BAER-KAPLANSKY CLASSES IN CATEGORIES

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The classical Baer-Kaplansky theorem states that any two torsion abelian groups having isomorphic endomorphism rings are isomorphic. An interesting topic of research has been to find other classes of abelian groups, and more generally, of modules, for which a Baer-Kaplansky-type theorem is still true. Such classes have been called *Baer-Kaplanksy classes* by Ivanov and Vámos [1].

Let $\mathcal{C}$ be a preadditive category and let $\mathcal{M}$ be a class of objects of $\mathcal{C}$. Following Ivanov and Vámos [1], $\mathcal{M}$ is called a *Baer-Kaplansky class* if for any two objects M and N of $\mathcal{M}$ such that $\mathrm{End}_{\mathcal{C}}(M) \cong \mathrm{End}_{\mathcal{C}}(N)$ (as rings), one has $M \cong N$. In this work we use functor categories techniques in order to relate Baer-Kaplansky classes in Grothendieck categories to Baer-Kaplansky classes in finitely accessible additive categories (in particular, the category of torsion-free abelian groups), exactly definable additive categories (in particular, the category of divisible abelian groups) and categories $\sigma[M]$ (in particular, the category of comodules over a coalgebra over a field).

Theorem 1. *Let $\mathcal{C}$ be a finitely accessible (an exactly definable) additive category. Let X and Y be objects of $\mathcal{C}$ such that X has a direct sum decomposition into indecomposable subobjects and there exists an IP-isomorphism $\Phi : \mathrm{End}_{\mathcal{C}}(X) \to \mathrm{End}_{\mathcal{C}}(Y)$. If one of the following conditions holds:*

1. *Y/X is pure-projective;*
2. *X is pure-injective;*

then X and Y are isomorphic.

Theorem 2. *Let $\mathcal{C}$ be a Krull-Schmidt finitely accessible (pure semisimple exactly definable) additive category. Then the class of finitely presented objects of $\mathcal{C}$ is Baer-Kaplansky if and only if the class of finitely presented indecomposable objects of $\mathcal{C}$ is Baer-Kaplansky.*

Theorem 3. *Let R be a ring with identity and let M be a pure semisimple left R-module. Then the class of finitely presented objects of $\sigma[M]$ is Baer-Kaplansky if and only if the class of (finitely presented) indecomposable objects of $\sigma[M]$ is Baer-Kaplansky.*

References

1. G. Ivanov and P. Vámos, *A characterization of FGC rings*, Rocky Mountain J. Math. **32** (2002), 1485–1492.

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