 2-APR-tilting operations on A are compatible with reflections of quiver Q.

Let Q be an ADE-quiver, $\widehat{Q}$ the extended one and $G < \mathrm{SL}(2)$ the corresponding finite subgroup. Then $\Lambda(\widehat{Q})$ is Morita equivalent to the skew group algebra $H * G$ where $H = K\langle x, y\rangle/([x, [x, y]], [y, [x, y]])$ is the Heisenberg algebra in two variables. The fixed subalgebra H^G is Gorenstein. Applying a result by Amiot-Iyama-Reiten [1], we obtain our version of algebraic McKay correspondences giving descriptions of the stable categories of CM-modules over H^G.

Theorem 2. *We have the following two equivalences of triangulated categories: $\underline{\mathrm{CM}}^{\mathbf{Z}} H^G \simeq \mathsf{D}^{\mathrm{b}}(A(Q))$, $\underline{\mathrm{CM}} H^G \simeq \mathsf{C}_2(A(Q))$ where C_2 denotes the 2-cluster category.*

References

1. Amiot, Claire; Iyama, Osamu; Reiten, Idun Stable categories of Cohen-Macaulay modules and cluster categories. Amer. J. Math. 137 (2015), no. 3, 813-857.
2. F. Cachazo, S. Katz and C. Vafa, Geometric Transition and $\mathcal{N} = 1$ Quiver Theories.
3. Etingof, Pavel; Latour, Frederic; Rains, Eric, On central extensions of preprojective algebras. J. Algebra 313 (2007), no. 1, 165-175.
4. Etingof, Pavel; Rains, Eric Central extensions of preprojective algebras, the quantum Heisenberg algebra, and 2-dimensional complex reflection groups. J. Algebra 299 (2006), no. 2, 570-588.

2010 *Mathematics Subject Classification.* 16G20, 16G70.