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ABSTRACTS

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Contents

1.

On balanced Auslander–Dlab–Ringel algebras

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Many algebras that appear in representation theory and algebraic geometry often fall into the following two classes at the same time. One is the class of Koszul algebras introduced by Priddy [6], and the other is the class of quasi-hereditary algebras introduced by Cline, Parshall, and Scott [3]. These two classes of algebras exhibit two different forms of dualities - the Koszul duality [2] and the Ringel duality [7]. One sufficient condition for the two dualities of a Koszul quasi-hereditary algebra to commute is given by Mazorchuk [5], and an algebra that satisfies such a condition is called a *balanced algebra* nowadays.

In ring theory, one interesting class of quasi-hereditary algebras is given by the *Auslander–Dlab–Ringel algebras*. The class was first studied by Auslander in [1] and subsequently by Dlab and Ringel in [4] to give a construction of algebras with finite global dimensions.

In this talk, we will explain in slightly more details about the aforementioned classes of algebras, and will give a sufficient condition for Auslander–Dlab–Ringel algebras to be balanced.

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2.

On the weakly Iwanaga–Gorenstein property of gendo algebras

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We explore the subject on the weakly Iwanaga–Gorenstein (abbr. IG) property of gendo algebras. Here, a gendo algebra means the ENDOmorphism algebra of a Generator [FK], which often has nice homological properties. From the Morita theoretic viewpoint, the endomorphism algebra of a progenerator admits the same module category as the original algebra. Auslander introduced the notion of representation dimensions and Auslander algebras, which are defined using gendo algebras. As is well-known, they give excelent relationships between representation theoretic properties and homological properties.

The notion of weakly IG algebras was introduced by Ringel–Zhang [RZ]. We say that a finite dimensional algebra over a field is *right IG* if the category of Cohen–Macaulay modules is Frobenius; hence the stable category admits a triangulated category structure. Dually, we define *left IG* algebras. A *weakly IG* algebra is defined to be right and left IG. For examle, an algebra with finite left selfinjective dimension is right IG, so an IG algebra, which has finite left and right selfinjective dimension, is wealy IG. Note that we do not know if a right IG algebra is left IG, and vice versa. The aim of this talk is to construct weakly IG algebras. Here is a main result.

Theorem 1. *Let Λ be a finite dimensional algebra over a field and M a finite dimensional right Λ-module. If Λ is representation-finite, then the gendo algebra $\mathrm{End}_\Lambda(\Lambda \oplus M)$ is wealy IG with finite CM representation type.*

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3.

g-polytopes of Brauer graph algebras

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Inspired by a work of Hille [2], Asashiba-Mizuno-Nakashima [1] studied simplicial complexes of two-term tilting complexes over finite dimensional symmetric algebras A. For $0 \leq j \leq n-1$, the set of j-dimensional faces consists of the set of g-vectors $\{g^{T_1}, \ldots, g^{T_{j+1}}\}$ for basic two-term pretilting complexes $T = \bigoplus_{i=1}^{j+1} T_i$ having $j+1$ indecomposable direct summands, where n is the number of simple modules of A. The *g-polytope* $\Delta(A)$ of A is given by $(n-1)$-dimensional faces

$$\Delta(A) := \bigcup_{T \in 2\text{-tilt} A} C_{\leq 1}(T) \ \subseteq \mathbb{R}^n,$$

where $C_{\leq 1}(T)$ is the convex hull of $n+1$ vectors $0, g^{T_1}, \ldots, g^{T_n}$ for a basic two-term tilting complex $T = \bigoplus_{i=1}^{n} T_i$. Note that the g-polytope can be regarded as a truncated version of g-vector cones since we have $C_{\leq 1}(T) = \{\sum_{i=1}^{n} a_i g^{T_i} \mid 0 \leq a_i \leq 1 \text{ for all } i = 1 \ldots, n\}$.

Due to the result of [1], the convexity and symmetry of g-polytopes are quite interesting in tilting mutation theory. One of their aims is to introduce the g-polytope as a new derived invariant of Brauer tree algebras. Note that Brauer tree algebras are *τ-tilting-finite* symmetric algebras, namely, having only finitely many isomorphism classes of basic two-term tilting complexes.

Theorem 1. [2] *Let G be a Brauer tree and A_G the associated Brauer tree algebra. Then $\Delta(A_G)$ is convex and satisfies $\Delta(A_G) = -\Delta(A_G)$. Therefore, if two Brauer tree algebras A_G and $A_{G'}$ are derived equivalent, then we have $\Delta(A_G) \cong \Delta(A_{G'})$.*

An aim of this talk is to give a generalization for non-τ-tilting-finite symmetric algebras. In this case, we mainly study the closure $\overline{\Delta}(A)$ rather than $\Delta(A)$ itself. Finally, we conclude that the closure of g-polytopes of Brauer graph algebras is invariant under iterated mutation.

Proposition 2. *Let A be a symmetric algebra. If any algebra B obtained by iterated mutation from A satisfies $\overline{\Delta}(B) = -\overline{\Delta}(B)$, then we have $\overline{\Delta}(A) \cong \overline{\Delta}(B)$.*

Theorem 3. *Let G be a Brauer graph and A_G the associated Brauer graph algebra. Then $\overline{\Delta}(A_G)$ is convex and satisfies $\overline{\Delta}(A_G) = -\overline{\Delta}(A_G)$. Therefore, if two Brauer graph algebras A_G and $A_{G'}$ are obtained by iterated mutation each other, then we have $\overline{\Delta}(A_G) \cong \overline{\Delta}(A_{G'})$.*

Furthermore, we determine all integral lattice points of $\overline{\Delta}(A_G)$. We use a geometric model of a classification of two-term tilting complexes over Brauer graph algebras established by Adachi-Aihara-Chan.

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4.

On cyclotomic quiver Hecke algebras of affine type

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We recall Fock representations over affine Lie algebras, which arose from the soliton theory, and categorification of integrable highest weight modules via cyclotomic quiver Hecke algebras. Then, I explain graded dimension formulas for the idempotent truncation of the cyclotomic quiver Hecke algebras, the Chuang-Rouquier derived equivalence and the Brundan-Kleshchev isomorphism theorem. In the last part, I explain two applications briefly. One is the classification of tame block algebras of Hecke algebras of classical type, the other is Specht module theory for affine type C. The latter is joint work with Euiyong Park and Liron Speyer.

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5.

WIDE SUBCATEGORIES AND LATTICES OF TORSION CLASSES

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This talk is based on joint work [1] with Calvin Pfeifer (Bonn).

Let $\mathcal{A}$ be a fixed essentially small abelian length category. A pair $(\mathcal{T}, \mathcal{F})$ of full subcategories $\mathcal{T}, \mathcal{F} \subset \mathcal{A}$ is called a *torsion pair* if

$$\mathcal{F} = \mathcal{T}^{\perp} = \{X \in \mathcal{A} \mid \mathsf{Hom}_{\mathcal{A}}(\mathcal{T}, X) = 0\},$$
$$\mathcal{T} = {}^{\perp}\mathcal{F} = \{X \in \mathcal{A} \mid \mathsf{Hom}_{\mathcal{A}}(X, \mathcal{F}) = 0\}.$$

One can show that a full subcategory $\mathcal{T} \subset \mathcal{A}$ is completed to a torsion pair $(\mathcal{T}, \mathcal{T}^{\perp})$ if and only if $\mathcal{T}$ is closed under extensions and factor objects. We call such subcategories $\mathcal{T}$ *torsion classes* in $\mathcal{A}$, and then, the set $\mathsf{tors}\,\mathcal{A}$ partially ordered by inclusion is a complete lattice, that is, meets and joins are well-defined for all subsets of $\mathsf{tors}\,\mathcal{A}$.

For two torsion classes $\mathcal{U} \subset \mathcal{T}$ in $\mathcal{A}$, we can consider the interval $[\mathcal{U}, \mathcal{T}]$ in $\mathsf{tors}\,\mathcal{A}$ and a full subcategory $\mathcal{W} := \mathcal{U}^{\perp} \cap \mathcal{T}$. The full subcategory $\mathcal{W}$ indicates the difference of the torsion classes $\mathcal{U} \subset \mathcal{T}$; more precisely,

$$\mathcal{T} = \mathcal{U} * \mathcal{W} := \{X \in \mathcal{A} \mid \text{there exists } 0 \to U \to X \to W \to 0 \text{ with } U \in \mathcal{U} \text{ and } W \in \mathcal{W}\}.$$

We call $[\mathcal{U}, \mathcal{T}]$ a *wide interval* if $\mathcal{W}$ is a *wide subcategory*, that is, $\mathcal{W}$ is closed under taking kernels, cokernels, and extensions. In this case, $\mathcal{W}$ is an abelian subcategory of $\mathcal{A}$ closed under extensions, so we have another complete lattice $\mathsf{tors}\,\mathcal{W}$.

A typical example of wide intervals is given by *τ-tilting reduction* established by Jasso [3] and Demonet–Iyama–Reading–Reiten–Thomas [2]; namely, let A be a finite-dimensional algebra over a field K, then a τ-rigid pair (N, Q) in the module category $\mathsf{mod}\,A$ gives a wide interval $[\mathsf{Fac}\,N, {}^{\perp}(\tau N) \cap Q^{\perp}]$. They showed that the wide interval $[\mathsf{Fac}\,N, {}^{\perp}(\tau N) \cap Q^{\perp}]$ is isomorphic to $\mathsf{tors}\,C_{N,Q}$ as a complete lattice, where $C_{N,Q}$ is a certain finite-dimensional K-algebra constructed from the τ-rigid pair (N, Q).

In our study, we were able to extend their result to all wide intervals.

Theorem 1. *Let $[\mathcal{U}, \mathcal{T}]$ be a wide interval in $\mathsf{tors}\,\mathcal{A}$ and $\mathcal{W} := \mathcal{U}^{\perp} \cap \mathcal{T}$. Then we have mutually inverse isomorphisms of complete lattices*

$$\Phi \colon [\mathcal{U}, \mathcal{T}] \to \mathsf{tors}\,\mathcal{W}, \quad \Psi \colon \mathsf{tors}\,\mathcal{W} \to [\mathcal{U}, \mathcal{T}]$$

given by $\Phi(\mathcal{V}) := \mathcal{U}^{\perp} \cap \mathcal{V}$ and $\Psi(\mathcal{X}) := \mathcal{U} * \mathcal{X}$ for any $\mathcal{V} \in [\mathcal{U}, \mathcal{T}]$ and any $\mathcal{X} \in \mathsf{tors}\,\mathcal{W}$.

In this talk, I would like to explain the detail of the theorem above. If time permits, I will give several characterizations of wide intervals obtained in our study.

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2010 *Mathematics Subject Classification*. 06A07, 18E40.

6.

2-categorical Cohen-Montgomery duality between categories with I-pseudo-actions and I-graded categories for a small category I

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Throughout this talk $\Bbbk$ denotes a commutative ring. We first note that a group pseudo-action of a group G on a category $\mathcal{C}$ defined by Deligne [2] and Drinfeld–Gelaki–Nikshych–Ostrik [3] is nothing but a pseudofunctor from G as a groupoid with a single object $*$ to the 2-category $\mathbf{CAT}$ of categories sending $*$ to $\mathcal{C}$. Thus if $\mathcal{C}$ is a small $\Bbbk$-category, then it is just a pseudofunctor $X\colon G \to \Bbbk\text{-}\mathbf{Cat}$ with $X(*) = \mathcal{C}$, where $\Bbbk\text{-}\mathbf{Cat}$ is the 2-category of small $\Bbbk$-categories. We denote by $G\text{-}\mathbf{Cat}$ the 2-category of small $\Bbbk$-categories with G-pseudo-actions, and by $G\text{-}\mathbf{GrCat}$ the 2-category of small G-graded $\Bbbk$-categories. By generalizing the main result in [1] it is possible to show that a 2-functor $?/G\colon G\text{-}\mathbf{Cat} \to G\text{-}\mathbf{GrCat}$ defined by extending the orbit category construction is a 2-equivalence with a 2-quasi-inverse $?\#G\colon G\text{-}\mathbf{GrCat} \to G\text{-}\mathbf{Cat}$ defined by extending the smash product. By replacing the group G by a small category I we extend this result. Denote by $\mathrm{Pfun}(I, \Bbbk\text{-}\mathbf{Cat})$ the 2-category of pseudofunctors $I \to \Bbbk\text{-}\mathbf{Cat}$, and by $I\text{-}\mathbf{GrCat}$ the 2-category of small I-graded $\Bbbk$-categories. Then we can generalize the Grothendieck construction to a 2-functor $\int_I\colon \mathrm{Pfun}(I, \Bbbk\text{-}\mathbf{Cat}) \to I\text{-}\mathbf{GrCat}$ and define the smash product 2-functor $?\#I\colon I\text{-}\mathbf{GrCat} \to \mathrm{Pfun}(I, \Bbbk\text{-}\mathbf{Cat})$ in such a way that they are 2-quasi-inverses to each other. Of course, if $I = G$ then we have $\int_I = ?/G$ and $?\#I = ?\#G$.

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7.

ON TWO SIDED HARADA RINGS AND QF RINGS

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Let R be a basic artinian ring, let $\{e_i\}_{i=1}^n$ be a complete set of orthogonal primitive idempotents of R and let $\{f_i\}_{i=1}^k \subseteq \{e_i\}_{i=1}^n$. A sequence $f_1R,\ f_2R,\ \ldots,\ f_kR$ is called a *right co-H-sequence* of R if the following (CHS1), (CHS2), (CHS3) hold:

(CHS1) For each $i = 1, 2, \ldots, k-1$, there exists an R-isomorphism $\xi_i : f_iR_R \to f_{i+1}J_R$.

(CHS2) The last term f_kR_R is injective.

(CHS3) $f_1R,\ f_2R,\ \ldots,\ f_kR$ is the longest sequence among the sequences which satisfy (CHS1),(CHS2), i.e., there does not exist an R-isomorphism: $fR_R \to f_1J_R$, where $f \in \{e_i\}_{i=1}^n$.

And, we call an artinian ring R *a left Harada ring* if there exists a basic set $\{e_{i,j}\}_{i=1,j=1}^{m\ \ n(i)}$ of orthogonal primitive idempotents of R' such that $e_{i,n(i)}R,\ e_{i,n(i)-1}R,\ \ldots,\ e_{i,1}R$ is a right co-H-sequence of R for all $i = 1, 2, \ldots, m$.

Left Harada ring is first studied by M. Harada in [1]. K. Oshiro further studied it and called the ring a left Harada ring (abbreviated left H-ring) in [2]. Many results on one sided Harada rings are given in [3].

In this talk, using a new concept "weak co-H-sequence", we characterize two sided Harada rings and give the relationship between two sided Harada rings and QF rings.

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8.

Recollement of comodule categories over coalgebra objects

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One approach [3] to categorify representation theory is to replace an algebra by certain nice "additive 2-category" $\mathcal{A}$, and finite dimensional modules by abelian categories (with finitely many simples and consists only of finite length objects) that are equipped with an action of $\mathcal{A}$.

It turns out that the categorified version of an short exact sequence is equivalent to specifying a recollement $(\mathbf{L}, \mathbf{M}, \mathbf{N})$ of abelian categories.

It is well-known that if $\mathbf{M}$ in a recollement of abelian categories (as shown above) is a module category, say $\mathrm{mod}(A)$, of an algebra A, then there will be an idempotent e of A so that $\mathbf{L} \simeq \mathrm{mod}(A/AeA)$ and $\mathbf{N} \simeq \mathrm{mod}(eAe)$.

However, unlike the special case of $\mathcal{A}$ being a tensor category where $\mathcal{A}$-modules can take the form of module categories [1], we can only guarantee an $\mathcal{A}$-module takes the form of a comodule category $\mathrm{comod}_{\mathcal{A}}(\mathrm{C})$ over a coalgebra object C in the collection of morphism categories of $\mathcal{A}$ [2]. In this talk, we explain the analogue of the characterisation of recollements of module categories in this more general setting.

This is a joint work with Vanessa Miemietz (arXiv: 1901.04685).

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9.

Unique Factorization property of non-UFDs

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A unique factorization domain (UFD) is an integral domain in which each nonzero nonunit can be written uniquely as a finite product of irreducible elements, and in this case, each irreducible element is a prime element. Let D be an integral domain and t be the so-called t-operation on D. As in [1], D is called a *weakly factorial domain* (WFD) if each nonzero nonunit of D can be written as a finite product of primary elements. Two primary elements a, b of D will be said to be *distinct* if $\sqrt{aD} \neq \sqrt{bD}$. Let D be a WFD, and note that if

$$x = x_1 \cdots x_n = a_1 \cdots a_m$$

are two finite products of distinct primary elements of D, then $n = m$ and $x_iD = a_iD$ for $i = 1, \ldots, n$ by reordering if necessary. Hence, each nonzero nonunit of a WFD can be written uniquely as a finite product of distinct primary elements.

Following [5], we say that a nonzero nonunit $x \in D$ is *homogeneous* if x is contained in a unique maximal t-ideal of D. Then, in this talk, we will say that D is a *homogeneous factorization domain* (HoFD) if each nonzero nonunit of D can be written as a finite product of pairwise t-comaximal homogeneous elements. The notion of HoFDs was first introduced in [2], where the authors called an HoFD a t-pure domain. Clearly, primary elements are homogeneous. Thus, the notion of HoFDs is a natural generalization of WFDs, and we have the following implications:

$$\text{UFD} \Rightarrow \text{Weakly factorial GCD-domain} \Rightarrow \text{WFD} \Rightarrow \text{HoFD}.$$

In this talk, we first show that the expression of an element of an HoFD is unique as in the case of WFDs. Then, among other things, we show that (1) a PvMD D is an HoFD if and only if $D[X]$, the polynomial ring over D, is an HoFD and (2) D is a weakly Matlis GCD-domain if and only if $D[X]$ is an HoFD with t-Spec$(D[X])$ treed. We also study the HoFD property of $A + XB[X]$ constructions, pullbacks, and semigroup rings. This talk is based on [3, 4].

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10.

Generalized inverses and clean decompositions

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Abstract: In this talk, we will give the relations between Drazin inverses (group inverses) with clean decompositions. Furthermore, we get the relations between Moore-Penrose inverses (core inverses, dual core inverses, pseudo core inverses) with $*$-clean decompositions. The expressions of these generalized inverses by using the clean ($*$-clean) decompositions are given.

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11.

The right core inverses of a product and a companion matrix

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Abstract: In this paper, characterizations of right core inverse by one-sided invertibility are given. The necessary and sufficient conditions, which guarantee that paq have right core inverses are investigated. Furthermore, characterizations of right core inverses of triangular matrices, 2×2 matrices and a companion matrix are considered.

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2010 *Mathematics Subject Classification.* 15A09.

