Density of g-vector cones from triangulated surfaces

Toshiya Yurikusa

Nagoya University

Email: m15049q@math.nagoya-u.ac.jp

This talk is based on [2]. Let A be a finite dimensional algebra over a field k. Adachi-Iyama-Reiten introduced τ-tilting theory which is generalization of tilting theory from the viewpoint of mutation. We denote

- $\mathrm{s}\tau\text{-tilt}\, A = \{$isomorphism classes of basic support τ-tilting A-modules$\}$,
- $\mathrm{s}\tau\text{-tilt}^+ A \subseteq \mathrm{s}\tau\text{-tilt}\, A$ consists of mutation equivalence classes containing A,
- $\mathrm{s}\tau\text{-tilt}^- A \subseteq \mathrm{s}\tau\text{-tilt}\, A$ ——————————————————————— 0.

Problem. $\mathrm{s}\tau\text{-tilt}\, A \setminus (\mathrm{s}\tau\text{-tilt}^+ A \cup \mathrm{s}\tau\text{-tilt}^- A) = ?$.

In this talk, we consider the Jacobian algebras defined from triangulated surfaces.

- (S, M) : a connected compact oriented Riemann surface with marked points.
- Q_T : a quiver associated with a triangulation T of (S, M).
- W : a non-degenerate potential of Q_T such that the associated Jacobian algebra $J = J(Q_T, W)$ is finite dimensional.

Remark 1. For the cluster algebra $\mathcal{A}(Q_T)$ associated with Q_T, there are bijections

$$\mathrm{s}\tau\text{-tilt}^+ J \leftrightarrow \{\text{clusters in } \mathcal{A}(Q_T)\} \leftrightarrow \{\text{tagged triangulations of } (S, M)\},$$

where if (S, M) is a closed surface with exactly one puncture, then tags are plain.

We give an answer of Problem for $A = J$.

Theorem 2. *We have* $\mathrm{s}\tau\text{-tilt}\, J = \mathrm{s}\tau\text{-tilt}^+ J \cup \mathrm{s}\tau\text{-tilt}^- J$. *More precisely, if* (S, M) *is a closed surface with exactly one puncture, then* $\mathrm{s}\tau\text{-tilt}\, J = \mathrm{s}\tau\text{-tilt}^+ J \sqcup \mathrm{s}\tau\text{-tilt}^- J$*; otherwise,* $\mathrm{s}\tau\text{-tilt}\, J = \mathrm{s}\tau\text{-tilt}^+ J = \mathrm{s}\tau\text{-tilt}^- J$.

The key ingredient to prove Theorem 2 is an invariant, called g-vector cone, of τ-tilting modules. The *g-vector cone* of a τ-tilting module M is a cone $C_J(M)$ in $K_0(J) \otimes_{\mathbb{Z}} \mathbb{R}$, where $K_0(J)$ is the Grothendieck group of J. They have the following property.

Theorem 3. [1, Theorem 2.4] *Any g-vector cone is of full-dimensional.*

The following is the main result in [2].

Theorem 4. *We have*

$$\overline{\bigcup_{M \in \mathrm{s}\tau\text{-tilt}^+ J \cup \mathrm{s}\tau\text{-tilt}^- J} C_J(M)} = K_0(J) \otimes_{\mathbb{Z}} \mathbb{R}\,.$$

Theorem 2 immediately follows from Theorems 3 and 4.

References

1. R. Dehy and B. Keller, *On the combinatorics of rigid objects in 2–Calabi–Yau categories*, Int. Math. Res. Not. Vol. 2008 (2008) Art. ID rnn029.
2. T. Yurikusa, *Density of g-vector cones from triangulated surfaces*, arXiv:1904.12479.

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