

# GORENSTEIN-PROJECTIVE AND SEMI-GORENSTEIN-PROJECTIVE MODULES

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ABSTRACT. Let  $A$  be an artin algebra. An  $A$ -module  $M$  will be said to be semi-Gorenstein-projective provided that  $\text{Ext}^i(M, A) = 0$  for all  $i \geq 1$ . All Gorenstein-projective modules are semi-Gorenstein-projective and only few and quite complicated examples of semi-Gorenstein-projective modules which are not Gorenstein-projective have been known. One of the aims of this talk is to provide conditions on  $A$  such that all semi-Gorenstein-projective left modules are Gorenstein-projective (such an algebra is called left weakly Gorenstein). In particular, in case there are only finitely many isomorphism classes of indecomposable left modules which are both semi-Gorenstein-projective and torsionless, then  $A$  is left weakly Gorenstein. This combines the thoughts of Y. Yoshino and R. Marczinzik. On the other hand, we exhibit a 6-dimensional algebra  $\Lambda$  with a semi-Gorenstein-projective module  $M$  which is not torsionless (thus not Gorenstein-projective). Actually, also the  $\Lambda$ -dual module  $M^*$  is semi-Gorenstein-projective. In this way, we show the independence of the total reflexivity conditions of L. L. Avramov and A. Martsinkovsky, thus completing a partial proof by D. A. Jorgensen and L. M. Şega. Since all the syzygy-modules of  $M$  and  $M^*$  are 3-dimensional, the example can be checked (and visualized) quite easily.

This talk is based on a joint work with Claus Michael Ringel.

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