12.

Functors between higher cluster categories of type A

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Isomorphism classes of indecomposable objects in the m-cluster category $\mathcal{C}^m(A_l)$ of type A_l are in bijection with so-called m-diagonals in a polygon P_N with $N = m(l+1)+2$ corners (Baur–Marsh 2008). When $m/m' = (l+1)/(l'+1) \in \mathbb{N}$, this gives rise to an injective map from the set of isomorphism classes of objects in $\mathcal{C}^{m'}(A_{l'})$ to the set of isomorphism classes of objects in $\mathcal{C}^m(A_l)$.

In the talk, we shall see that this embedding of objects comes from a functor between the two cluster categories. While the functor in question is not full and faithful, it factors as a full and faithful functor composed with a covering of $\mathcal{C}^m(A_l)$.

cancelled

13.

Recent Results in Clean Rings

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A ring is called *clean* if every element can be written as the sum of a unit and an idempotent, and a ring is called *strongly clean* if such a unit and idempotent can be chosen so as to commute. Clean and strongly clean rings (and their variants) have been an object of much study in recent years, and there are many interesting open questions. In this talk, we will outline some recent work on the topic.

2010 *Mathematics Subject Classification*. .

14.

On Enochs conjecture

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Abstract

In the late 1990s, E. E. Enochs asked whether each covering class of modules is closed under direct limits. This problem is still open in general. In this talk, a brief introduction to Enochs conjecture will be given and some results on this subject will be reviewed and discussed.

Key Words: Covering class; Direct limit; Coherent ring; Absolutely pure module.

2010 Mathematics Subject Classification: 16D10; 16D50; 16P70.

15.

Questions and counterexamples on strongly clean rings

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Recall that an element of a ring is said to be strongly clean if it can be written as the sum of an idempotent and a unit that commute. The behavior of strong cleanness with respect to power series rings (and, more generally, rings complete with respect to an ideal) has been studied by many authors over the past 15 years or so. We will present a counterexample about strongly clean elements in power series rings, and present some related open questions.

2010 *Mathematics Subject Classification*. .

16.

TRIANGULARIZATION OF MATRICES AND POLYNOMIAL MAPS

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In this talk, we present conditions for a set of matrices satisfying a permutation identity to be simultaneously triangularizable. A set S of $n \times n$ matrices over an algebraically closed field is called σ-permutable if S satisfies a permutation identity:

$$A_1 A_2 \cdots A_r = A_{\sigma(1)} A_{\sigma(2)} \cdots A_{\sigma(r)}, \text{ for all } A_1, A_2, \ldots, A_r \in S,$$

for some nonidentity permutation $\sigma \in S_r$. We prove that S is triangularizable if S is σ-permutable for some nonidentity permutation $\sigma \in S_r$ with $\Delta(\sigma) = 1$, where $\Delta(\sigma) = \gcd\{|\sigma(i) - i| \mid 1 \le i \le r\}$. As applications, we generalize the Radjavi's result on triangularization of matrices with permutable trace and results of Yan and Tang on linear triangularization of polynomial maps. Joint work with Yueyue Li and Yan Tian.

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2010 *Mathematics Subject Classification.* 15A21, 14R15.

17.

Almost Gorenstein Rees algebras

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My talk is based on the recent research jointly with S. Goto, N. Matsuoka, M. Rahimi, H. L. Truong, and K.-i. Yoshida ([3, 4, 5, 6, 7]). The purpose of this talk is to investigate the question of when the Rees algebras of ideals are almost Gorenstein rings. Almost Gorenstein rings are one of the candidates for a class of Cohen-Macaulay rings which may not be Gorenstein but still good, hopefully next to the Gorenstein rings. The notion of these local rings dates back to the paper [1] of V. Barucci and R. Fröberg in 1997, where they dealt with one-dimensional analytically unramified local rings and developed a beautiful theory. However, since their notion is not flexible enough to analyze analytically ramified rings, in 2013 S. Goto, N. Matsuoka, and T. T. Phuong [2] extended the notion to arbitrary Cohen-Macaulay local rings but still of dimension one. Finally, in 2015 S. Goto, R. Takahashi and N. Taniguchi [9] proposed the definition of almost Gorenstein graded/local rings of higher dimension.

Possessing in [8] one of its roots, the theory of Rees algebras has been satisfactorily developed and nowadays one knows many Cohen-Macaulay Rees algebras. Among them Gorenstein Rees algebras are rather rare ([10]). Nevertheless, although they are not Gorenstein, some of Cohen-Macaulay Rees algebras are still good and could be *almost Gorenstein graded* rings, which we would like to report in this talk.

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2010 *Mathematics Subject Classification.* 13H10, 13H15, 13A30.

18.

The Jordan-Hölder property, Grothendieck monoids and Bruhat inversions

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The *Jordan-Hölder theorem* for modules says that the ways in which a module can be built up from simple modules are essentially unique. We may say that the category of modules (with finite length) satisfies *the Jordan-Hölder property*, abbreviated by (JHP). The aim of my talk is to investigate (JHP) in the setting of Quillen's *exact categories*.

As in the case of module categories, we can define simple objects, composition series and (JHP) in exact categories. Typical examples are extension-closed subcategories of $\mathsf{mod}\,\Lambda$ for an artin algebra Λ, and in this case, all objects have at least one composition series. However, it turns out that there exists many categories which does not satisfies (JHP), as well as those which does.

It is known that (JHP) implies the free-ness of the Grothendieck group, but the converse does not hold: for "nice" categories such as functorially finite torsion(-free) classes, their Grothendieck groups are free of finite rank, but (JHP) fails in some cases. Thus it is natural to consider a more sophisticated invariant than Grothendieck groups. Then I define *Grothendieck monoids*, which is a commutative monoid subject to the same universal property as the Grothendieck group. Then we have the following result:

Theorem 1. *Let $\mathcal{E}$ be an exact category. Then $\mathcal{E}$ satisfies (JHP) if and only if its Grothendieck monoid $\mathsf{M}(\mathcal{E})$ is a free monoid.*

As an application, we have the following numerical criterion.

Corollary 2. *Let $\mathcal{E}$ be a "nice" exact category. Then $\mathcal{E}$ satisfies (JHP) if and only if the number of indecomposable projective objects is equal to that of simple objects.*

We apply this to the representation theory of type A_n quiver Q by using combinatorics on the symmetric group S_{n+1}. It is known that torion-free classes of $\mathsf{mod}\,kQ$ are in bijection with c-sortable elements w of S_{n+1} ([1, 3]). Let $\mathcal{F}(w)$ be the corresponding torsion-free class. Then we obtain the following purely combinatorial description of simples and criterion for (JHP).

Theorem 3. *Simple objects in $\mathcal{F}(w)$ are in bijection with Bruhat inversions, or Bruhat lower covers, of w. In particular, $\mathcal{F}(w)$ satisfies (JHP) if and only if the number of Bruhat inversions of w is equal to that of supports of w.*

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2010 *Mathematics Subject Classification.* 18E10, 16G10.

19.

ARITHMETIC MODULES OVER GENERALIZED DEDEKIND DOMAINS

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Let D be an integrally closed domain with its quotient field K and M be a finitely generated torsion-free D-module. In [1], we showed that M is a generalized Dedekind module (G-Dedekind module for short), and M is a Dedekind module if and only if M is a multiplication module and KM is isomorphic to K as K-modules in case D is a Dedekind domain. These results show ,in arithmetic module theory, that G-Dedekind modules are more important than Dedekind modules.

We assume that D is a Noetherian G-Dedekind domain (by G-Dedekind domain we mean any v-ideal is invertible). Then we have the following results:

(1) If M is a projective D-module, then M is a G-Dedekind module.

(2) In case M is not a projective D-module, then M is generally a Krull module, that is, for each v-submodule N of M, $(N^{-}N)_v = M$.

2000 Mathematics Subject Classification: 13A15, 13A18,13E05,13E15

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2010 *Mathematics Subject Classification*. .

20.

Ideal Approximation Theory

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In the general setting of an exact category, the idea of the classical theory of approximations is to select a suitable subcategory to approximate arbitrary objects by the ones from this subcategory. While the idea of ideal approximation theory is to give morphisms and ideals of categories equal status as objects and subcategories.

Ideal Approximation Theory for exact categories is devoted to the study of precovering ideals, and the dual notion of preenveloping ideals, with emphasis on the notion of a special precovering (respectively, special preenveloping) ideal. A main technical tool to develop this theory is the mono-epi exact structure on the category of morphisms over an exact category which is introduced and analyzed in [5]. Along the way, several important results, such as ideal versions of Salce's Lemma [4, 5], Wakamatsu's Lemma [5], Eklof's Lemma [3] and Bongartz's Lemma [6] which are fundamental tools in classical theory, and an analogy of Ghost's Lemma [5] in triangulated categories, have been derived. The ideal approximation theory has been used to the study of ring and representation theory. For examples, (1) it is used to give an affirmative answer to an question asked by Benson and Gnacadja concerning sharp upper bounds for the phantom number of a finite group [5]; and (2) it is used to prove a partial dual of a result of Xu [10]: if R is a right coherent ring, and the class of pure projective right R-modules is closed under extensions, then every FP-projective module is pure projective [3].

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2010 *Mathematics Subject Classification.* 18E10; 18G15; 18G25; 16G70; 16N20.

21.

A functorial approach to monomorphism categories for species

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We investigate abstract versions of the monomorphism category as studied by Ringel and Schmidmeier. We prove that analogues of the kernel and cokernel functor send almost split sequences over the path algebra and the preprojective algebra to split or almost split sequences in the monomorphism category. This is based on the joint work with Julian Külshammer, Chrysostomos Psaroudakis and Sondre Kvamme.

2010 *Mathematics Subject Classification.* 16G70.

22.

cancelled

On the Galois Linear Maps

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ABSTRACT

In this paper we discuss some properties of the Galois linear maps. As an application we provide some equivalent conditions for Hopf algebras and Hopf (co)quasigroups.

Mathematics Subject Classifications (2000): 16W30.

23.

COHOMOLOGY RINGS AND APPLICATION IN HYPERNORMAL FORM FOR A CLASS OF 4-DIMENSIONAL VECTOR FIELDS

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Homological algebra was established in the 1940s, which is a powerful tool for solving the problem in ring theory [1]. With respect to good properties, homological algebra is attracting widespread interest in fields such as group theory[2], algebraic topology and differential equations [3], etc. One of the major challenges in ordinary differential equations is further reduction of vector fields, in which the theory of normal form plays an important role [4].An interesting insight concerning the application of cohomology rings in differential equations is presented.

In this paper, the application of cohomology rings theory in the research of hypernormal form (unique normal form, simplest normal form) and the associated coefficients for a class of four-dimensional vector fields is investigated. Based on the theory of cohomology rings, hypernormal form for four-dimensional vector fields is obtained by using the method of the combination of Hilbert series, new grading function and multiple Lie brackets. With aid of the method of combining multiple Lie brackets with parametric transformation, the corresponding relations of coefficients between original vector fields and its further reduction are given.

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2010 *Mathematics Subject Classification.* 70K45, 30G05, 15B33.

24.

COHEN-MACAULAY MODULES OVER YONEDA ALGEBRAS

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For a ring Λ and a Λ-module M, the abelian group

$$\Gamma = \bigoplus_{i \geq 0} \mathrm{Ext}^i_\Lambda(M, M)$$

with the Yoneda product is called the Yoneda algebra, which has widely been studied, for example, in the theory of Koszul duality.

We investigate the properties of Yoneda algebras Γ in the following setup:

- Λ is a finite dimensional algebra of finite representation type.
- M is an additive generator for the module category.

In the talk, we will give some fundamental results on these Γ, such as coherence, Gorenstein property, and a description of the stable category of Cohen-Macaulay Γ-modules.

2010 *Mathematics Subject Classification.* 16G50, 16E65, 18E30.

25.

Noncommutative Auslander Theorem and noncommutative quotient singularities

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Let k be an algebraically closed field of characteristic zero. Let G be a small subgroup of $\mathrm{GL}(n,k)$, and let $S=k[x_1,\ldots,x_n]$ be the polynomial algebra. Then G acts on S naturally. There is a natural isomorphism of algebras $S*G\cong\mathrm{End}_{S^G}(S)$, where $S*G$ is the skew group algebra, and S^G is the fixed subalgebra of S. This result is usually called Auslander Theorem (cf. [1, 2]). Auslander Theorem was generalized to noncommutative settings (cf. [3, 4]). In this talk, I will report some progresses in noncommutative Auslander Theorem, and their applications to noncommutative McKay correspondence (cf. [9, 5, 6]) and noncommutative resolutions for singularities (cf. [8, 10]). Some progress on singularities of noncommutative quadric hypersurfaces are also included in this talk (cf. [11, 7]).

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26.

On Representation of a Ring with unity on a Module over a Ring with Unity

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The representation of rings on finite dimension vector spaces has been generalized to the representation of rings on modules over a commutative ring. Let S be a commutative ring with unity and M an S-module. A representation of ring R with unity on an S-module M is a ring homomorphism from R to the ring of endomorphisms of M. An S-module associated with a representation of R is called a representation module of R. For any ring homomorphism $f\colon R \to S$, we define a representation of ring R with unity on M via f, and it is called an f-representation of ring R which is a special case of the representation of ring R on an S-module. This S-module associated with the f-representation of ring R is called an f-representation module of R.

The result of our study is generalized Schur's Lemma. If an S-module is a representation module of ring R then it is is an R-S-bimodule, and every R-S-bimodule is a representation module of R. However, a bimodule is not necessarily an f-representation module of the ring. Furthermore, for two ring homomorphisms f, g from R to S, we obtained a sufficient condition of the equivalent of an f-representation and a g-representation. We also find some a sufficient condition of a module homomorphism becomes a morphism from an f-representation to a g-representation. This study also reveals that the sufficient condition of f-representation of the ring R on a finite dimension free module over a principal ideal domain S is decomposable and completely reducible.

In the case of S not commutative, we give the sufficient condition of the S-module M becomes the representation module of R. The category of f-representation modules of ring R is Abelian and Morita equivalent to the category of modules over an R-algebra. Thus, if the category of modules over the R-algebra which is equivalent to the category of f-representation modules of R satisfies the Krull-Schmidt Theorem, then the category of f-representation modules of R also satisfies Krull-Schmidt's Theorem.

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2010 *Mathematics Subject Classification.* 16G10, 16W20, 12E15.

27.

A remark on graded countable Cohen-Macaulay representation type

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In the representation theory of Cohen-Macaulay algebras, a classifying the algebras with respect to the complexity of the classification of maximal Cohen-Macaulay modules over them is a major subject. It has been conjectured that all Cohen-Macaulay algebras split into three classes, that is, *Cohen-Macaulay discrete* (including *finite*), *Cohen-Macaulay tame* and *Cohen-Macaulay wild*. For graded cases, the studies are investigated by Eisenbud and Herzog [4], Stone [5], Drozd and Tovpyha [3] and so on. In this talk, we shall give a remark on graded countable Cohen-Macaulay representation type.

Let $R = \oplus_{i=0}^{\infty} R_i$ be a commutative positively graded ring with $R_0 = k$ an algebraically closed field. Let S be a graded Noetherian normalization. That is, S is a graded polynomial subring of R such that R is a finitely generated graded S-module. A finitely generated graded R-module M is said to be maximal Cohen-Macaulay (MCM) if M is graded free as a graded S-module. We say that a graded Cohen-Macaulay ring R is of graded countable CM representation type if there are infinitely but only countably many isomorphism classes of indecomposable graded Cohen-Macaulay R-modules up to shift.

Theorem 1. *Let R be of graded countable CM representation type. For each graded free S-module F there are finitely many isomorphism classes of MCM R-modules which are isomorphic to F as graded S-modules.*

To prove the theorem we consider the analogy of a module variety for finitely generated modules over a finite dimensional algebra, which was introduced by Dao and Shipman [2].

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28.

On the gendo-symmetric algebra of a trivial extension algebra

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In representation theory of algebras, endomorphism algebras play important roles. For example, the endomorphism algebra of a progenerator is Morita equivalent to the original algebra. More generally, the endomorphism algebra of a tilting module is derived equivalence to the original algebra. When a given algebra is representation-finite, the endomorphism algebra of the additive generator in the module category is the Auslander algebra [A]. Thus, endomorphism algebras are interesting subjects of study.

Our purpose is to investigate the representation types of endomorphism algebras. However, in most cases, endomorphism algebras are representation-infinite. On the other hand, the endomorphism algebra of a generator is expected to be easy deal with. Therefore, we consider the endomorphism algebra of a generator over a symmetric algebra, so-called a *gendo-symmetric algebra* [FK]. In particular, our aim is to determine when a gendo-symmetric algebra is representation-finite. In the case, we also study the structure of the Auslander-Reiten quiver.

Our main result can be stated as follows. Let B be the trivial extension algebra of an algebra A and X an indecomposable non-projective B-module. Consider the gendo-symmetric algebra $\Lambda := \mathrm{End}_B(B \oplus X)$ given by the generator $B \oplus X$. In this talk, we give a complete description of Λ being representation-finite. Moreover, we construct the stable Auslander-Reiten quiver of Λ.

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29.

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Some results on Noetherian Warfield domains

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Let R be a Noetherian domain. It is proved that R is a NWF domain if and only if, for every maximal ideal M of R, both M and M^2 can be generated by two elements. A sufficient condition under which all ideals of a domain are SG-projective is also given in this article.

30.

The Extension Dimension of Abelian Categories

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Let $\mathcal{A}$ be an abelian category having enough projective objects and enough injective objects. We prove that if $\mathcal{A}$ admits an additive generating object, then the extension dimension and the weak resolution dimension of $\mathcal{A}$ are identical, and they are at most the representation dimension of $\mathcal{A}$ minus two. By using it, for a right Morita ring Λ, we establish the relation between the extension dimension of the category mod Λ of finitely generated right Λ-modules and the representation dimension as well as the global dimension of Λ. In particular, we give an upper bound for the extension dimension of mod Λ in terms of the projective dimension of certain class of simple right Λ-modules and the radical layer length of Λ. It is a joint work with Junling Zheng and Xin Ma.

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31.

On the 2-test modules of projectivity and weakly $\mathfrak{m}$-full ideals

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Throughout this talk, let R be a commutative noetherian local ring with maximal ideal $\mathfrak{m}$ and residue field k. All modules considered in this paper are assumed to be finitely generated. The notion of a strong test module for projectivity has been introduced and studied by Ramras [3]. An R-module M is called a *strong test module for projectivity* if every R-module N with $\mathrm{Ext}^1_R(N, M) = 0$ is projective. The residue field k and the unique maximal ideal $\mathfrak{m}$ are typical examples of a strong test module for projectivity.

Definition 1. Let M be a non-zero module and let n be a positive integer.

