On an open problem concerning the small finitistic dimension of a commutative ring

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Let R be a commutative ring with identity and let $\mathcal{Q}$ be the set of finitely generated semiregular ideals of R. A $\mathcal{Q}$-torsion-free R-module M is called a Lucas module if $\mathrm{Ext}_R^1(R/J, M) = 0$ for any $J \in \mathcal{Q}$. And R is called a DQ-ring if every ideal of R is a Lucas module. It is proved that if the small finitistic dimension of R is zero, then R is a DQ ring. In terms of a trivial extension, we construct a total ring of quotients $R = D \propto H$ which is not a DQ ring. Thus in this case, the small finitistic dimension of R is not zero. Then this fact gives a negative answer to an open problem posed by Cahen *et al.*.

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