On Thompson's group F and its group algebra

Tsunekazu Nishinaka
University of Hyogo
Email: nishinaka@econ.u-hyogo.ac.jp

We have studied about group algebras of non-noetherian groups and showed that they are often primitive if base groups have non-abelian free subgroups. Our main method was two edge-colored graph theory. In general our method using these graphs seems to be effective for a group algebra of a group with a non-abelian free subgroup. But there exist some non-Noetherian groups with no non-abelian free subgroups such as Thompson's group F. In this talk, we first introduce an application of (undirected) two edge-colored graphs to group algebras of non-Noetherian groups and then improve our graph theory in order to be able to investigate group algebras of Thompson's group F. Finally, we introduce an application our graph theory to a problem on group algebras of Thompson's group F.

Definition 1 (Thompson's group F)**.** We define Thompson's group F as the group (under composition) of thosehomeomorphisms of the interval $[0, 1]$, which satisfy the following conditions:

1. they are piecewise linear and orientation-preserving,
2. in the pieces where the maps are linear, the slope is always a power of 2, and
3. the breakpoints are dyadic, i.e., they belong to the set $D \times D$, where $D = [0, 2] \cap \mathbb{Z}[\frac{1}{2}]$.

Thompson's group F has a following presentation:

$$\langle x_0, x_1, x_2, \cdots x_n, \cdots , \mid x_i^{-1} x_j x_i = x_{j+1}, \text{ for } i < j \rangle.$$

In this talk, we consider the following property (P) on the group algebra KG of a group G over a field K:

(P) There exist elements $a, b \in KG \setminus \{0\}$ such that $ax + by \neq 0$ for any $x, y \in KG \setminus \{0\}$.

We can see that many group algebras of non-noetherian groups satisfy the property (P) but have not known it on group algebras of Thompson's group F yet. We introduce a new approach to the problem.

References

1. J. Alexander and T. Nishinaka, *Non-noetherian groups and primitivity of their group algebras*, J. Algebra **473** (2017), 221–246.
2. J. W. Cannon, W. J. Floyd, and W. R.Parry, *Introductory notes on Richard Thompson's groups*, Enseign. Math. **42(2)**(1996), 215–256.

2010 *Mathematics Subject Classification.* 16S34, 20C07, 20E25, 20E06, 05C15.