(1) M is called *n-test module for projectivity* if every module X with $\mathrm{Ext}^{1\sim n}_R(X, M) = 0$ is projective.

(2) M is called *n-Tor-test module for projectivity* if every module X with $\mathrm{Tor}^R_{1\sim n}(X, M) = 0$ is projective.

The main results in this talk are the following three theorems.

Theorem 2. *If M is an n-Tor-test module for projectivity then $M, \Omega_R M, \Omega^2_R M, \ldots, \Omega^n_R M$ are n-test modules for projectivity.*

Theorem 3. *If I is weakly $\mathfrak{m}$-full and $\mathrm{Tor}^R_1(M, R/I) = 0$ then a free covering $0 \to N \to F \to M \to 0$ induces a short exact sequence $0 \to N/IN \to F/IF \to M/IM \to 0$ satisfying $\mathrm{depth}_R N/IN > 0$. Moreover, if I is $\mathfrak{m}$-primary then M is projective.*

Theorem 4. *Suppose I is weakly $\mathfrak{m}$-full and $\mathrm{depth}_R R/I = 0$. If $\mathrm{Tor}^R_n(M, R/I) = 0$ and $\mathrm{depth}_R(\mathrm{Tor}^R_{n-1}(M, R/I)) > 0$ then $\mathrm{proj.dim}_R M < n - 1$ for all positive integer n.*

These theorems induce the following corollaries.

Corollary 5. [1] *Let R be a local ring and let I be an $\mathfrak{m}$-primary ideal of R. If I is weakly $\mathfrak{m}$-full then R/I is a 1-Tor-test module for projectivity.*

Corollary 6. *Let R be a local ring and let I be an $\mathfrak{m}$-primary ideal of R. If I is weakly $\mathfrak{m}$-full then R/I and I are strong test modules for projectivity.*

Corollary 7. [2] *Suppose I is weakly $\mathfrak{m}$-full and $\mathrm{depth}_R R/I = 0$, the following statements hold.*

(1) *R/I is a 2-Tor-test module for projectivity.*
(2) *R/I and I are 2-test modules for projectivity.*

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32.

Ulrich ideals in hypersurfaces

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The purpose of this talk is to investigate the structure and ubiquity of Ulrich ideals in a hypersurface ring.

In a Cohen-Macaulay local ring $(R, \mathfrak{m})$, an $\mathfrak{m}$-primary ideal I is called an Ulrich ideal in R if there exists a parameter ideal Q of R such that $I \supsetneq Q$, $I^2 = QI$, and I/I^2 is R/I-free. The notion of Ulrich ideal/module dates back to the work [3] in 2014, where S. Goto, K. Ozeki, R. Takahashi, K.-i. Watanabe, and K.-i. Yoshida introduced the notion, generalizing that of maximally generated maximal Cohen-Macaulay modules ([1]), and started the basic theory. The maximal ideal of a Cohen-Macaulay local ring with minimal multiplicity is a typical example of Ulrich ideals, and the higher syzygy modules of Ulrich ideals are Ulrich modules. In [3, 4], all Ulrich ideals of Gorenstein local rings of finite CM-representation type with dimension at most 2 are determined by means of the classification in the representation theory.

Nevertheless, even for the case of hypersurface rings, there seems known only scattered results which give a complete list of Ulrich ideals, except the case of finite CM-representation type and the case of several numerical semigroup rings. Therefore, in this talk, we focus our attention on a hypersurface ring which is not necessarily finite CM-representation type.

In what follows, unless otherwise specified, let $(S, \mathfrak{n})$ be a Cohen-Macaulay local ring with $\dim S = d+1$ $(d \geq 1)$, and $f \in \mathfrak{n}$ a non-zero divisor on S. We set $R = S/(f)$. For each $a \in S$, let $\overline{a}$ denote the image of a in R. We denote by $\mathcal{X}_R$ the set of Ulrich ideals in R. We then have the following, which characterizes Ulrich ideals in a hypersurface ring.

Theorem 1. *Suppose that $(S, \mathfrak{n})$ is a regular local ring with $\dim S = d+1$ $(d \geq 1)$ and $0 \neq f \in \mathfrak{n}$. Set $R = S/(f)$. Then we have*

$$\mathcal{X}_R = \left\{ (\overline{a_1}, \overline{a_2}, \cdots, \overline{a_d}, \overline{b}) \;\middle|\; \begin{array}{l} a_1, a_2, \ldots, a_d, b \in \mathfrak{n} \text{ be a system of parameters of } S, \\ \text{and there exist } x_1, x_2, \ldots, x_d \in (a_1, a_2, \cdots, a_d, b) \text{ and } \varepsilon \in U(S) \\ \text{such that } b^2 + \displaystyle\sum_{i=0}^{d} a_i x_i = \varepsilon f. \end{array} \right\},$$

where $U(S)$ denotes the set of unit elements of S.

Let $a_1, \ldots, a_d, b \in \mathfrak{n}$ be a system of parameters of S, so that $b^2 + \sum_{i=1}^{d} a_i x_i = \varepsilon f$ with $x_1, \ldots, x_d \in (a_1, \cdots, a_d, b)$ and $\varepsilon \in U(S)$. Then $I = (\overline{a_1}, \overline{a_2}, \cdots, \overline{a_d}, \overline{b}) \in \mathcal{X}_R$, with a reduction $Q = (\overline{a_1}, \overline{a_2}, \cdots, \overline{a_d})$ by Theorem 1. By [3, Corollary 7.2], in the exact sequence $0 \to Q \xrightarrow{\iota} I \to R/I \to 0$, the free resolution of I induced from minimal free resolutions of Q and R/I is also minimal. We construct this resolution, by using the relation $b^2 + \sum_{i=1}^{d} a_i x_i = \varepsilon f$. We set

$$K = \mathrm{K}_\bullet(a_1, \ldots, a_d; S) = (K_\bullet, \partial_\bullet^K) \text{ and } L = \mathrm{K}_\bullet(x_1, \ldots, x_d; S) = (K_\bullet, \partial_\bullet^L)$$

are Koszul complexes of S generated by $a_1, \ldots, a_d$ and $x_1, \ldots, x_d$. We define $G = (G_\bullet, \partial_\bullet)$ by $G_0 = K_0$, $G_i = K_i \oplus G_{i-1} = S^{\oplus \sum_{j=0}^{i} \binom{d}{j}}$ for $i \geq 1$, and

$$\partial_1 = \left[\begin{array}{c|c} \partial_1^K & b \end{array}\right], \partial_2 = \left[\begin{array}{c|c} \partial_2^K & -bE_d \mid {}^t\partial_1^L \\ \hline O & \partial_1 \end{array}\right], \text{ and}$$

$$\partial_i = \left[\begin{array}{c|c} \partial_i^K & (-1)^{i-1}bE_{\binom{d}{i-1}} \mid {}^t\partial_{i-1}^L \mid O \\ \hline O & \partial_{i-1} \end{array}\right] \text{ for } i \geq 3.$$

We notice that $\partial_i = \partial_{d+1}$ for any $i \geq d+1$. Set $F = (F_\bullet, \overline{\partial_\bullet}) = (G_\bullet \otimes R, \partial_\bullet \otimes R)$. We then have the following.

Theorem 2. *$F : \cdots \to F_i \xrightarrow{\overline{\partial_i}} F_{i-1} \to \cdots \to F_1 \xrightarrow{\overline{\partial_1}} F_0 = R \xrightarrow{\varepsilon} R/I \to 0$ is a minimal free resolution of R/I.*

As a consequence, we get a matrix factorization of d-th syzygy module of R/I, which is an Ulrich module with respect to I (see [3, Definition 1.2]).

Corollary 3. *Let $M = \operatorname{Im} \overline{\partial_d}$. Then $0 \to G_{d+2} \xrightarrow{\partial_{d+1}} G_{d+1} \xrightarrow{\tau} M \to 0$ is exact as S-modules and $\partial_{d+1}^2 = gE_{2^d}$, where $\tau : G_{d+1} \xrightarrow{\varepsilon} F_{d+1} \xrightarrow{\overline{\partial_d}} M$. Therefore ∂_{d+1} gives a matrix factorization of M.*

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33.

Hochschild cohomology of Beilinson algebras of graded down-up algebras

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Let k be an algebraically closed field of char $k = 0$. A graded k-algebra $A(\alpha, \beta) := k\langle x, y\rangle/(x^2y - \beta yx^2 - \alpha xyx,\ \ xy^2 - \beta y^2x - \alpha yxy), \deg x = m, \deg y = n \in \mathbb{N}^+$ with parameters $\alpha, \beta \in k$ is called a *graded down-up algebra*. It is known that a graded down-up algebra $A = (\alpha, \beta)$ is a noetherian AS-regular algebra of dimension 3 if and only if $\beta \neq 0$ ([4]). By the special case of [5, Theorem 4.14], if $A = A(\alpha, \beta)$ is a graded down-up algebra with $\beta \neq 0$, then the Beilinson algebra ∇A of A is extremely Fano of global dimension 2, and there exists an equivalence of triangulated categories $\mathsf{D^b}(\mathsf{tails}\, A) \cong \mathsf{D^b}(\mathsf{mod}\, \nabla A)$, where $\mathsf{tails}\, A$ is the noncommutative projective scheme of A in the sense of [1].

The aim of our talk is to investigate the Hochschild cohomology groups $\mathrm{HH}^i(\nabla A)$ of ∇A of a graded down-up algebra $A = A(\alpha, \beta)$ with $\beta \neq 0$. If $\deg x = \deg y = 1$, then a description of the Hochschild cohomology group $\mathrm{HH}^i(\nabla A)$ of ∇A has been obtained using a geometric technique ([2, Table 2]). In this talk, for $\deg x = 1, \deg y = n \geq 2$, we give the dimension formula of $\mathrm{HH}^i(\nabla A)$ for each $i \geq 0$. In this case, the Beilinson algebra ∇A of A is given by the following quiver Q with relations $f_i = 0$ $(1 \leq i \leq n)$, $g = 0$:

$$Q := \begin{tikzcd} 1 \arrow[r, "x_1"] \arrow[rrrr, bend right, "y_1"'] & 2 \arrow[r, "x_2"] \arrow[rrrr, bend right, "y_2"'] & \cdots \arrow[r, "x_{n-1}"] & n \arrow[r, "x_n"] \arrow[rrr, bend right, "y_n"'] & n+1 \arrow[r, "x_{n+1}"] \arrow[rrr, bend right, "y_{n+1}"'] & n+2 \arrow[r, "x_{n+2}"] \arrow[rrr, bend right, "y_{n+2}"'] & \cdots \arrow[r, "x_{2n}"] & 2n+1 \arrow[r, "x_{2n+1}"] & 2n+2 \end{tikzcd},$$

$$f_i := x_i x_{i+1} y_{i+2} - \beta y_i x_{i+n} x_{i+n+1} - \alpha x_i y_{i+1} x_{i+n+1},$$
$$g := x_1 y_2 y_{n+2} - \beta y_1 y_{n+1} x_{2n+1} - \alpha y_1 x_{n+1} y_{n+2}.$$

In particular, it turns out from our dimension formula that the group structure of $\mathrm{HH}^i(\nabla A)$ depends on the values of $\alpha^2 + 4\beta$ and $\delta_n := \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha & 1 \\ \beta & 0 \end{pmatrix}^n \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ([3, Theorem 1.4]). Using the fact that Hochschild cohomology is invariant under derived equivalence, our result implies the following: Let $A = A(\alpha, \beta)$ and $A' = A(\alpha', \beta')$ be graded down-up algebras with $\deg x = 1, \deg y = n \geq 1$, where $\beta \neq 0, \beta' \neq 0$. If $\delta_n := \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha & 1 \\ \beta & 0 \end{pmatrix}^n \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$ and $\delta'_n := \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha' & 1 \\ \beta' & 0 \end{pmatrix}^n \begin{pmatrix} 1 \\ 0 \end{pmatrix} \neq 0$, then $\mathsf{D^b}(\mathsf{tails}\, A) \ncong \mathsf{D^b}(\mathsf{tails}\, A')$ ([3, Corollary 1.5]).

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34.

The Hochschild cohomology of a class of exceptional periodic selfinjective algebras of polynomial growth

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This talk is based on joint work with G. Zhou and W. Lyu. It is known that the non-standard periodic representation-infinite selfinjective algebras of polynomial growth are socle deformations of the corresponding periodic standard algebras, and every such an algebra Λ is geometric socle deformation of excactly one representation-infinite standard algebra Λ' of polynomial growth. These algebras Λ and Λ' are called exceptional periodic algebras of polynomial growth in [1]. In [2], their Hochschild cohomology groups $\mathrm{HH}^i(\Lambda)$ and $\mathrm{HH}^i(\Lambda')$ for $i = 0, 1, 2$ are determined, and it is shown that Λ and Λ' are not derived equivalent.

In this talk, we determine the Hochschild cohomology ring of a class of exceptional periodic selfinjective algebras of polynomial growth.

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35.

McCoy property over Jacobson radicals

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Abstract In this paper, we introduce a new class of rings related to Jacobson radicals called feckly McCoy rings. A ring R is called right feckly McCoy if the factor ring $R/J(R)$ is a right McCoy ring. We mainly study the structural property of right feckly McCoy rings, and elaborate upon some special extensions of rings with McCoy property over Jacobson radicals. It is also brought into focus the Ore extension of right feckly McCoy rings under some conditions. Moreover, we generalize the annihilator of a ring to the Jacobson radical, and study some special properties of rings with annihilators over the Jacobson radical under the right feckly McCoy condition.

keywords McCoy ring, Jacobson radical, feckly McCoy ring, polynomial extension, annihilator

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Cohen-Macaulay differential graded modules and negative Calabi-Yau configurations

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We introduce the class of Cohen-Macaulay (=CM) dg (=differential graded) modules over Gorenstein dg algebras and study their basic properties. We show that the category of CM dg modules forms a Frobenius extriangulated category, in the sense of Nakaoka and Palu, and it admits almost split extensions. We also study representation-finite d-self-injective dg algebras A in detail. In particular, we classify the Auslander-Reiten (=AR) quivers of CMA for those A in terms of $(-d-1)$-Calabi-Yau (=CY) configurations, which are Riedtmann's configurations for the case $d = 0$. For any given $(-d-1)$-CY configuration C, we show there exists a d-self-injective dg algebra A, such that the AR quiver of CMA is given by C.

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37.

The characteristic variety of an elliptic algebra

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This is based on joint work with Alex Chirvasitu and S. Paul Smith [2, 3, 4].

There have been several attempts to define the space associated to a noncommutative ring. For a graded algebra over a field $\Bbbk$, one established approach is to look at $\operatorname{QGr} A$, the category of graded A-modules modulo the full subcategory consisting of torsion modules. When the algebra A is commutative and finitely generated in degree one, the category $\operatorname{QGr} A$ is equivalent to the category of quasi-coherent sheaves on $\operatorname{Proj} A$. Thus, for a noncommutative algebra A, we may consider $\operatorname{QGr} A$ as the category of "quasi-coherent sheaves" on the associated "noncommutative projective scheme".

To understand $\operatorname{QGr} A$, the first things one should look at are objects coming from point modules:

Definition 1. Let A be a nonnegatively graded $\Bbbk$-algebra that is finitely generated in degree one. A graded A-module M is called a *point module* if it is cyclic and satisfies

$$\dim_{\Bbbk} M_i = \begin{cases} 1 & \text{if } i \geq 0, \\ 0 & \text{if } i < 0. \end{cases}$$

Artin-Tate-Van den Bergh [1] showed that the point modules are parametrized by a space called the *point scheme*, which is defined as an inverse limit of schemes. Each point module defines a simple object in $\operatorname{QGr} A$. Point modules have played a crucial role in the study of Artin-Schelter regular algebras.

In 1989, Feigin and Odesskii introduced a family of algebras $Q_{n,k}(E,\tau)$ parametrized by an elliptic curve E over $\mathbb{C}$, a closed point $\tau \in E$, and coprime integers $n > k \geq 1$. This is a huge generalization of higher dimensional Sklyanin algebras, and provides flat deformations of polynomial algebras when τ varies.

The aim of this talk is to describe the major component of the point scheme of the elliptic algebra $Q_{n,k}(E,\tau)$, which we call the *characteristic variety*. For a higher dimensional Sklyanin algebra, the characteristic variety is the elliptic curve E and it is the only non-discrete irreducible component of the point scheme. For other elliptic algebras, the characteristic variety depends on the negative continued fraction of the rational number n/k and is realized as the quotient of a product of copies of E by a finite group.

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Tate-Hochschild cohomology from the singularity category

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The singularity category (or stable derived category) was introduced by Buchweitz [1] in 1986 and rediscovered in a geometric context by Orlov [8] in 2003. It measures the failure of regularity of an algebra or scheme. Following Buchweitz, one defines the Tate-Hochschild cohomology of an algebra as the Yoneda algebra of the identity bimodule in the singularity category of bimodules. In recent work, Zhengfang Wang [9] has shown that Tate-Hochschild cohomology is endowed with the same rich structure as classical Hochschild cohomology: a Gerstenhaber [5] bracket in cohomology and a B-infinity structure [3] at the cochain level. This suggests that Tate-Hochschild cohomology might be isomorphic to the classical Hochschild cohomology of a (differential graded) category, in analogy with a theorem of Lowen-Van den Bergh [7] in the classical case. We show that indeed, at least as a graded algebra, Tate- Hochschild cohomology is the classical Hochschild cohomology of the singularity category with its canonical dg enhancement. In joint work with Zheng Hua [4], we have applied this to prove a weakened version of a conjecture by Donovan-Wemyss [2] on the reconstruction of a (complete, local, compound Du Val) singularity from its contraction algebra, i.e. the algebra representing the non commutative deformations of the exceptional fiber of a resolution.

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39.

A new semistar operation on a commutative ring and its applications

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In this talk, a new semistar operation, called the q-operation, on a commutative ring R is introduced in terms of the ring $Q_0(R)$ of finite fractions. It is defined as the map $q : \mathcal{F}_q(R) \to \mathcal{F}_q(R)$ by $A \mapsto A_q := \{x \in Q_0(R) \mid$ there exists some finitely generated semiregular ideal J of R such that $Jx \subseteq A\}$ for any $A \in \mathcal{F}_q(R)$, where $\mathcal{F}_q(R)$ denotes the set of nonzero R-submodules of $Q_0(R)$. The main superiority of this semistar operation is that it can also act on R-modules. And we can also get a new hereditary torsion theory τ_q induced by a (Gabriel) topology $\{I \mid I$ is an ideal of R with $I_q = R_q\}$. Based on the existing literature of τ_q-Noetherian rings by Golan and Bland *et al.*, in terms of the q-operation, we can study them in more detailed and deep module-theoretic point of view, such as τ_q-analogue of the Hilbert basis theorem, Krull's principal ideal theorem, Cartan-Eilenberg-Bass theorem, and Krull intersection theorem.

