Constructions of rejective chains

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Let $\mathcal{C}$ be a Krull–Schmidt category. In [2], a chain $\mathcal{C} = \mathcal{C}_0 \supset \mathcal{C}_1 \supset \cdots \supset \mathcal{C}_n = 0$ of subcategories of $\mathcal{C}$ is called a *total right rejective chain* if the following conditions hold:

(a) $\mathcal{C}_i$ is a *right rejective subcategory* of $\mathcal{C}$;

(b) the Jacobson radical of the factor category $\mathcal{C}_{i-1}/[\mathcal{C}_i]$ is zero.

In this talk, we give various examples of total right rejective chains. It is known that total right rejective chains are deeply related right-strongly quasi-hereditary algebras which are a special class of quasi-hereditary algebras introduced by Ringel [3].

Proposition 1 ([4, Theorem 3.22])**.** *Let A be an artin algebra. Then A is right-strongly quasi-hereditary if and only if the category* $\mathsf{proj}A$ *has a total right rejective chain*

The following theorem is one of main results of this talk. One is a refinement of [1, Proposition 1.6], and the other is a refinement of [1, Proposition 2.3] and [5, Proposition 3.1].

Theorem 2. *Let A be an artin algebra. If A is a locally hereditary algebra or a Nakayama algebra with heredity ideal, then the category* $\mathsf{proj}A$ *admits a total right rejective chain. In particular, the following statements hold.*

(1) *If A is a locally hereditary algebra, then A is right-strongly quasi-hereditary.*

(2) *Let A be a Nakayama algebra. Then A is a right-strongly quasi-hereditary algebra if and only if there exists a heredity ideal of A.*

Next, we study Δ-good module category $\mathcal{F}(\Delta)$ using rejective chains. In [6], it is shown that if the category $\mathcal{F}(\Delta)$ over a quasi-hereditary algebra A has an additive generator M, then the endomorphism algebra $\mathrm{End}_A(M)$ is quasi-hereditary. Motivated by this result, we give the following proposition.

Proposition 3. *Let A be a quasi-hereditary algebra and $\mathcal{F}(\Delta)$ the Δ-good module category. Assume that $\mathcal{F}(\Delta)$ has an additive generator M and multiplicity-free. Then the category $\mathcal{F}(\Delta)$ admits a total right rejective chain. In particular, the endomorphism algebra $\mathrm{End}_A(M)$ is a right-strongly quasi-hereditary algebra.*

References

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2010 *Mathematics Subject Classification.* Primary 16G10; Secondary 18A40.