Noncommutative Matrix Factorizations and Knörrer's Periodicity Theorem

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In commutative ring theory, Knörrer's periodicity theorem is a powerful tool to study Cohen-Macaulay representation theory over hypersurfaces, and matrix factorizations are essential ingredients to prove the theorem. In order to study noncommutative quadric hypersurfaces, which are major objects of study in noncommutative algebraic geometry, we introduce a notion of noncommutative matrix factorization and prove the following noncommutative graded versions of Eisenbud's theorem [1] and Knörrer's periodicity theorem [2].

Theorem 1 ([3])**.** *If S is a graded quotient algebra of a noetherian AS-regular alegebra, $f \in S_d$ is a homogeneous regular normal element of degree $d > 0$, and $A = S/(f)$, then*

$$\mathrm{NMF}_S^{\mathbb{Z}}(f)/\operatorname{add}\{(1,f)\} \cong \mathrm{TR}_S^{\mathbb{Z}}(A)$$

$$\underline{\mathrm{NMF}}_S^{\mathbb{Z}}(f) := \mathrm{NMF}_S^{\mathbb{Z}}(f)/\operatorname{add}\{(1,f),(f,1)\} \cong \mathrm{TR}_S^{\mathbb{Z}}(A)/\operatorname{add}\{A\} =: \underline{\mathrm{TR}}_S^{\mathbb{Z}}(A)$$

where $\mathrm{NMF}_S^{\mathbb{Z}}(f)$ is the category of noncommutative graded matrix factorizations of f over S, $\mathrm{TR}^{\mathbb{Z}}(A)$ is the category of finitely generated graded totally reflexive modules over A, and $\mathrm{TR}_S^{\mathbb{Z}}(A) := \{M \in \mathrm{TR}^{\mathbb{Z}}(A) \mid \mathrm{pd}_S(M) < \infty\}$.

Theorem 2 ([4])**.** *Assume that the base field is algebraically closed of characteristic not 2. Let S be a noetherian AS-regular algebra and $f \in S_{2e}$ a homogeneous regular normal element of even degree $2e > 0$. If there exists a graded algebra automorphism σ of S such that $af = f\sigma^2(a)$ for every $a \in S$, then*

$$\underline{\mathrm{TR}}^{\mathbb{Z}}(S/(f)) \cong \underline{\mathrm{NMF}}_S^{\mathbb{Z}}(f) \cong \underline{\mathrm{NMF}}_{S[u;\sigma][v;\sigma]}^{\mathbb{Z}}(f+u^2+v^2) \cong \underline{\mathrm{TR}}^{\mathbb{Z}}(S[u;\sigma][v;\sigma]/(f+u^2+v^2))$$

where $S[u;\sigma][v;\sigma]$ is the Ore extension of S by σ with $\deg u = \deg v = e$.

If time permits, we discuss applications to noncommutative quadric hypersurfaces.

References

1. D. Eisenbud, *Homological algebra on a complete intersection, with an application to group representations*, Trans. Amer. Math. Soc. **260** (1980), 1, 35–64.
2. H. Knörrer, *Cohen-Macaulay modules on hypersurface singularities I*, Invent. Math. **88** (1987), 153–164.
3. I. Mori and K. Ueyama, *Noncommutative matrix factorizations with an application to skew exterior algebras*, preprint.
4. I. Mori and K. Ueyama, *Noncommutative Knörrer's periodicity theorem and noncommutative quadric hypersurfaces*, preprint.

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