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40.

CHARACTERIZATIONS OF RADICALS IN SKEW POLYNOMIAL AND SKEW LAURENT POLYNOMIAL RINGS

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Throughout this paper, all rings are assumed to be associative with 1. We let R denote an arbitrary ring, and let σ be any automorphism of R. By $R[x;\sigma]$ ($R[x,x^{-1};\sigma]$) we mean the skew (Laurent) polynomial ring over R, subject to the (left) skewing condition $xr = \sigma(r)x$ ($x^{-1}r = \sigma^{-1}(r)x^{-1}$) for each $r \in R$.

Pearson and Stephenson [4] characterized the prime radical of a skew polynomial ring as $P(R[x;\sigma]) = (P(R) \cap P_\sigma(R)) + P_\sigma(R)xR[x;\sigma]$ where $P_\sigma(R)$ is the intersection of all strongly σ-prime ideals of R, which is the so-called σ-prime radical of R. The prime radical of a skew Laurent polynomial ring was investigated by Cheon et al. [1]. They showed that $P(R[x,x^{-1};\sigma]) = P_{(\sigma,\sigma^{-1})}(R)[x,x^{-1};\sigma]$, where $P_{(\sigma,\sigma^{-1})}(R)$ denotes the intersection of all (σ,σ^{-1})-prime ideals of R.

On the other hand, Ferrero [2] characterized the generalized nilradical of skew polynomial and skew Laurent polynomial rings as $N(R[x;\sigma]) = N(R) + N_\sigma(R)xR[x;\sigma]$ and $N(R[x,x^{-1};\sigma]) = N_\sigma(R)[x,x^{-1};\sigma]$, where $N_\sigma(R)$ is the intersection of all the σ-ideals of R which are also completely prime ideals. Ferrero also showed that $s(R[x;\sigma]) = s(R) + s_\sigma(R)xR[x;\sigma]$, where $s(R)$ ($s_\sigma(R)$) denotes the (σ-)strongly prime radical of R in [3].

For the continuation of the study of radicals of skew (Laurent) polynomial rings, in this paper, we conduct the study showing that radicals between the prime radical and the generalized nilradical have similar forms to the above formulas.

In particular, we first give a complete description of the Levitzki radical of a skew (Laurent) polynomial ring through the prime ideals and skewed prime ideals in the base ring. We next provide formulas similar to the above expression of radicals for the strongly prime radical and the uniformly strongly prime radical of these rings.

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41.

Twist automorphisms of quantum unipotent cells and dual canonical bases

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Quantum unipotent cell is introduced by De Concini-Procesi [3] as a quantum analogue of the coordinate ring of unipotent cells and they proved an isomorphism between quantum analogue of coordinate ring of intersection of unipotent subgroup and shifted Gaussian cells in finite type. In this talk, we construct quantum analogue of twist automorphism whose classical counterpart is introduced by Berenstein-Fomin-Zelevinsky [1] and Berenstein-Zelevinsky [2] in the study of total positivity for Schubert varieties. We prove the quantum twist automorphism preserves the dual canonical basis of quantum unipotent cells. Furthemore quantum cluster monomials is also preserved under the quantum twist automorphism in symmetric case using the additive categorification by Geiss-Leclerc-Schröer [4]. This is a joint work [5] with Hironori Oya.

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A characterization of local rings of countable representation type

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All the contents of this article are taken from joint work with Justin Lyle and Ryo Takahashi [3]. We refer the reader to it for the details.

Cohen–Macaulay representation theory has been studied widely and deeply for more than four decades. Buchweitz, Greuel and Schreyer [2] proved that the local hypersurfaces of finite (resp. countable) CM-representation type,(that is, Cohen–Macaulay local rings possessing finitely/infinitely-but-countably many nonisomorphic indecomposable maximal Cohen–Macaulay modules) are precisely the local hypersurfaces of type (A_n) with $n \geq 1$, (D_n) with $n \geq 4$, and (E_n) with $n = 6, 7, 8$ (resp. (A_∞) and (D_∞)).

In this talk, we introduce another representation type, namely, *finite* CM_+*-representation type*. We say that a Cohen–Macaulay local ring has finite CM_+-representation type if there exist only finitely many isomorphism classes of indecomposable maximal Cohen–Macaulay modules that are *not* locally free on the punctured spectrum. Then, Araya, Iima and Takahashi [1] observed that the local hypersurfaces of type (A_∞) and (D_∞) has finite CM_+-representation type. Thus, it is natural to ask the following question.

Conjecture 1. *Let R be a complete local Gorenstein ring of dimension d not having an isolated singularity. Then the following two conditions are equivalent.*

(1) *The ring R has finite CM_+-representation type.*
(2) *There exist a complete regular local ring S and a regular system of parameters $x_0, \dots, x_d$ such that R is isomorphic to*

$$S/(x_0^2 + x_2^2 + \cdots + x_d^2) \quad or \quad S/(x_0^2 x_1 + x_2^2 + \cdots + x_d^2).$$

We give a complete answer to this conjecture in dimension one.

Theorem 2. *Let R be a homomorphic image of a regular local ring. Suppose that R does not have an isolated singularity but is Gorenstein. If $\dim R = 1$, the following are equivalent.*

(1) *The ring R has finite CM_+-representation type.*
(2) *There exist a regular local ring S and a regular system of parameters x, y such that R is isomorphic to $S/(x^2)$ or $S/(x^2 y)$.*

When either of these two conditions holds, the ring R has countable CM*-representation type.*

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43.

An application of a theorem of Sheila Brenner for Hochschild extension algebras of a truncated quiver algebra

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Brenner [1] studied the number of indecomposable direct summands of the middle term of an almost split sequence starting with a simple module, and she showed how to determine this number for an artin algebra. As a consequence of this result she obtains, for a self-injective artin algebra, the number of indecomposable direct summands of $\operatorname{rad} P/\operatorname{soc} P$, where P is indecomposable projective. Moreover, Fernández-Platzeck [2] gave simple interpretation for of them the trivial extension algebra of an algebra. Their description is given in terms of oriented cycles in the ordinary quiver of the trivial extension algebra. In this talk, we will give a similar interpretation of a theorem of Sheila Brenner for Hochschild extension algebras which is a generalization of trivial extension algebras.

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44.

Mutations for star-to-tree complexes and pointed Brauer trees

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Throughout this talk, let k be an algebraically closed field, G_0 a Brauer star of type (e, m) and B a Brauer star algebra over k associated to G_0.

Let us begin with the definition of the two-restricted tilting complex for the Brauer star algebra B and the fact on this complex.

Definition 1. [2] Let $\hat{T}$ be a tilting complex over a Brauer star algebra B. We call $\hat{T}$ a two-restricted tilting complex if any indecomposable direct summand of $\hat{T}$ is a shift of the following elementary complex, where the first nonzero term is in degree 0.

- $S_i : 0 \to Q_i \to 0,$
- $T_{jk} : 0 \to Q_j \xrightarrow{h_{jk}} Q_k \to 0,$

where the map h_{jk} has maximal rank among homomorphisms from Q_j to Q_k.

Theorem 2. [2] *There is a one-to-one correspondence between the set of multiplicity-free two-restricted tilting complexes for the Brauer star algebra B and the set of pointed Brauer trees of type (e, m).*

On the other hand, in [1], it is shown that any representation-finite symmetric algebra is tilting-connected, so any Brauer tree algebra is a tilting-connected algebra. Hence, for any two-restricted tilting complex $\hat{T}$ for the Brauer star algebra B, there must exist a sequence of irreducible mutations converts B to $\hat{T}$. Regarding this fact, in [3] they give a sequence of irreducible mutations converts B to $\hat{T}$ in the case that $\hat{T}$ corresponds to the pointed Brauer tree with the reverse pointing or the left alternating pointing.

In this talk, for any two-restricted tilting complex $\hat{T}$, we give an algorithm to find such a sequence of mutations from the pointed Brauer tree to which $\hat{T}$ corresponds.

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45.

The Auslander-Reiten conjecture for non-Gorenstein rings

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The purpose of this talk is to study the vanishing of cohomology. The Auslander-Reiten conjecture is one of the long-standing conjectures about the vanishing, that is, for a Noetherian ring R and a finitely generated R-module M, $\mathrm{Ext}^i_R(M, M \oplus R) = 0$ for all $i > 0$ implies that M is a projective R-module. In this talk, we focus on the Auslander-Reiten conjecture for the case where R is commutative. In that case, the following result is fundamental.

Fact 1. Suppose that R is a commutative Noetherian local ring. Let Q be an ideal of R generated by a regular sequence on R. Then the Auslander-Reiten conjecture holds for R if and only if it holds for R/Q.

Motivated by this result, we explore the Auslander-Reiten conjecture for R/Q^ℓ in connection with that for R, where ℓ is a positive integer. Let us note that Q^ℓ do not preserve some homological properties, for example, Gorensteinness. Therefore R/Q^ℓ gives a new class of rings which satisfy the Auslander-Reiten conjecture. As a result of this talk, we have an affirmative answer to this question for the case where R is Gorenstein and ℓ is bounded above by the number of minimal generators of Q. Furthermore, we have two applications of the result. To state the applications, let us recall some notations.

Definition 2. (1) **(determinantal ring)** Let $s \le t$ be positive integers and $A[\mathbf{X}] = A[X_{ij}]_{1 \le i \le s, 1 \le j \le t}$ a polynomial ring over a commutative ring A. Let $\mathbb{I}_s(\mathbf{X})$ denote the ideal of $A[\mathbf{X}]$ generated by the maximal minors of the matrix (X_{ij}). Then $A[\mathbf{X}]/\mathbb{I}_s(\mathbf{X})$ is called a *determinantal ring over A*.

(2) **(Ulrich ideal)** Let $(R, \mathfrak{m})$ be a Cohen-Macaulay local ring and I an $\mathfrak{m}$-primary ideal. Then I is an *Ulrich ideal* if
  (a) I is not a parameter ideal, but $I^2 = \mathfrak{q}I$ for some parameter ideal $\mathfrak{q}$.
  (b) I/I^2 is a free R/I-module.

With these notations, we have the following, which is a goal of this talk.

Theorem 3. *The following assertions are true.*

(1) *Suppose A is either a complete intersection or a Gorenstein normal domain. Then the Auslander-Reiten conjecture holds for the determinantal ring $A[\mathbf{X}]/\mathbb{I}_s(\mathbf{X})$ if $2s \le t + 1$.*

(2) *Let R be a Cohen-Macaulay local ring. If there is an Ulrich ideal such that R/I is a complete intersection, then the Auslander-Reiten conjecture holds for R.*

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46.

Partial group actions and partial Galois extensions

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The talk will be based on the paper [1]. Let (S, α) be a set with a partial action of a group G. We shall present some results on partial orbits and partial stabilizers. If (S, α) is a partial Galois extension, we also study the partial Galois extensions in (S, α) generated by central idempotents of S. Let $\mathcal{M}_K$ denote the set of minimal elements of the Boolean ring generated by certain central idempotents of S associated to K. It is invariant under the partial action α restricted to K, denoted α_K. We will show how to construct partial Galois extensions in (S, α) via partial orbits in the α_K-invariant subset $\mathcal{M}_K$.

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47.

On CRP rings

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We study the one-sided regularity of matrices in upper triangular matrix rings in relation with the structure of diagonal entries. We consider next a ring theoretic condition that ab being regular implies ba being also regular for elements a, b in a given ring. Rings with such a condition are said to be *commutative at regular product* (simply, *CRP* rings). CRP rings are shown to be contained in the class of directly finite rings, and we prove that if R is a directly finite ring that satisfies the descending chain condition for principal right ideals or principal left ideals, then R is CRP. We obtain in particular that the upper triangular matrix rings over commutative rings are CRP.

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The 8th China-Japan-Korea International Conference

Title:
Rudimentary rings: Rings have a faithful indecomposable endoregular module

Gangyong Lee (Chungnam National University)

Jacobson, in 1945, introduced the notion of primitive rings and proved the structure theorem for primitive rings as an analogue of the Wedderburn-Artin structure theorem for semisimple artinian rings. The existence of a faithful simple module plays a crucial role in studying primitive rings. The study of the class of primitive rings has been a topic of wide interest.

Now, we introduce the notion of a rudimentary ring as a generalization of a primitive ring. A ring R is called *right rudimentary* if there exists a faithful right R-module M such that $\mathrm{End}_R(M)$ is a division ring. We provide results on this new concept and give a number of examples that delimit our results and the notions. Szele showed that there is no noncommutative division ring as the endomorphism ring of an abelian group (as a $\mathbb{Z}$-module). We extend this result on matrix rings over a commutative ring.

(This is a joint work with Cosmin Roman and Xiaoxiang Zhang)

49.

A note on Skolem-Noether algebras

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Abstract

The paper was motivated by Kovacs' paper [**?**], Isaacs' paper [**?**] and a recent paper [**?**] concerning Skolem-Noether algebras due to Brešar et al.. Let K be a unital commutative ring, not necessarily a field. Given a unital K-algebra S, where K is contained in the center of S, $n \in \mathbb{N}$, the goal of this paper is to study the question: when can a homomorphism $\phi\colon \mathrm{M}_n(K) \to \mathrm{M}_n(S)$ be extended to an inner automorphism of $\mathrm{M}_n(S)$? As an application of main results proved in the paper, it is proved that if S is a semilocal algebra with a central separable subalgebra R, then any homomorphism from R into S can be extended to an inner automorphism of S.

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Key words and phrases: Skolem-Noether algebra, (inner) automorphism, matrix algebra, central simple algebra, central separable algebra, semilocal ring, UFD, stably finite, Dedekind-finite.

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Speaker: Tsiu-Kwen Lee

50.

The center subalgebra of the quantized enveloping algebra of a simple Lie algebra revisited

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Let $\mathfrak{g}$ be a finite dimensional simple complex Lie algebra and $U = U_q(\mathfrak{g})$ the quantized enveloping algebra (in the sense of Jantzen) with q being generic. In this paper, we show that the center $Z(U_q(\mathfrak{g}))$ of the quantum group $U_q(\mathfrak{g})$ is isomorphic to a monoid algebra, and that $Z(U_q(\mathfrak{g}))$ is a polynomial algebra if and only if $\mathfrak{g}$ is of type A_1, B_n, C_n, D_{2k+2}, E_7, E_8, F_4 or G_2. Moreover, when $\mathfrak{g}$ is of type A_n, then $Z(U_q(\mathfrak{g}))$ is isomorphic to a quotient algebra of a polyno-mial algebra described by n-sequences; when $\mathfrak{g}$ is of type D_n with n odd, then $Z(U_q(\mathfrak{g}))$ is isomorphic to a quotient algebra of a polynomial algebra in $n+1$ variables with one relation; when $\mathfrak{g}$ is of type E_6, then $Z(U_q(\mathfrak{g}))$ is isomorphic to a quotient algebra of a polynomial algebra in fourteen variables with eight relations;

51.

Rota-Baxter H-operators and pre-Lie H-pseudoalgebras over a cocommutative Hopf algebra H

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ABSTRACT

The aim of this paper is to study the Rota-Baxter H-operators and H-pseudoalgebras of different types over a cocommutative Hopf algebra H. Firstly, we introduce the concept of a Rota-Baxter H-operator on an H-pseudoalgebra, and give some basic properties and examples. Then, we obtain a large number of pre-Lie (resp. associative) H-pseudoalgebras from the ordinary Rota-Baxter algebras. Finally, the annihilation algebras of the left pre-Lie H-pseudoalgebras are discussed.

52.

Relative coherent modules and semihereditary modules

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Given a positive integer n, a left R-module M is called n-coherent (resp. n-semihereditary) if every n-generated submodule of M is finitely presented (resp. projective). We investigate the properties of n-coherent modules and n-semihereditary modules. Various results are developed, many extending known results.

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53.

DG polynomial algebras and their homological properties

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In [1], we introduce and study differential graded (DG for short) polynomial algebras. In brief, a DG polynomial algebra $\mathcal{A}$ is a connected cochain DG algebra such that its underlying graded algebra $\mathcal{A}^{\#}$ is a polynomial algebra $k[x_1, x_2, \cdots, x_n]$ with $|x_i| = 1$, for any $i \in \{1, 2, \cdots, n\}$.

We describe all possible differential structures on DG polynomial algebras; compute their DG automorphism groups; study their isomorphism problems; and show that they are all homologically smooth and Gorestein DG algebras. Furthermore, it is proved that the DG polynomial algebra $\mathcal{A}$ is a Calabi-Yau DG algebra when its differential $\partial_{\mathcal{A}} \neq 0$ and the trivial DG polynomial algebra $(\mathcal{A}, 0)$ is Calabi-Yau if and only if n is an odd integer.

Beside these, I will also present our most recent works [2] on the various invariants of DG polynomial algebras.

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54.

AS-regularity of geometric algebras of plane cubic curves

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A geometric algebra $A = \mathcal{A}(E, \sigma)$ introduced by Mori [4] is a quadratic algebra which determines and is determined by the pair (E, σ) where E is a projective scheme and $\sigma \in \operatorname{Aut} E$. In noncommutative algebraic geometry, AS-regular algebras are the most important class of algebras to study, and Artin-Tate-Van den Bergh [1] showed that every 3-dimensional quadratic AS-regular algebra is a geometric algebra where E is $\mathbb{P}^2$ or a cubic curve in $\mathbb{P}^2$. In this talk, we study its converse.

Suppose that E is a cubic curve in $\mathbb{P}^2$. If E is singular, then a geometric algebra $A = \mathcal{A}(E, \sigma)$ is AS-regular for almost all $\sigma \in \operatorname{Aut} E$ by [3]. This is not the case if E is smooth. If E is smooth, then we choose a suitable $\tau \in \operatorname{Aut} E$ of finite order as in [2] so that every $\sigma \in \operatorname{Aut} E$ can be written as $\sigma = \sigma_p \tau^i$ where $p \in E$, $i \in \mathbb{Z}_{|\tau|}$ and $|\tau|$ is the order of τ. One of the main results of [3] is that we characterize AS-regularity of a geometric algebra $A = \mathcal{A}(E, \sigma_p \tau^i)$ in terms of the pair (p, i). It turns out that if $i = 0$, then A is always an AS-regular algebra (called a Sklyanin algebra). On the other hand, if $i \neq 0$, then A is hardly ever AS-regular.

If time permits, we will explain how to check AS-regularity using a twist of a superpotential in the sense of [5].

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55.

Efficient generation of ideals in core subalgebras of the polynomial ring $k[t]$ over a field k

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This is a joint work [1] with Naoki Endo, Shiro Goto, and Yuki Yamamoto.

In this talk, we find efficient systems of generators for ideals in certain subalgebras R of the polynomial ring $S = k[t]$ with one indeterminate t over a field k. The class of subalgebras which we explore in this talk naturally includes the semigroup rings $k[H]$ of numerical semigroups H.

Let R be a k-subalgebra of S. We say that R is a core of S, if $t^{c_0}S \subseteq R$ for some integer $c_0 > 0$. If R is a core of S, then

$$k[t^{c_0}, t^{c_0+1}, \dots, t^{2c_0-1}] \subseteq R \subseteq S,$$

and a given k-subalgebra R of S is a core of S if and only if $R \supseteq k[H]$ for some numerical semigroup H. Therefore, once R is a core of S, R is a finitely generated k-algebra of dimension one, and S is a birational module-finite extension of R with $t^{c_0}S \subseteq R : S$. Typical examples of cores are, of course, the semigroup rings $k[H]$ of numerical semigroups H. However, cores of S do not necessarily arise as semigroup rings for some numerical semigroups.

Let R be a core k-subalgebra of S. Take $f \in R$ such that $f(0) = 1$. We consider the ideal $I = fS \cap R$.

Problem 1. (1) Determine the minimal number of generators of I.
(2) Find a system of generators of I.

The problem (1) is already known by classical results given by O. Forster [2] and R. G. Swan [3]. In this talk, we give a method to solve the problem (2) and we recover the known results on the problem (1) in our situation.

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On a cubical generalization of preprojective algebras

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In this abstract K denotes a field of char $K = 0$ and Q denotes a finite acyclic quiver.

Recall that the preprojective algebra $\Pi(Q) = K\overline{Q}/(\rho)$ is the path algebra $K\overline{Q}$ of the double quiver $\overline{Q}$ of Q with the mesh relation $\rho = \sum_{\alpha \in Q_1} \alpha\alpha^* - \alpha^*\alpha$. It is an important mathematical object having rich representation theory and plenty of applications. In this joint work with M. Herschend, we study a cubical generalization $\Lambda = \Lambda(Q) := K\overline{Q}/([a, \rho] \mid a \in \overline{Q}_1)$ where $[-,+]$ is the commutator. We note that our algebra Λ is a special case of algebras $\Lambda_{\lambda,\mu}$ introduced by Etingof-Rains [4], which is a special case of algebras Λ_Φ introduced by Cachazo-Katz-Vafa [2]. However, our algebra Λ of very special case has intriguing properties, among other things it provides the universal Auslander-Reiten triangle.

We may equip Λ with a grading by setting $\deg \alpha = 0, \deg \alpha^* := 1$ for $\alpha \in Q_1$. Λ_1 We introduce an algebra to be $A = A(Q) := \begin{pmatrix} KQ & \Lambda_1 \\ 0 & KQ \end{pmatrix}$ where Λ_1 is the degree 1-part of Λ.

We note that Etingof-Latour-Rains [3] showed that if Q is a ADE-quiver, then Λ is symmetric. We summarize existing results on the algebras Λ and A.

Theorem 1. *(1) Λ is finite dimensional if and only if Q is an ADE-quiver if and only if A is 2-representation finite algebra. Assume that this is the case. Then Λ is a stably 3-Calabi-Yau symmetric algebra. Moreover we have an isomorphism $\Lambda \cong \bigoplus_{M \in \operatorname{ind} KQ} M \otimes_K M$ of KQ-modules.*

(2) Λ is infinite dimensional if and only if Q is not an ADE-quiver A is 2-representation infinite algebra. Assume this is the case. Then Λ is graded coherent and 3-Calabi-Yau.

(3) In any case, the 2-quasi-Veronese algebra of Λ is isomorphic to the 3-preprojective algebra of A. 2-APR-tilting operations on A are compatible with reflections of quiver Q.

Let Q be an ADE-quiver, $\widehat{Q}$ the extended one and $G < \mathrm{SL}(2)$ the corresponding finite subgroup. Then $\Lambda(\widehat{Q})$ is Morita equivalent to the skew group algebra $H * G$ where $H = K\langle x, y\rangle/([x,[x,y]],[y,[x,y]])$ is the Heisenberg algebra in two variables. The fixed subalgebra H^G is Gorenstein. Applying a result by Amiot-Iyama-Reiten [1], we obtain our version of algebraic McKay correspondences giving descriptions of the stable categories of CM-modules over H^G.

Theorem 2. *We have the following two equivalences of triangulated categories: $\underline{\mathrm{CM}}^{\mathbf{Z}} H^G \simeq \mathsf{D}^{\mathrm{b}}(A(Q))$, $\underline{\mathrm{CM}} H^G \simeq \mathsf{C}_2(A(Q))$ where C_2 denotes the 2-cluster category.*

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Noncommutative Matrix Factorizations and Knörrer's Periodicity Theorem

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In commutative ring theory, Knörrer's periodicity theorem is a powerful tool to study Cohen-Macaulay representation theory over hypersurfaces, and matrix factorizations are essential ingredients to prove the theorem. In order to study noncommutative quadric hypersurfaces, which are major objects of study in noncommutative algebraic geometry, we introduce a notion of noncommutative matrix factorization and prove the following noncommutative graded versions of Eisenbud's theorem [1] and Knörrer's periodicity theorem [2].

Theorem 1 ([3]). *If S is a graded quotient algebra of a noetherian AS-regular alegebra, $f \in S_d$ is a homogeneous regular normal element of degree $d > 0$, and $A = S/(f)$, then*

$$\mathrm{NMF}_S^{\mathbb{Z}}(f)/\operatorname{add}\{(1,f)\} \cong \mathrm{TR}_S^{\mathbb{Z}}(A)$$

$$\underline{\mathrm{NMF}}_S^{\mathbb{Z}}(f) := \mathrm{NMF}_S^{\mathbb{Z}}(f)/\operatorname{add}\{(1,f),(f,1)\} \cong \mathrm{TR}_S^{\mathbb{Z}}(A)/\operatorname{add}\{A\} =: \underline{\mathrm{TR}}_S^{\mathbb{Z}}(A)$$

where $\mathrm{NMF}_S^{\mathbb{Z}}(f)$ is the category of noncommutative graded matrix factorizations of f over S, $\mathrm{TR}^{\mathbb{Z}}(A)$ is the category of finitely generated graded totally reflexive modules over A, and $\mathrm{TR}_S^{\mathbb{Z}}(A) := \{M \in \mathrm{TR}^{\mathbb{Z}}(A) \mid \mathrm{pd}_S(M) < \infty\}$.

Theorem 2 ([4]). *Assume that the base field is algebraically closed of characteristic not 2. Let S be a noetherian AS-regular algebra and $f \in S_{2e}$ a homogeneous regular normal element of even degree $2e > 0$. If there exists a graded algebra automorphism σ of S such that $af = f\sigma^2(a)$ for every $a \in S$, then*

$$\underline{\mathrm{TR}}^{\mathbb{Z}}(S/(f)) \cong \underline{\mathrm{NMF}}_S^{\mathbb{Z}}(f) \cong \underline{\mathrm{NMF}}_{S[u;\sigma][v;\sigma]}^{\mathbb{Z}}(f+u^2+v^2) \cong \underline{\mathrm{TR}}^{\mathbb{Z}}(S[u;\sigma][v;\sigma]/(f+u^2+v^2))$$

where $S[u;\sigma][v;\sigma]$ is the Ore extension of S by σ with $\deg u = \deg v = e$.

If time permits, we discuss applications to noncommutative quadric hypersurfaces.

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An application of Hochschild cohomology to the moduli of subalgebras of the full matrix ring II

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Definition 1. We say that a subsheaf $\mathcal{A}$ of $\mathcal{O}_X$-algebras of $\mathrm{M}_n(\mathcal{O}_X)$ is a *mold* of degree n on a scheme X if $\mathrm{M}_n(\mathcal{O}_X)/\mathcal{A}$ is a locally free sheaf. We denote by $\mathrm{rank}\mathcal{A}$ the rank of $\mathcal{A}$ as a locally free sheaf.

Proposition 2. *The following contravariant functor is representable by a closed subscheme of the Grassmann scheme* $\mathrm{Grass}(d, n^2)$*:*

$$\begin{array}{lcccl} \mathrm{Mold}_{n,d} & : & (\mathbf{Sch})^{op} & \to & (\mathbf{Sets}) \\ & & X & \mapsto & \left\{ \mathcal{A} \,\middle|\, \mathcal{A} \textit{ is a mold of degree } n \textit{ on } X \textit{ with } \mathrm{rank}\mathcal{A} = d \right\}. \end{array}$$

The following theorem is one of the main results.

Theorem 3 ([1])**.** *Let S be a locally noetherian scheme. For a rank d mold $\mathcal{A}$ of degree n on S, denote by $\tau_{\mathcal{A}} : S \to \mathrm{Mold}_{n,d} \otimes_{\mathbb{Z}} S$ the morphism induced by $\mathcal{A}$. Set $\mathcal{A}(x) := \mathcal{A} \otimes_{\mathcal{O}_S} k(x) \subseteq \mathrm{M}_n(k(x))$, where $k(x)$ is the residue field of a point $x \in S$. Put $\mathrm{PGL}_{n,S} := \mathrm{PGL}_n \otimes_{\mathbb{Z}} S$. Let us define the S-morphism $\phi_{\mathcal{A}} : \mathrm{PGL}_{n,S} \to \mathrm{Mold}_{n,d} \otimes_{\mathbb{Z}} S$ by $P \mapsto P\mathcal{A}P^{-1}$. Then $\phi_{\mathcal{A}}$ is smooth if and only if $H^1(\mathcal{A}(x), \mathrm{M}_n(k(x))/\mathcal{A}(x)) = 0$ for each $x \in S$.*

Let k be an algebraically closed field. There are 26 types of k-subalgebras of $\mathrm{M}_3(k)$ up to inner automorphisms of $\mathrm{M}_3(k)$. For all types of k-subalgebras A of $\mathrm{M}_3(k)$, we have calculated Hochschild cohomology $H^i(A, \mathrm{M}_n(k)/A)$. We introduce several results not only for an algebraically closed field k but also for any commutative ring R.

Theorem 4 ([1])**.** *Set* $\mathrm{N}_3(R) := \left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix} \,\middle|\, a, b, c, d \in R \right\} \subset \mathrm{M}_3(R)$ *for a commutative ring R. Then*

$$H^i(\mathrm{N}_3(R), \mathrm{M}_3(R)/\mathrm{N}_3(R)) = \begin{cases} R^2 & (i = 0) \\ R^{i+1} & (i > 0). \end{cases}$$

Theorem 5 ([1])**.** *Set* $\mathrm{S}_4(R) := \left\{ \begin{pmatrix} a & b & c \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix} \,\middle|\, a, b, c \in R \right\} \subset \mathrm{M}_3(R)$ *for a commutative ring R. Then*

$$H^i(\mathrm{S}_4(R), \mathrm{M}_3(R)/\mathrm{S}_4(R)) = \begin{cases} R^4 & (i = 0) \\ R^{3 \cdot 2^i} & (i > 0). \end{cases}$$

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59.

Pure derived categories and weak balanced big Cohen-Macaulay modules

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Let R be a commutative noetherian ring of finite Krull dimension. In the first half of this talk, we give a new approach to reach the pure derived category of flat R-modules. Motivated by Neeman [5], Murfet and Salarian [3] defined the pure deived category as the Verdier quotient $\mathsf{K}(\mathsf{Flat}\, R)/\mathsf{K}_{\mathsf{pac}}(\mathsf{Flat}\, R)$ of the homotopy category of complexes of flat R-modules by the subcategory of pure acyclic complexes. There is a general theory due to Gillespie [1] that yields complete cotorsion pairs in the level of complexes, and it is possible to deduce from his work that the pure derived category is triangulated equivalent to the homotopy category $\mathsf{K}(\mathsf{FlCot}\, R)$ of complexes of flat cotorsion modules, where we say that an R-module M is cotorsion if $\mathrm{Ext}^1_R(F, M) = 0$ for any flat R-module F.

On the other hand, our main tool is a Čech complex of functors introduced in the previous work [4] with Yoji Yoshino. The Čech complex is constructed from localizations and completions with respect to prime ideals, and it yields a triangulated functor $\mathsf{K}(\mathsf{Flat}\, R) \to \mathsf{K}(\mathsf{FlCot}\, R)$. We prove that this functor is a left adjoint to the inclusion functor $\mathsf{K}(\mathsf{FlCot}\, R) \to \mathsf{K}(\mathsf{Flat}\, R)$, and this adjoint pair naturally induces the triangulated equivalence $\mathsf{K}(\mathsf{Flat}\, R)/\mathsf{K}_{\mathsf{pac}}(\mathsf{Flat}\, R) \xrightarrow{\cong} \mathsf{K}(\mathsf{FlCot}\, R)$. Moreover, using this fact, we concretely illustrate correspondence between different stable categories.

In the second half of this talk, we provide a reasonable framework to study an infinite version of Cohen-Macaulay representation theory. Following Holm [2], we say that an R-module M is *weak balanced big Cohen-Macaulay* if any system of parameters of the maximal ideal $\mathfrak{m}$ is a weak regular sequence on M, where $M/\mathfrak{m}M$ can be zero. If R is a Gorenstein local ring, then the subcategory $\mathsf{K}_{\mathsf{ac}}(\mathsf{FlCot}\, R)$ of acyclic complexes can be identified with the stable category of weak balanced big Cohen-Macaulay cotorsion modules modulo flat cotorsion modules. We explain that this stable category is suitable to develop Puninski's work [6].

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60.

Nilpotent polynomials with non-nilpotent coefficients

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It is well known that the coefficients of nilpotent polynomials over noncommutative rings generally are not all nilpotent. We show that this remains true even under extremely strong restrictions on the set of nilpotents in the coefficient ring. If R is a ring and its set of nilpotents, $\mathrm{Nil}(R)$, satisfies $\mathrm{Nil}(R)^2 = 0$, then in general $\mathrm{Nil}(R[x]) \not\subseteq \mathrm{Nil}(R)[x]$. This is proven by constructing an explicit polynomial example. The smallest possible degree of such a polynomial is seven. Related problems are raised.

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On Thompson's group F and its group algebra

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We have studied about group algebras of non-noetherian groups and showed that they are often primitive if base groups have non-abelian free subgroups. Our main method was two edge-colored graph theory. In general our method using these graphs seems to be effective for a group algebra of a group with a non-abelian free subgroup. But there exist some non-Noetherian groups with no non-abelian free subgroups such as Thompson's group F. In this talk, we first introduce an application of (undirected) two edge-colored graphs to group algebras of non-Noetherian groups and then improve our graph theory in order to be able to investigate group algebras of Thompson's group F. Finally, we introduce an application our graph theory to a problem on group algebras of Thompson's group F.

Definition 1 (Thompson's group F)**.** We define Thompson's group F as the group (under composition) of thosehomeomorphisms of the interval $[0, 1]$, which satisfy the following conditions:
1. they are piecewise linear and orientation-preserving,
2. in the pieces where the maps are linear, the slope is always a power of 2, and
3. the breakpoints are dyadic, i.e., they belong to the set $D \times D$, where $D = [0, 2] \cap \mathbb{Z}[\frac{1}{2}]$.

Thompson's group F has a following presentation:

$$\langle x_0, x_1, x_2, \cdots x_n, \cdots,\ |\ x_i^{-1} x_j x_i = x_{j+1},\ \text{for } i < j \rangle.$$

In this talk, we consider the following property (P) on the group algebra KG of a group G over a field K:

(P) There exist elements $a, b \in KG \setminus \{0\}$ such that $ax + by \neq 0$ for any $x, y \in KG \setminus \{0\}$.

We can see that many group algebras of non-noetherian groups satisfy the property (P) but have not known it on group algebras of Thompson's group F yet. We introduce a new approach to the problem.

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General heart construction and the Gabriel-Quillen embedding

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Recently, the notion of extriangulated category was introduced in [5] as a simultaneous generalization of triangulated category and exact category. A typical example of extriangulated categories is the cotorsion class of a cotorsion pair over a triangulated category.

Our first aim is to provide an analog of the following Gabriel-Quillen embedding theorem for extriangulated categories. It shows that any skeletally small exact category $\mathcal{C}$ can be embedded in the category $\mathsf{Lex}\,\mathcal{C}$ of left exact functors from $\mathcal{C}$ to the category Ab of abelian groups. More precisely, the canonical inclusion $R : \mathsf{Lex}\,\mathcal{C} \to \mathsf{Mod}\,\mathcal{C}$ admits a left adjoint Q and hence we have a localization sequence:

$$\begin{tikzcd} \operatorname{Ker} Q \arrow[r] & \mathsf{Mod}\,\mathcal{C} \arrow[r, "Q"] \arrow[l, bend left] & \mathsf{Lex}\,\mathcal{C}. \arrow[l, bend left, "R"] \end{tikzcd}$$

Moreover, the composed functor $E_{\mathcal{C}} : \mathcal{C} \hookrightarrow \mathsf{Mod}\,\mathcal{C} \xrightarrow{Q} \mathsf{Lex}\,\mathcal{C}$, which is called the Gabriel-Quillen embedding functor, is exact and fully faithful. We show a “finitely presented” version of the theorem for some extriangulated categories with weak-kernels, especially, there exists a Gabriel-Quillen type functor $E_{\mathcal{C}} : \mathcal{C} \to \mathsf{lex}\,\mathcal{C}$, where $\mathsf{lex}\,\mathcal{C}$ denotes the category of the finitely presented left exact functors from $\mathcal{C}$ to Ab. Using the functor $E_{\mathcal{C}}$, we provide necessary and sufficient conditions for an extriangulated category $\mathcal{C}$ to be exact and abelian, respectively.

Our main result is an application for a cotorsion pair $(\mathcal{U}, \mathcal{V})$ in a triangulated category $\mathcal{T}$. In [4, 1], it was proved that there exists an abelian category $\underline{\mathcal{H}}$ associated to the cotorsion pair, called the heart. This result was shown for two extremal cases [2, 3], namely, t-structures and 2-cluster tilting subcategories. Since the cotorsion class $\mathcal{U}$ has a natural extriangulated structure, we have the Gabriel-Quillen type functor $E_{\mathcal{U}} : \mathcal{U} \to \mathsf{lex}\,\mathcal{U}$. Our result provides a good understanding for a construction of the heart, in particular, we have an equivalence $\underline{\mathcal{H}} \simeq \mathsf{lex}\,\mathcal{U}$.

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63.

The chain conditions on ideals in composite generalized power series rings

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Let R be a commutative ring with identity and $(\Gamma, \leq)$ a strictly ordered monoid. Let R be a commutative ring with identity and $(\Gamma, \leq)$ a strictly ordered monoid. We denote by $[\![R^{\Gamma,\leq}]\!]$ the set of all mappings $f : \Gamma \to R$ such that $\mathrm{supp}(f) := \{\alpha \in \Gamma \mid f(\alpha) \neq 0\}$ is an artinian and narrow subset of Γ. With pointwise addition, $[\![R^{\Gamma,\leq}]\!]$ is an (additive) abelian group. Moreover, for every $\alpha \in \Gamma$ and $f, g \in [\![R^{\Gamma,\leq}]\!]$, the set $X_\alpha(f,g) := \{(\beta,\gamma) \in \Gamma \times \Gamma \mid \alpha = \beta + \gamma, f(\beta) \neq 0, \text{ and } g(\gamma) \neq 0\}$ is finite; so this allows to define the operation of *convolution*:

$$(fg)(\alpha) = \textstyle\sum_{(\beta,\gamma)\in X_\alpha(f,g)} f(\beta)g(\gamma).$$

Then $[\![R^{\Gamma,\leq}]\!]$ is a commutative ring (under these operations) with unit element $\mathbf{e}$, namely $\mathbf{e}(0) = 1$ and $\mathbf{e}(\alpha) = 0$ for all $\alpha \in \Gamma^*$, which is called the *ring of generalized power series* of Γ over R, which is first introduced by P.Ribenboim.

Let $D \subseteq E$ be an extension of commutative rings with identity, I a nonzero proper ideal of D, $(\Gamma, \leq)$ a strictly ordered monoid, and $\Gamma^* = \Gamma \setminus \{0\}$. Set $D + [\![E^{\Gamma^*,\leq}]\!] = \{f \in [\![E^{\Gamma,\leq}]\!] \mid f(0) \in D\}$ and $D + [\![I^{\Gamma^*,\leq}]\!] = \{f \in [\![D^{\Gamma,\leq}]\!] \mid f(\alpha) \in I \text{ for all } \alpha \in \Gamma^*\}$. Then $D \subsetneq D + [\![I^{\Gamma^*,\leq}]\!] \subsetneq [\![D^{\Gamma,\leq}]\!] \subseteq D + [\![E^{\Gamma^*,\leq}]\!] \subseteq [\![E^{\Gamma,\leq}]\!]$.

In this talk, we give some conditions for the rings $D + [\![E^{\Gamma^*,\leq}]\!]$ and $D + [\![I^{\Gamma^*,\leq}]\!]$ to satisfy the ascending chain condition on principal ideals (ACCP) or Noetherian.

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64.

Relationships between quantized algebras and their semiclassical limits

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A Poisson $\mathbb{C}$-algebra R appears in classical mechanical system and its quantized algebra appearing in quantum mechanical system is a $\mathbb{C}[[\hbar]]$-algebra $Q = R[[\hbar]]$ with star product $*$ such that for any $a, b \in R \subseteq Q$,

$$a * b = ab + B_1(a,b)\hbar + B_2(a,b)\hbar^2 + \dots$$

subject to

$$\{a,b\} = \hbar^{-1}(a * b - b * a)|_{\hbar=0}, \qquad \cdots \quad (**)$$

where $B_i : R \times R \longrightarrow R$ are bilinear products. The given Poisson algebra R is recovered from its quantized algebra Q by $R = Q/\hbar Q$ with Poisson bracket $(**)$, which is called its semiclassical limit. But it seems that the star product in Q is complicate and that Q is difficult to understand at an algebraic point of view since it is too big. For instance, if λ is a nonzero element of $\mathbb{C}$ then $\hbar - \lambda$ is a unit in Q and thus a so-called deformation of R, $Q/(\hbar - \lambda)Q$, is trivial. Hence it seems that we need an appropriate $\mathbb{F}$-subalgebra A of Q such that A contains all generators of Q, $\hbar \in A$ and A is understandable at an algebraic point of view, where $\mathbb{F}$ is a subring of $\mathbb{C}[[\hbar]]$.

Here we discuss how to find nontrivial deformations from quantized algebras and how similar quantized algebras are to their semiclassical limits. Results are illustrated by examples.

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On liftable DG modules over a commutative DG algebra

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This talk is based on a join work with Yuji Yoshino [4].

M.Auslander, S.Ding and Ø.Solberg [1] studied liftings and weak liftings of finitely generated modules over a commutative Noetherian algebra. Recently, S.Nasseh and S.Sather-Wagstaff [2] and S.Nasseh and Y.Yoshino [3] extended them to the case of differential graded (=DG) modules over differential graded (=DG) algebras.

Let $A \to B$ be a homomorphism of DG algebras over a commutative ring R. A DG B-module N is said to be *liftable to* A if there is a DG A-module M such that $N \cong B \otimes_A M$. In this case, M is called a *lifting of* N to A.

We concern a lifting problem in the situation $A \to B$ where $B = A\langle X | dX = t\rangle$ is an extended DG R-algebra of A by the adjunction of a variable X which kills a cycle t in A. The both papers [2, 3] treated the lifting problem or the weak lifting problem in such cases but with the assumption that the degree of X is odd. In this case, B is a Koszul complex over A.

In this talk, we study the lifting problem in the situation $A \to B = A\langle X | dX = t\rangle$ where B is obtained from adding a variable X of *even* degree. It should be noted that underlying graded algebra of B is a free algebra over A with a divided powers variable X. Let N be a semi-free DG B-module. The aim of this talk is to construct an obstruction for liftings of N to A as an element of $\mathrm{Ext}_B^{|X|+1}(N, N)$. To do this, we introduce a certain operator on the set of graded R-linear endomorphisms $\mathrm{End}_R(N)$, which is called the *j-operator*. The notion of the j-operator was first introduced by J.Tate in the paper [5]. We show that N is liftable to A if and only if the obstruction of N vanishes. The following is our main result of this talk.

Theorem 1. *Let A be a DG R-algebra and $B = A\langle X | dX = t\rangle$ be an extended DG R-algebra of A by the adjunction of a variable X of even positive degree. Assume that a DG B-module N is semi-free.*

(1) *If N is bounded below and $\mathrm{Ext}_B^{|X|+1}(N, N) = 0$, then N is liftable to A.*
(2) *If N is liftable to A and $\mathrm{Ext}_B^{|X|}(N, N) = 0$, then a lifting of N is unique up to DG A-isomorphisms.*

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THE STRUCTURE OF SALLY MODULES AND NORMAL HILBERT COEFFICIENTS

KAZUHO OZEKI

This talk is based on a joint work with S. K. Masuti and M. E. Rossi, and H. L. Truong.

The Sally module of an ideal is an important tool to interplay between Hilbert coefficients and the properties of the associated graded ring. In this talk we give new insights on the structure of the Sally module. We apply these results characterizing the almost minimal value of the first and the second normal Hilbert coefficients in an analytically unramified Cohen-Macaulay local ring.

Let $(R, \mathfrak{m})$ be an analytically unramified Cohen-Macaulay local ring of dimension $d > 0$ with infinite residue field $R/\mathfrak{m}$ and I an $\mathfrak{m}$-primary ideal of R. Let $\overline{I}$ denote the integral closure of I. Consider the so called *normal filtration* $\{\overline{I^n}\}_{n\in\mathbb{Z}}$ and we are interested in the corresponding Hilbert-Samuel polynomial. It is well-known that there are integers $\overline{\mathrm{e}}_i(I)$, called the *normal Hilbert coefficients* of I, such that for $n \gg 0$

$$\ell_R(R/\overline{I^{n+1}}) = \overline{\mathrm{e}}_0(I)\binom{n+d}{d} - \overline{\mathrm{e}}_1(I)\binom{n+d-1}{d-1} + \cdots + (-1)^d\overline{\mathrm{e}}_d(I).$$

Let us choose a parameter ideal J of R which forms a reduction of I. We set $\overline{\mathrm{r}}_J(I) := \min\{r \geq 0 \mid \overline{I^{n+1}} = J\overline{I^n} \text{ for all } n \geq r\}$.

Suppose that $d \geq 2$. Then by [1] it is known that

$$\overline{\mathrm{e}}_2(I) \geq \overline{\mathrm{e}}_1(I) - \overline{\mathrm{e}}_0(I) + \ell_R(R/\overline{I}) \geq \ell_R(\overline{I^2}/J\overline{I})$$

hold true and if either of the inequalities is an equality, then $\overline{\mathrm{r}}_J(I) \leq 2$, in particular the associated graded ring $\overline{G}(I) = \bigoplus_{n\geq 0} \overline{I^n}/\overline{I^{n+1}}$ is Cohen-Macaulay. Thus the ideals I with $\overline{\mathrm{e}}_1(I) = \overline{\mathrm{e}}_0(I) - \ell_R(R/\overline{I}) + \ell_R(\overline{I^2}/J\overline{I})$ and/or $\overline{\mathrm{e}}_2(I) = \overline{\mathrm{e}}_1(I) - \overline{\mathrm{e}}_0(I) + \ell_R(R/\overline{I})$ enjoy nice properties.

In this talk we present the structure of the Sally module in the case the first or the second normal coefficient is almost minimal, that is the equality $\overline{\mathrm{e}}_1(I) = \overline{\mathrm{e}}_0(I) - \ell_R(R/\overline{I}) + \ell_R(\overline{I^2}/J\overline{I}) + 1$ or $\overline{\mathrm{e}}_2(I) = \overline{\mathrm{e}}_1(I) - \overline{\mathrm{e}}_0(I) + \ell_R(R/\overline{I}) + 1$ holds true. We present in these cases the structure of the Sally module and, in particular, we investigate the depth of associated graded ring $\overline{G}(I)$.

As the title outlines, an important tool in this talk is the Sally module introduced by W. V. Vasconcelos [2]. The aim of this talk was to define a module in between the associated graded ring and the Rees algebra taking care of important information coming from a minimal reduction. Actually, a more detailed information comes from the graded parts of a suitable filtration $\{C^{(i)}\}$ of the Sally module that was introduced by M. Vaz Pinto in [3]. In this talk we prove some important results on $C^{(2)}$ which will be key ingredients for proving the main result. Some of them are stated in a very general setting. Our hope is that these will be successfully applied to give new insights to problems related to the normal Hilbert coefficients, for instance [1].

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67.

The Construction of a Continuous Linear Representation From a Topological Group Into Topological Module Space Over Principle Ideal Domain

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Let ρ_c be a continuous homomorphism from a topological group G into $GL_c(V)$ of all continuous bijective transformations. Homomorphism ρ_c like above is called a continuous linear representation from a topological group G into a topological vector space V. Because a module space is a vector space over a ring, not a field, we have construct a topological module space firstly. In this paper, we will construct a continuous linear representation from a topological group into a topological module space over principle ideal domain
.

Keywords: *topological group, topological module space, continuous linear representation.*

68.

Noetherian-like properties in polynomial and power series rings

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There are several Noetherian-like properties, e.g., Noetherian spectrum, Laskerian, strong-finite-type (SFT), piecewise Noetherian property. We investigate the stability of such properties under polynomial and power series extensions. In particular, we show that for a nonzero SFT prime ideal P of a Prüfer domain D, the following statements are equivalent: (1) $D[\![X]\!]_{P[\![X]\!]}$ is Noetherian; (2) $\operatorname{ht} P = 1$ and $\bar{k}[\![X]\!] = \bar{D}[\![X]\!]_{\bar{D}\setminus(0)}$, where $\bar{D} = D/P$ and $\bar{k}$ is the quotient field of $\bar{D}$; (3) $D[\![X]\!]_{P[\![X]\!]}$ is a valuation domain. As a corollary, we also show that for a Prüfer domain D, $D[\![X]\!]$ is piecewise Noetherian if and only if D is Noetherian.

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69.

A NOTE ON FI-SEMI INJECTIVE MODULES

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In this paper, the concept of $FI-M-$principally injective and $FI-$semi-injective (fully invariant -semi injective) modules are introduce and establishes some important results. Also investigate the characterizations of PP-ring and commutative semi-simple rings in term of $FI-$semi-injective modules.

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70.

On the monoid of ideals of orders in quadratic number fields

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This talk is based on a joint work with J. Brantner and A. Geroldinger. Let $\mathcal{O}$ be an order in a quadratic number field. A proper nonzero ideal of $\mathcal{O}$ is called an ideal atom if it is not the product of two proper ideals of $\mathcal{O}$. It is well-known that every nonzero ideal of $\mathcal{O}$ can be written as a finite product of ideal atoms. In this talk we investigate the structure of product decompositions of ideals of orders in quadratic numbers fields into ideal atoms. We discuss and determine several factorization theoretical invariants of the monoid of ideals of $\mathcal{O}$, like the elasticity, the unions of sets of lengths, the set of catenary degrees and the delta set. Furthermore, we apply the aforementioned results to characterize when the minimum of the delta set of $\mathcal{O}$ is bigger than one.

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Is Ware's problem true or not ?

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R. Ware gave the following problem in his paper: *Endomorphism rings of projective modules*, Trans. Amer. Math. Soc. **155** (1971), 233-256.

Problem: If a projective right R-module P has unique maximal submodule L, then L is the largest maximal submodule of P.

In the paper, A. Facchini, D. Herbera, I. Sakhajev *Finitely Generated Flat Modules and a Characterization of Semiperfect Rings*, Comm. in Algebra, Vol.**31** No.9(2003), 4195–214 asserts this problem is negative by showing the following properties:

Let $_RU$ be auniserial R-module and $S = \mathrm{End}_R(U)$ an endomorphism ring of $_RU$. Then he following conditions are equivalent.

(1) *U_S is not quasi-small.*

(2) *U_S is countable generated and a simple left R-module $_RR/K$ is flat and $\sum_{f \in K} f(U_S) = U_S$. Here $K = \{f \in S | f$ is not epimorphism$\}$.*

In this case, $_RK$ is an infinitely generated projective module with unique maximal submodule.

Here, U_S is called quasi-small if $U \cong T$ for a direct summand T of $\bigoplus_{i \in \Gamma} M_i$, then there is a finite subset $\Delta \subset \Gamma$ such that $T \subset \bigoplus_{i \in \Delta} M_i$. We emark T is a direct summand of $\bigoplus_{i \in \Delta} M_i$.

In this talk, we give some interesting example:

Example: *Let F be a field Z a commutative F-algebra with bases $\{v_x \mid 0 < x \leq 1\}$ with the multiplication $v_x \cdot v_y = v_{xy}$*

which seems to be a counter example of the above properties.

Also we report Ware's problem is true by using Nakayama-Azumaya Lemma for projective modules.

Furter, we invesitage sturctures of a module with unique maximal submodule.

One structure theorem is:

Theorem: *Let R be a ring and M a right R-module with unique maximal submodule L. then M is indecomposable or $M = M_1 \oplus M_2$ such that*

M_1 has unique maximal submodule and M_2 does not have any maximal submodules.

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72.

Q-graded Hopf quasigroups

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Abstract. Firstly, we introduce a class of new algebraic systems which generalize Hopf quasigroups and Hopf $\pi-$algebras called Q-graded Hopf quasigroups, and research some properties of them. Secondly, we define the representations of Q-graded Hopf quasigroups, i.e Q-graded Hopf quasimodules, research the construction method and fundamental theorem of them. Thirdly, we research the smash products of Q-graded Hopf quasigroups.

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73.

When is a quasi-discrete module quasi-projective?

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In this talk, we firstly introduce the concept "d-square full" modules related to "d-square free" modules. A module M is called *d-square free* if, whenever its factor module is isomorphic to $N^2 = N \oplus N$ for some module N, then $N = 0$ ([1], [2] (cf.[3])). A module M is called *d-square full* if, for any proper submodule X of M, there exist a proper submodule Y of M with $X \subseteq Y$ and an epimorphism $f : M \to (M/Y)^2$. Secondly, we show some basic properties of these modules. Finally, using the concept and results, we consider the problem "when is a quasi-discrete module quasi-projective?"

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ON THE RADIUS OF THE CATEGORY OF EXTENSIONS OF MATRIX FACTORIZATIONS

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This is a joint work with Ryo Takahashi (Nagoya University, Kansas University).

1. Introduction

Rouquier [5] has introduced the notion of the dimension of a triangulated category. As an analogue for abelian categories, Dao and Takahashi [2, 3] have introduced the notions of the dimension and radius of a full subcategory of an abelian category with enough projective objects. Our purpose of this talk is studying the dimension and radius of a full subcategory of the category of finitely generated modules over a commutative noetherian ring, and the dimension of the singularity category of a commutative noetherian ring.

For a noetherian ring R we denote by $\mathsf{D}_{\mathsf{sg}}(R)$ the singularity category of R, i.e., the Verdier quotient of the bounded derived category of $\operatorname{\mathsf{mod}} R$ by perfect complexes. Our main result yields the following corollary, which gives rise to an inequality of the dimensions of the singularity categories of 1-dimensional hypersurfaces. This corollary refines a recent result of Kawasaki, Nakamura and Shimada [4, Theorem 4.5], which assumes that the elements $x_1, \dots, x_n$ are powers of distinct prime elements and that the local ring S is complete.

Corollary 1. *Let S be a regular local ring of dimension two and $x_1, \dots, x_n \in S$. Then one has*

$$\dim \mathsf{D}_{\mathsf{sg}}(S/(x_1 \cdots x_n)) \leq \sup_{1 \leq i \leq n} \{\dim \mathsf{D}_{\mathsf{sg}}(S/(x_i))\} + 1.$$

In particular, if $S/(x_i)$ has finite CM-representation type for $1 \leq i \leq n$, then

$$\dim \mathsf{D}_{\mathsf{sg}}(S/(x_1 \cdots x_n)) \leq 1.$$

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75.

Action functor formalism

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Given a monoidal category $\mathcal{C} = (\mathcal{C}, \otimes, 1)$, we denote its Drinfeld center by $\mathcal{Z}(\mathcal{C})$. If the forgetful functor $U : \mathcal{Z}(\mathcal{C}) \to \mathcal{C}$ admits a right adjoint, say R, then the adjoint object of $\mathcal{C}$ is defined by $A_{\mathcal{C}} := UR(1)$. Our main concern is the case where $\mathcal{C}$ is a finite tensor category in the sense of Etingof-Ostrik [1]. Some fundamental results on finite-dimensional Hopf algebras have been extended to the setting of finite tensor categories by using the adjoint object and the adjunction $U \dashv R$ [3, 4]. Here a naive question arises: Why is the adjoint object useful for this kind of problems? As the adjoint object is defined in terms of the tensor product of $\mathcal{C}$, there is no obvious reason why it relates to somewhat ring-theoretic or representation-theoretic problems.

In this talk, I introduce an abstract framework connecting the adjoint object and several ring-theoretic notions and review how results on Hopf algebras are extended to the setting of finite tensor categories. Let $\mathcal{C}$ be a finite tensor category. A key ingredient is the 'action' functor $\rho : \mathcal{C} \to \mathrm{Rex}(\mathcal{C})$ defined by $\rho(X) = X \otimes (-)$, where $\mathrm{Rex}(\mathcal{C})$ is the category of right exact linear endofunctors on $\mathcal{C}$. It turns out that ρ has a right adjoint, say ρ^{ra}, and the adjoint object $A_{\mathcal{C}}$ is isomorphic to $\rho^{\mathrm{ra}}(\mathrm{id}_{\mathcal{C}})$. If we pick an arbitrary algebra L such that $\mathcal{C} \approx L$-mod, then $\mathrm{Rex}(\mathcal{C}) \approx L$-bimod. Some ring-theoretic notions can be formulated in terms of the category of bimodules. If a ring-theoretic notion which we aim to investigate has such a description, then one can transport it to the category $\mathcal{C}$ through the equivalence L-bimod $\approx \mathrm{Rex}(\mathcal{C})$ and the functor $\rho^{\mathrm{ra}} : \mathrm{Rex}(\mathcal{C}) \to \mathcal{C}$. This allows us to discuss relations between the notion and the adjoint object.

As explained in [5], this formalism has a lot of applications. For example, $\mathrm{Ext}^{\bullet}_{\mathcal{C}}(1, A_{\mathcal{C}})$ is shown to be isomorphic to the Hochschild cohomology $\mathrm{HH}^{\bullet}(L)$. Noteworthy, this result extends the $\mathrm{SL}_2(\mathbb{Z})$-action on the Hochschild cohomology of a ribbon factorisable Hopf algebra to the setting of non-semisimple modular tensor categories. Under the assumption that the double dual functor on $\mathcal{C}$ is isomorphic to the identity functor, $\mathrm{Ext}^{\bullet}_{\mathcal{C}}(A_{\mathcal{C}}, 1)$ is shown to be dual to Hochschild homology $\mathrm{HH}_{\bullet}(L)$ by a similar argument and an abstract treatment of the Nakayama functor established in [2]. Thus, under the same assumption, $\mathrm{Hom}_{\mathcal{C}}(A_{\mathcal{C}}, 1)$ is isomorphic to the space of symmetric linear forms on L. I will show further applications of this kind of techniques. If time permits, I will talk about a generalization to modules over a finite tensor category.

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76.

ELLIPTIC ALGEBRAS

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Ongoing work with Alex Chirvasitu (SUNY, Buffalo) and Ryo Kanda (Osaka).

This talk concerns the elliptic algebras $Q_{n,k}(E,\tau)$ defined by Odesskii and Feigin in 1989. Each $Q_{n,k}(E,\tau)$ is a connected graded $\mathbb{C}$-algebra, usually not commutative, depending on a pair of relatively prime integers $n > k \geq 1$, an elliptic curve $E = \mathbb{C}/\Lambda$, and a translation automorphism $z \mapsto z + \tau$ of E. At first glance, its definition as the free algebra $\mathbb{C}\langle x_0, \ldots, x_{n-1}\rangle$ modulo the n^2 relations

$$\sum_{r \in \mathbb{Z}_n} \frac{\theta_{j-i+r(k-1)}(0)}{\theta_{j-i-r}(-\tau)\theta_{kr}(\tau)} x_{j-r}x_{i+r} \qquad (i,j) \in \mathbb{Z}_n^2$$

reveals nothing. Here the $\theta_\alpha(z)$, $\alpha \in \mathbb{Z}_n$, are theta functions of order n that are quasi-periodic with respect to the lattice Λ. For a fixed (n,k,E) the $Q_{n,k}(E,\tau)$'s form a flat family of deformations of the polynomial ring $\mathbb{C}[x_0, \ldots, x_{n-1}]$. They are Koszul algebras so their Koszul duals form a flat family of finite dimensional algebras that are deformations of the exterior algebra $\wedge(\mathbb{C}^n)$. In a sequence of fascinating papers Feigin and Odesskii proved and claimed that the $Q_{n,k}(E,\tau)$'s have a number of remarkable properties. The ingredients that appear in the study of these algebras indicate the richness of the subject:

- the quantum Yang-Baxter equation with spectral parameter;
- the negative continued fraction expansion for $\frac{n}{k}$;
- a distinguished invertible sheaf $\mathcal{L}_{n/k}$ on $E^g = E \times \cdots \times E$, where g is the length of the continued fraction;
- the Fourier-Mukai transform $\mathbf{R}\mathrm{pr}_{1*}(\mathcal{L}_{n/k} \otimes^{\mathbf{L}} \mathrm{pr}_g^*(\,\cdot\,))$ is an auto-equivalence of the bounded derived category $\mathsf{D}^b(\mathrm{coh}(E))$ that provides a bijection $\mathcal{E}(1,0) \to \mathcal{E}(k,n)$ where $\mathcal{E}(r,d)$ is the set of isomorphism classes of indecomposable bundles of rank r and degree d on E;
- identities for theta functions in one and in g variables;
- the variety $X_{n/k}$ defined as the image of the morphism $|\mathcal{L}_{n/k}| : E^g \to \mathbb{P}^{n-1} = \mathbb{P}(H^0(E^g, \mathcal{L}_{n/k})^*)$, and an automorphism $\sigma : X_{n/k} \to X_{n/k}$ defined in terms of τ and the continued fraction;
- $X_{n/k} \cong E^g/\Sigma_{n/k}$, the quotient modulo the action of a subgroup of the symmetric group Σ_{g+1} defined in terms of the location of the 2's in the continued fraction;
- a homomorphism $Q_{n,k}(E,\tau) \to B(X_{n/k}, \sigma, \mathcal{L}_{n/k}) = B(E^g, \sigma, \mathcal{L}_{n/k})^{\Sigma_{n/k}}$ where $B(\cdot,\cdot,\cdot)$ is a twisted homogeneous coordinate ring à la Artin-Tate-Van den Bergh;
- when $X_{n/k}$ is E^g, an adjoint triple of functors $i^* \dashv i_* \dashv i^!$ where $i_* : \mathrm{Qcoh}(E^g) \to \mathrm{QGr}(Q_{n,k}(E,\tau))$ plays the role of a direct image functor for a morphism $E^g \to \mathrm{Proj}_{nc}(Q_{n,k}(E,\tau))$ in the sense of non-commutative algebraic geometry;
- a similar result when $X_{n/k}$ is the symmetric power S^gE;

The algebras $Q_{n,1}(E,\tau)$ when $n = 3,4$ are the 3- and 4-dimensional Sklyanin algebras discovered by Artin-Schelter (1986) and Sklyanin (1982) and studied by Artin-Tate-Van den Bergh and Smith-Stafford and Levasseur-Smith. For $n \geq 5$, a lot is known about $Q_{n,1}(E,\tau)$ due to work of Tate-Van den Bergh and Staniszkis.

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77.

POSITIVELY GRADED RINGS ARE MAXIMAL ORDERS AND GENERALIZED DEDEKIND RINGS

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Let $R = \oplus_{n\in\mathbb{Z}_0} R_n$ be a positively graded ring which is a sub-ring of strongly graded ring of type $\mathbb{Z}$, where R_0 is a Noetherian prime rings. We define a concept of $\mathbb{Z}_0-$invariant maximal order and show that R is a maximal order if and only if R_0 is a $\mathbb{Z}_0-$invariant maximal order. If R is a maximal order, then we completely describe all $v-$invertible ideals. As an application, we show that R is a generalized Dedekind prime if and anly if R_0 is a $\mathbb{Z}_0-$invariant generalized Dedekind prime rings. We give example of $\mathbb{Z}_0-$invariant generalized Dedekind prime rings but neither generalized Dedekind prime rings nor maximal orders.

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78.

Constructions of rejective chains

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Let $\mathcal{C}$ be a Krull–Schmidt category. In [2], a chain $\mathcal{C} = \mathcal{C}_0 \supset \mathcal{C}_1 \supset \cdots \supset \mathcal{C}_n = 0$ of subcategories of $\mathcal{C}$ is called a *total right rejective chain* if the following conditions hold:

(a) $\mathcal{C}_i$ is a *right rejective subcategory* of $\mathcal{C}$;

(b) the Jacobson radical of the factor category $\mathcal{C}_{i-1}/[\mathcal{C}_i]$ is zero.

In this talk, we give various examples of total right rejective chains. It is known that total right rejective chains are deeply related right-strongly quasi-hereditary algebras which are a special class of quasi-hereditary algebras introduced by Ringel [3].

Proposition 1 ([4, Theorem 3.22])**.** *Let A be an artin algebra. Then A is right-strongly quasi-hereditary if and only if the category* $\mathsf{proj}A$ *has a total right rejective chain*

The following theorem is one of main results of this talk. One is a refinement of [1, Proposition 1.6], and the other is a refinement of [1, Proposition 2.3] and [5, Proposition 3.1].

Theorem 2. *Let A be an artin algebra. If A is a locally hereditary algebra or a Nakayama algebra with heredity ideal, then the category* $\mathsf{proj}A$ *admits a total right rejective chain. In particular, the following statements hold.*

(1) *If A is a locally hereditary algebra, then A is right-strongly quasi-hereditary.*

(2) *Let A be a Nakayama algebra. Then A is a right-strongly quasi-hereditary algebra if and only if there exists a heredity ideal of A.*

Next, we study Δ-good module category $\mathcal{F}(\Delta)$ using rejective chains. In [6], it is shown that if the category $\mathcal{F}(\Delta)$ over a quasi-hereditary algebra A has an additive generator M, then the endomorphism algebra $\mathrm{End}_A(M)$ is quasi-hereditary. Motivated by this result, we give the following proposition.

Proposition 3. *Let A be a quasi-hereditary algebra and $\mathcal{F}(\Delta)$ the Δ-good module category. Assume that $\mathcal{F}(\Delta)$ has an additive generator M and multiplicity-free. Then the category $\mathcal{F}(\Delta)$ admits a total right rejective chain. In particular, the endomorphism algebra $\mathrm{End}_A(M)$ is a right-strongly quasi-hereditary algebra.*

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79.

BAER-KAPLANSKY CLASSES IN CATEGORIES

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The classical Baer-Kaplansky theorem states that any two torsion abelian groups having isomorphic endomorphism rings are isomorphic. An interesting topic of research has been to find other classes of abelian groups, and more generally, of modules, for which a Baer-Kaplansky-type theorem is still true. Such classes have been called *Baer-Kaplanksy classes* by Ivanov and Vámos [1].

Let $\mathcal{C}$ be a preadditive category and let $\mathcal{M}$ be a class of objects of $\mathcal{C}$. Following Ivanov and Vámos [1], $\mathcal{M}$ is called a *Baer-Kaplansky class* if for any two objects M and N of $\mathcal{M}$ such that $\mathrm{End}_{\mathcal{C}}(M) \cong \mathrm{End}_{\mathcal{C}}(N)$ (as rings), one has $M \cong N$. In this work we use functor categories techniques in order to relate Baer-Kaplansky classes in Grothendieck categories to Baer-Kaplansky classes in finitely accessible additive categories (in particular, the category of torsion-free abelian groups), exactly definable additive categories (in particular, the category of divisible abelian groups) and categories $\sigma[M]$ (in particular, the category of comodules over a coalgebra over a field).

Theorem 1. *Let $\mathcal{C}$ be a finitely accessible (an exactly definable) additive category. Let X and Y be objects of $\mathcal{C}$ such that X has a direct sum decomposition into indecomposable subobjects and there exists an IP-isomorphism $\Phi : \mathrm{End}_{\mathcal{C}}(X) \to \mathrm{End}_{\mathcal{C}}(Y)$. If one of the following conditions holds:*

1. *Y/X is pure-projective;*
2. *X is pure-injective;*

then X and Y are isomorphic.

Theorem 2. *Let $\mathcal{C}$ be a Krull-Schmidt finitely accessible (pure semisimple exactly definable) additive category. Then the class of finitely presented objects of $\mathcal{C}$ is Baer-Kaplansky if and only if the class of finitely presented indecomposable objects of $\mathcal{C}$ is Baer-Kaplansky.*

Theorem 3. *Let R be a ring with identity and let M be a pure semisimple left R-module. Then the class of finitely presented objects of $\sigma[M]$ is Baer-Kaplansky if and only if the class of (finitely presented) indecomposable objects of $\sigma[M]$ is Baer-Kaplansky.*

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Knörrer's periodicity for skew quadric hypersurfaces

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It is well-known that A is the homogeneous coordinate ring of a smooth quadric hypersurface in $\mathbb{P}^{n-1}$ if and only if $A \cong k[x_1, \dots, x_n]/(x_1^2 + \cdots + x_n^2)$. Applying the graded Knörrer's periodicity theorem, we have

$$\underline{\mathrm{CM}}^{\mathbb{Z}}(A) \cong \begin{cases} \underline{\mathrm{CM}}^{\mathbb{Z}}(k[x_1]/(x_1^2)) \cong \mathcal{D}^b(\operatorname{mod} k) & \text{if } n \text{ is odd,} \\ \underline{\mathrm{CM}}^{\mathbb{Z}}(k[x_1, x_2]/(x_1^2 + x_2^2)) \cong \mathcal{D}^b(\operatorname{mod} k^2) & \text{if } n \text{ is even.} \end{cases}$$

In this talk, we study a skew version of this equivalence.

Let $S = k\langle x_1, \dots, x_n\rangle/(x_i x_j - \varepsilon_{ij} x_j x_i)$ be a (± 1)-skew polynomial algebra generated in degree 1 where $\varepsilon_{ii} = 1, \varepsilon_{ij} = \varepsilon_{ji} = \pm 1$. Then $f = x_1^2 + \cdots + x_n^2$ is a homogeneous regular central element in S, so $A = S/(f)$ is an example of a homogeneous coordinate ring of a noncommutative quadric hypersurface in the sense of [2]. In this talk, we introduce graphical methods to compute $\underline{\mathrm{CM}}^{\mathbb{Z}}(S/(f))$. To do this, we associate each (± 1)-skew polynomial algebra S with a certain graph G. We present the four operations, called mutation, relative mutation, Knörrer reduction, and two points reduction for G, and show that they are powerful in computing $\underline{\mathrm{CM}}^{\mathbb{Z}}(S/(f))$. In fact, by using these four graphical methods, we can completely compute $\underline{\mathrm{CM}}^{\mathbb{Z}}(S/(f))$ up to $n \le 6$. As a result, in the case $n \le 6$, we see $\underline{\mathrm{CM}}^{\mathbb{Z}}(S/(f))$ is equivalent to one of $\mathcal{D}^b(\operatorname{mod} k^{2^i})$ where $0 \le i \le 5$. Moreover we also see that if $n \le 6$, then the structure of $\underline{\mathrm{CM}}^{\mathbb{Z}}(S/(f))$ is determined by the number of irreducible components of the point scheme of S that are isomorphic to $\mathbb{P}^1$. (From this it follows that the conjecture proposed in [3] holds true for $n \le 6$.)

This talk is based on the results of [1].

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81.

A Batalin-Vilkovisky differential on the complete cohomology ring of a Frobenius algebra

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In the 1980s, Buchweitz [1] introduced the notion of singularity category in order to provide a framework for Tate cohomology of Gorenstein algebras. Recently, under this framework, Wang [3] has defined the r-th *Tate-Hochschild cohomology group* of a Noetherian algebra A over a field k as

$$\underline{\mathrm{Ext}}^r_{A\otimes_k A^{\mathrm{op}}}(A,A) := \mathrm{Hom}_{\mathcal{D}_{\mathrm{sg}}(A\otimes_k A^{\mathrm{op}})}(A, A[r]),$$

where $r \in \mathbb{Z}$ and $\mathcal{D}_{\mathrm{sg}}(A\otimes_k A^{\mathrm{op}})$ is the singularity category of $A\otimes_k A^{\mathrm{op}}$. He also discovered a Gerstenhaber structure on the Tate-Hochschild cohomology ring

$$\underline{\mathrm{Ext}}^\bullet_{A\otimes_k A^{\mathrm{op}}}(A,A) := \bigoplus_{r\in\mathbb{Z}} \underline{\mathrm{Ext}}^r_{A\otimes_k A^{\mathrm{op}}}(A,A).$$

In 1957, Nakayama [2] introduced the complete cohomology groups $\widehat{\mathrm{HH}}^*(A,A)$ of a Frobenius algebra A over a field k, which is analogous to Tate cohomology of a finite group. It is known that the complete cohomology is isomorphic to the Tate-Hochschild cohomology. Wang [3] proved that there is a graded commutative product $\star$, called $\star$-product, on the complete cohomology such that the complete cohomology ring is isomorphic to Tate-Hochschild cohomology ring. Moreover, he showed that the complete cohomology ring of a symmetric algebra has a Batalin-Vilkovisky (BV) structure by using Tradler's BV differential and Connes operator. In particular, the BV differential generates the Gerstenhaber bracket on the Tate-Hochschild cohomology.

In this talk, we explain how to construct a BV structure on the complete cohomology of a Frobenius algebra whose Nakayama automorphism is diagonalizable.

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82.

ON GENERALIZED DEDEKIND MODULES OVER GENERALIZED DEDEKIND DOMAIN

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We introduce the notion of G-Dedekind modules, as the generalization of Dedekind modules. A module M is called a generalized Dedekind module (a G-Dedekind module for short) if any v-submodule of M is invertible. Let D be a Noetherian G-Dedekind domain and M a G-Dedekind D-module. We denote as $M[x]$ the polynomial $D[x]$-module in an indeterminate x and $K(x)$ the quotient field of $K[x]$, which is the quotient ring of $D[x]$. We show that $M[x]$ is a also a G-Dedekind $D[x]$-module.
Keywords : G-Dedekind modules, G-Dedekind domains, polynomial modules, invertible submodules.

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83.

Boolean Graphs - A Survey

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Abstract

A Boolean graph is the zero divisor graph of a Boolean ring. For a positive integer n, let $[n] = \{1, 2, \ldots, n\}$, and $2^{[n]}$ the power set of $[n]$. A finite Boolean graph B_n is isomorphic to a graph defined on the vertex set $2^{[n]} \setminus \{[n], \emptyset\}$, where two vertices are adjacent if and only if their meet is empty. In this paper, we give a survey of some works done in the area of research related to Boolean graphs, in both graph theoretic and algebraic aspects. We also introduce some most recent works by the author and others.

84.

The classification of Leibniz conformal algebras of rank three

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Abstract: Leibniz conformal algebras of rank one are either Virasoro Lie conformal algebras or abelian Lie conformal algebras. The classification of Lie conformal algebras of rank two had been completed. In this talk, I will introduce some progress on the classification of Leibniz conformal algebras of rank three.

85.

ON A PROBLEM OF SOCLE-DEFORMATIONS OF SELF-INJECTIVE ORBIT ALGEBRAS

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This is a report from joint work with A. Skowroński, [1] [2].

By an algebra we mean a basic, connected, finite-dimensional associative algebra with identity over a field K. For an algebra A, we consider finite-dimensional right A-modules, and denote by $\operatorname{mod} A$ the category of finite-dimensional right A-modules.

An algebra A is called *selfinjective* if A is injective in $\operatorname{mod} A$, and then $\operatorname{soc}(A) := \operatorname{soc}(A_A) = \operatorname{soc}({}_AA)$. Selfinjective algebras A and A' are said to be *socle equivalent* if the quotient algebras $A/\operatorname{soc}(A)$ and $A'/\operatorname{soc}(A')$ are isomorphic, in this case, A is also called a *socle deformation* of A'.

Let $\widehat{B}$ be the repetitive algebra of an algebra B, which is an infinite dimensional K-algebra with $\bigoplus_{i\in\mathbb{Z}}(B \oplus D(B))$ as a K-vector space, where $D(B) = \operatorname{Hom}_K(B, K)$. For some group G of automorphisms of $\widehat{B}$ regarded as a K-category, we have the category $\widehat{B}/G$ whose objects are by definition all G-orbits of objects of $\widehat{B}$, and $\widehat{B}/G$ as an algebra is finite dimensional selfinjective, called an *orbit algebra* of B. Important classes of socle deformations A of a selfinjective orbit algebra $\widehat{B}/G$ are of finite representation type (C. Riedtmann, 1980-83) and of polynomial growth (A, Skowroński, 1989) over an algebraically closed field K, in those cases B may be chosen as an algebra of finite global dimension and G an infinite cyclic group. In fact, B is a quasi-tilted algebra (more precisely, a tilted algebra for A of representation-finite or representation-domestic type (special case of polynomial growth)).

Problem: *Determine the selfinjective algebras A over a filed K socle equivalent to an orbit algebra $\widehat{B}/G$ of an algebra B of finite global dimension and G an infinite cyclic group.*

It should be noted that the problem asserts that the study of selfinjective algebras A determined in the problem may be reduced to the study of algebras B of finite global dimension, and it seems to be difficult even in the case of representation-finite selfinjective algebras over a (not necessarily algebraically closed) field.

In my talk, in view of the above facts by Riedtmann and Skowroń'ski, we consider the case where B is a tilted algebra and G is a cyclic group generated by an automorphism of the form $\varphi\nu_{\widehat{B}}$ where $\nu_{\widehat{B}}$ and φ are the Nakayama and a positive automprphisms respectively, and a solution to the case and applications are explained.

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Happel's functor and homologically well-graded Iwanaga-Gorenstein algebras

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In representation theory of algebras, derived category and stable category are two major classes of triangulated categories. It has been shown by many researchers that those different kinds of triangulated categories are related in various cases.

Happel [2] established the following relationship. For a finite dimensional algebra Λ over a field, one has a trivial extension $\mathrm{T}(\Lambda) = \Lambda \oplus D(\Lambda)$. $\mathrm{T}(\Lambda)$ is a graded self-injective algebra, and so the stable category $\underline{\mathrm{mod}}^{\mathbb{Z}}\, \mathrm{T}(\Lambda)$ of $\mathbb{Z}$-graded $\mathrm{T}(\Lambda)$-modules has a structure of triangulated category. In this setting, he constructed a fully faithful functor

$$\mathcal{H} : \mathsf{D}^{\mathrm{b}}(\mathrm{mod}\,\Lambda) \to \underline{\mathrm{mod}}^{\mathbb{Z}}\, \mathrm{T}(\Lambda).$$

He also showed that $\mathcal{H}$ gives an equivalence precisely when $\mathrm{gl.dim}\,\Lambda < \infty$.

This functor $\mathcal{H}$ can be generalized as follows. We start from a finitely graded Iwanaga-Gorenstein algebra A, and replace $\underline{\mathrm{mod}}^{\mathbb{Z}}\, \mathrm{T}(\Lambda)$ with the stable category $\underline{\mathsf{CM}}^{\mathbb{Z}} A$ of $\mathbb{Z}$-graded Cohen-Macaulay A-modules. Also we replace Λ with the Beilinson algebra ∇A of A. Then there is a functor

$$\mathcal{H} : \mathsf{D}^{\mathrm{b}}(\mathrm{mod}\,\nabla A) \to \underline{\mathsf{CM}}^{\mathbb{Z}}\, A.$$

Note that this is not fully faithful in general.

In my talk, we study when this functor $\mathcal{H}$ is fully faithful or gives an equivalence. For this purpose, we introduce *homologically well-graded* Iwanaga-Gorenstein algebra, which can be characterized as a finitely graded algebra posses a homological symmetry. Our main result is that this class of algebras is precisely the class of finitely graded Iwanaga-Gorenstein algebras A that $\mathcal{H}$ is fully faithful. We also identify the class that $\mathcal{H}$ gives an equivalence. Our results recover some of results shown in previous works [1, 2, 3].

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87.

Auslander-Bridger theory for projective complexes over commutative Noetherian rings

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Let R be a commutative Noetherian ring and let $\mathcal{K}(R)$ be the homotopy category of all complexes of finitely generated projective modules over R. For any $X \in \mathcal{K}(R)$ the R-dual complex $X^* = \mathrm{Hom}_R(X, R)$ is defined and the operation $(-)^*$ gives the duality on $\mathcal{K}(R)$. The main theorem of this talk is the following:

Main Theorem [2] *Let $X \in \mathcal{K}(R)$ and assume that R is a generically Gorenstein ring. Then, X is acyclic if and only if X^* is acyclic.*

Recall that R is called a generically Gorenstein ring if the total ring of quotients is Gorenstein. This theorem includes the Tachikawa conjecture and the dependence of totally reflexivity conditions for modules over a generically Gorenstein ring.

To prove this theorem we need to develop and establish the Auslander-Bridger type theory for $\mathcal{K}(R)$. Precisely speaking, we have a natural mapping $\rho^i_{X,R} : H^{-i}(X^*) \to H^i(X)^*$ for $X \in \mathcal{K}(R)$ and $i \in \mathbb{Z}$. We say that a complex $X \in \mathcal{K}(R)$ is ***torsion-free** (resp. ***reflexive**) if $\rho^i_{X,R}$ are injective (resp. bijective) mappings for all $i \in \mathbb{Z}$. Let $\mathrm{Add}(R)$ be the additive full subcategory of $\mathcal{K}(R)$ consisting of all split complexes. We can show that $\mathrm{Add}(R)$ is functorially finite in $\mathcal{K}(R)$ and hence every complex in $\mathcal{K}(R)$ is resolved by complexes in $\mathrm{Add}(R)$. Define $\underline{\mathcal{K}(R)}$ to be the factor category $\mathcal{K}(R)/\mathrm{Add}(R)$. Then we are able to define the syzygy functor Ω and the cosyzygy functor Ω^{-1} on $\underline{\mathcal{K}(R)}$, and as a result we have an adjoint pair (Ω^{-1}, Ω) of functors. Then we can show that X is *torsion-free iff $X \cong \Omega^{-1}\Omega X$ in $\underline{\mathcal{K}(R)}$. And under the assumption that R is generically Gorenstein, X is *reflexive iff $X \cong \Omega^{-2}\Omega^2 X$ in $\underline{\mathcal{K}(R)}$.

There is a triangles of the form

$$\Delta^{(n,0)}(X) \to \Omega^{-n}\Omega^n(X) \to X \to \Delta^{(n,0)}(X)[1],$$

for $X \in \mathcal{K}(R)$ and $n > 0$, where $\Delta^{(n,0)}(X)$ has a finite $\mathrm{Add}(R)$-resolution of length at most $n-1$. This is one of the key theorems in order to prove Main Theorem. The second key observation is that any syzygy complex $\Omega^r X$ ($\forall r > 0$) is *torsion-free if $H(X^*) = 0$.

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Density of g-vector cones from triangulated surfaces

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This talk is based on [2]. Let A be a finite dimensional algebra over a field k. Adachi-Iyama-Reiten introduced τ-tilting theory which is generalization of tilting theory from the viewpoint of mutation. We denote

- $\mathrm{s}\tau\text{-tilt}\, A = \{$isomorphism classes of basic support τ-tilting A-modules$\}$,
- $\mathrm{s}\tau\text{-tilt}^+ A \subseteq \mathrm{s}\tau\text{-tilt}\, A$ consists of mutation equivalence classes containing A,
- $\mathrm{s}\tau\text{-tilt}^- A \subseteq \mathrm{s}\tau\text{-tilt}\, A$ ―――――――――――――――――――――――― 0.

Problem. $\mathrm{s}\tau\text{-tilt}\, A \setminus (\mathrm{s}\tau\text{-tilt}^+ A \cup \mathrm{s}\tau\text{-tilt}^- A) = ?$.

In this talk, we consider the Jacobian algebras defined from triangulated surfaces.

- (S, M) : a connected compact oriented Riemann surface with marked points.
- Q_T : a quiver associated with a triangulation T of (S, M).
- W : a non-degenerate potential of Q_T such that the associated Jacobian algebra $J = J(Q_T, W)$ is finite dimensional.

Remark 1. For the cluster algebra $\mathcal{A}(Q_T)$ associated with Q_T, there are bijections

$$\mathrm{s}\tau\text{-tilt}^+ J \leftrightarrow \{\text{clusters in } \mathcal{A}(Q_T)\} \leftrightarrow \{\text{tagged triangulations of } (S, M)\},$$

where if (S, M) is a closed surface with exactly one puncture, then tags are plain.

We give an answer of Problem for $A = J$.

Theorem 2. *We have* $\mathrm{s}\tau\text{-tilt}\, J = \mathrm{s}\tau\text{-tilt}^+ J \cup \mathrm{s}\tau\text{-tilt}^- J$. *More precisely, if (S, M) is a closed surface with exactly one puncture, then* $\mathrm{s}\tau\text{-tilt}\, J = \mathrm{s}\tau\text{-tilt}^+ J \sqcup \mathrm{s}\tau\text{-tilt}^- J$*; otherwise,* $\mathrm{s}\tau\text{-tilt}\, J = \mathrm{s}\tau\text{-tilt}^+ J = \mathrm{s}\tau\text{-tilt}^- J$.

The key ingredient to prove Theorem 2 is an invariant, called g-vector cone, of τ-tilting modules. The *g-vector cone* of a τ-tilting module M is a cone $C_J(M)$ in $K_0(J) \otimes_{\mathbb{Z}} \mathbb{R}$, where $K_0(J)$ is the Grothendieck group of J. They have the following property.

Theorem 3. [1, Theorem 2.4] *Any g-vector cone is of full-dimensional.*

The following is the main result in [2].

Theorem 4. *We have*

$$\overline{\bigcup_{M \in \mathrm{s}\tau\text{-tilt}^+ J \cup \mathrm{s}\tau\text{-tilt}^- J} C_J(M)} = K_0(J) \otimes_{\mathbb{Z}} \mathbb{R}\,.$$

Theorem 2 immediately follows from Theorems 3 and 4.

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89.

GORENSTEIN-PROJECTIVE AND SEMI-GORENSTEIN-PROJECTIVE MODULES

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Abstract. Let A be an artin algebra. An A-module M will be said to be semi-Gorenstein-projective provided that $\mathrm{Ext}^i(M, A) = 0$ for all $i \geq 1$. All Gorenstein-projective modules are semi-Gorenstein-projective and only few and quite complicated examples of semi-Gorenstein-projective modules which are not Gorenstein-projective have been known. One of the aims of this talk is to provide conditions on A such that all semi-Gorenstein-projective left modules are Gorenstein-projective (such an algebra is called left weakly Gorenstein). In particular, in case there are only finitely many isomorphism classes of indecomposable left modules which are both semi-Gorenstein-projective and torsionless, then A is left weakly Gorenstein. This combines the thoughts of Y. Yoshino and R. Marczinzik. On the other hand, we exhibit a 6-dimensional algebra Λ with a semi-Gorenstein-projective module M which is not torsionless (thus not Gorenstein-projective). Actually, also the Λ-dual module M^* is semi-Gorenstein-projective. In this way, we show the independence of the total reflexivity conditions of L. L. Avramov and A. Martsinkovsky, thus completing a partial proof by D. A. Jorgensen and L. M. Şega. Since all the syzygy-modules of M and M^* are 3-dimensional, the example can be checked (and visualized) quite easily.

This talk is based on a joint work with Claus Michael Ringel.

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90.

Tilting modules over Auslander-Gorenstein algebras

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For a finite dimensional algebra Λ and a non-negative integer n, we characterize when the set $\mathrm{tilt}_n\Lambda$ of additive equivalence classes of tilting modules with projective dimension at most n has a minimal (or equivalently, minimum) element. This generalize results of Happel-Unger. Moreover, for an n-Gorenstein algebra Λ with $n \geq 1$, we construct a minimal element in $\mathrm{tilt}_n\Lambda$. As a result, we give equivalent conditions for a k-Gorenstein algebra to be Iwanaga-Gorenstein. Moreover, for an 1-Gorenstein algebra Λ and its factor algebra $\Gamma = \Lambda/(e)$, we show that there is a bijection between $\mathrm{tilt}_1\Lambda$ and the set $\mathrm{sttilt}\Gamma$ of isomorphism classes of basic support τ-tilting Γ-modules, where e is an idempotent such that $e\Lambda$ is the additive generator of projective-injective Λ-modules. This is a joint work with Osamu Iyama.

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91.

SEMIBRICKS, WIDE SUBCATEGORIES AND RECOLLEMENTS

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The notion of (semi)bricks, regarded as a generalization of (semi)simple modules, which correspond bijectively to wide subcategories, appeared in a paper of Ringel in 1976. In recently years, there has been several new developments motivated by links to τ-tilting theory studied by Asai. In this talk, we mainly apply the gluing techniques for (semi)bricks and reduction techniques for wide subcategories along a recollement introduced by Beilinson-Bernstein-Deligne. Gluing support τ-tilting modules through a formula(perhaps compatible with gluing semibricks, t-structures, co-t-structures, silting objects) is left open.

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DIRECTED PARTIAL ORDERS OVER NON-ARCHIMEDEAN FIELDS

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Abstract. Let F be a non-archimedean linearly ordered field, and $C = F+F\sqrt{-1}$. In this talk, we classify all directed partial orders on C with $1 > 0$ via bounded semigroups of F^+ and those with $1 \not> 0$ via special convex subsets of F^+. We note that none of these directed partial orders is a lattice order on C, which gives the Birkhoff-Pierce problem a negative answer in this case.(Joint with Jingjing Ma and liusan Wu)

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93. On an open problem concerning the small finitistic dimension of a commutative ring

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Let R be a commutative ring with identity and let $\mathcal{Q}$ be the set of finitely generated semiregular ideals of R. A $\mathcal{Q}$-torsion-free R-module M is called a Lucas module if $\mathrm{Ext}^1_R(R/J, M) = 0$ for any $J \in \mathcal{Q}$. And R is called a DQ-ring if every ideal of R is a Lucas module. It is proved that if the small finitistic dimension of R is zero, then R is a DQ ring. In terms of a trivial extension, we construct a total ring of quotients $R = D \propto H$ which is not a DQ ring. Thus in this case, the small finitistic dimension of R is not zero. Then this fact gives a negative answer to an open problem posed by Cahen *et al.*.

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94.

The structure of connected (graded) Hopf algebras

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In this talk, we will present a structure theorem for connected graded Hopf algebras over a field of characteristic 0 by claiming the existence of a family of homogeneous generators and a total order on the index set that satisfy some excellent conditions. The approach to the structure theorem is constructive based on the combinatorial properties of Lyndon words and the standard bracketing on words. As consequences of the structure theorem, we will show that connected graded Hopf algebras of finite Gelfand-Kirillov dimension over a field of characteristic 0 are all iterated Hopf Ore extensions of the base field as well as some keystone facts of connected Hopf algebras over a field of characteristic 0.

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95.

QUATERNION RING AND APPLICATION IN HYPERNORMAL FORM OF 4 DIMENSIONAL SEMI-SIMPLE NONLINEAR DYNAMICAL SYSTEMS

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Quaternion in the real number domain was first proposed in 1843 by Hamilton, whose purpose was to find a way to study spatial geometry similar to complex number in solving plane problems [1]. As a famous example, quaternion plays an important role in the ring theory. Many experts and scholars have studied the theory and related properties of quaternion ring [2, 3]. Due to its unique properties and advantages [4], quaternion ring theory has potential application prospects in many fields [5].

In the field of nonlinear dynamics, further reduction of normal form or hypernormal form (unique normal form, simplest normal form) has become one of the most important topics [6, 7]. However, there are still very few results for further reduction of normal forms for higher dimensional systems. One of the main difficulties is that the matrices in the computation of normal forms are usually very large and which makes the computation very difficult. In this paper, we present a new method of expressing and simplifying high dimensional nonlinear dynamical systems by introducing the quaternion ring theory, and investigate the hypernormal form of a 4 dimensional semi-simple nonlinear dynamical system. The main technique used to the computation is the combination of a new grading function and multiple Lie brackets. The introduction of quaternion ring theory helps to reduce the computation of large size matrices in the study of hypernormal forms.

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96.

Structures of Irreducible Yetter-Drinfeld Modules over Quasi-Triangular Hopf Algebras

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(This is a joint work with Dr. Liu Zhimin)

Let (H, R) be a finite dimensional semisimple and cosemisimple quasi-triangular Hopf algebra over a field k. In this talk, by using the Majid's transmuted braided group of H and Ostrik's theorem on characterizing module categories over monoidal categories, we present a structure theorem of irreducible objects of the Yetter-Drinfeld H-module category.

Our structure theorem generalizes the results of Dijkgraaf-Pasquier-Roche and Gould on Yetter-Drinfeld modules over finite group algebras.

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97.

Lie solvability in matrix algebras

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If an algebra $\mathcal{A}$ satisfies the polynomial identity

$$[x_1, y_1][x_2, y_2] \cdots [x_{2^m}, y_{2^m}] = 0$$

(for short, $\mathcal{A}$ is D_{2^m}), then $\mathcal{A}$ is trivially Lie solvable of index $m+1$ (for short, $\mathcal{A}$ is Ls_{m+1}). We will show that the converse holds for subalgebras of the upper triangular matrix algebra $U_n(R)$, R any commutative ring, and $n \geq 1$.

We will also consider two related questions, namely whether, for a field F, an Ls_2 subalgebra of $M_n(F)$, for some n, with (F-)dimension larger than the maximum dimension $2 + \left\lfloor \frac{3n^2}{8} \right\rfloor$ of a D_2 subalgebra of $M_n(F)$, exists, and whether a D_2 subalgebra of $U_n(F)$ with (the mentioned) maximum dimension, other than the typical D_2 subalgebras of $U_n(F)$ with maximum dimension, which were exhibited in [1] and refined in [3], exists. Partial results with regard to these two questions are obtained.

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