The 57th Symposium on Ring Theory

and Representation Theory

ABSTRACT

Okayama University of Science, Okayama

September 8 – 12, 2025

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Tilting modules for algebraic groups Ⅲ

An extension theorem of semibricks

Sota Asai

Bricks and semibricks are generalizations of simple modules and semisimple modules, respectively, via Schur's Lemma.

Namely, let Λ be a finite-dimensional algebra over an algebraically closed field K, and $\mathsf{mod}\,\Lambda$ be the category of finitely generated right Λ-modules. Then, a module $B \in \mathsf{mod}\,\Lambda$ is called a *brick* if its endomorphism algebra $\mathsf{End}_\Lambda(B)$ is isomorphic to K, and we write $\mathsf{brick}\,\Lambda$ for the set of isoclasses of bricks. Moreover, a (finite or infinite) subset $\mathcal{S} \subset \mathsf{brick}\,\Lambda$ is called a *semibrick* if $\mathsf{Hom}_\Lambda(B_1, B_2) = 0$ for any distinct $B_1, B_2 \in \mathcal{S}$, and we write $\mathsf{sbrick}\,\Lambda$ for the set of semibricks.

Clearly, any subset of a semibrick is a semibrick again. Also, $\mathsf{sbrick}\,\Lambda$ is a partially ordered set with respect to inclusions. Therefore, it is natural to consider *maximal* semibricks.

In general, many algebras admit an infinite semibrick $\mathcal{S}$, that is, a semibrick consisting of infinitely many isoclasses of bricks. In this case, any finite subset $\mathcal{S}' \subsetneq \mathcal{S}$ is never maximal. On the other hand, the set of isoclasses of all simple Λ-modules is a maximal finite semibrick. I considered when a finite semibrick is maximal, and obtained a nice necessary condition in terms of representation varieties.

Let $\Lambda \simeq KQ/I$ with a finite quiver Q and an admissible ideal $I \subset KQ$. Then, Λ-modules are identified with quiver representations of Q satisfying the relations associated with I. For each fixed dimensional vector $d = (d_i)_{i \in Q_0}$, we have the *representation variety* $\mathsf{rep}(Q, I, d)$, which can be considered as the set of all Λ-modules M whose dimension vectors $\underline{\mathsf{dim}}\,M$ are d. The direct product $\prod_{i \in Q_0} \mathsf{GL}_K(d_i)$ acts on $\mathsf{rep}(Q, I, d)$, so we can consider the *orbit* $\mathcal{O}_M$ of each point $M \in \mathsf{rep}(Q, I, d)$, which is equal to $\{N \in \mathsf{rep}(Q, I, d) \mid M \simeq N \text{ as } \Lambda\text{-modules}\}$. Each orbit $\mathcal{O}_M$ is contained in a single irreducible component of $\mathsf{rep}(Q, I, d)$.

Now. I can state the following main result of this talk, which is an extension theorem of semibricks.

Theorem 1. *Assume that $\Lambda \simeq KQ/I$ is a finite-dimensional algebra over an algebraically closed field K. Let $\mathcal{S} \in \mathsf{sbrick}\,\Lambda$ be a finite semibrick. Then, we have the following assertions.*

(1) *Let B be a brick in $\mathcal{S}$, and $\mathcal{Z}$ be an irreducible component of $\mathsf{rep}(Q, I, \underline{\mathsf{dim}}\,B)$ such that $B \in \mathcal{Z}$. If the orbit $\mathcal{O}_M$ is not dense in $\mathcal{Z}$, then there exists an infinite semibrick $\mathcal{S}'$ such that $\mathcal{S} \subsetneq \mathcal{S}'$ and $\mathcal{S}' \setminus \mathcal{S} \subset \mathcal{Z}$.*

(2) *If $\mathcal{S}$ is maximal in $\mathsf{sbrick}\,\Lambda$, then for any $B \in \mathcal{S}$, the orbit $\mathcal{O}_B$ is an open dense subset of some irreducible component of $\mathsf{rep}(Q, I, \underline{\mathsf{dim}}\,B)$.*

This is a much stronger version of results in [1]. The results in the case that Λ is hereditary is used to prove the Jordan-Dedekind property of the bounded derived category $\mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,\Lambda)$ in [4]. If time permits, I would like to explain a sketch of my proof, which is based on [3, 2].

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On the Auslander–Reiten theory for extended module categories of proper connective DG-algebras

Nao Mochizuki and Marvin Plogmann

Auslander–Reiten theory in the module category has played a central role in the representation theory of finite-dimensional algebras. In this talk, we introduce a generalization of Auslander–Reiten theory for module categories to the framework of proper connective DG-algebras via the use of d-extended module categories. This talk is based on [3].

The d-extended module category is a generalization of the module category over a finite-dimensional algebra. It is defined as a full subcategory of the derived category of DG-algebras and naturally carries the structure of an extriangulated category [4]. We demonstrate that this category admits Auslander–Reiten–Serre duality and Auslander–Reiten sequences, formulated in [1].

These results are already known in the following cases:

(1) For finite-dimensional algebras [5],
(2) For d-self-injective DG-algebras [2],
(3) For homologically smooth DG-algebras [1].

Furthermore, we provide an explicit description of the Auslander–Reiten–Serre duality and discuss the first Brauer–Thrall conjecture in this setting.

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Minimal projective resolution and magnitude homology of geodetic metric spaces

Shun Wakatsuki, Yasuhiko Asao, Aaron Chan

Magnitude homology is an invariant of metric spaces, introduced by Hepworth–Willerton [3] and Leinster–Shulman. It reflects geometric information such as a certain kind of convexity in the metric space and has been studied as an interesting invariant. It can be described as a Tor over a graded ring σX, which is obtained as a quotient of a certain path algebra. In this talk, we introduce results obtained by explicitly constructing a minimal projective resolution over σX. Koszulness of σX is also discussed.

We begin by giving the definition of the magnitude homology of a metric space (X, d). Let $\mathbb{K}$ be a commutative ring. We denote the free module spanned by the Cartesian product of X (as a set) by $\mathsf{MC}_n(X) = \mathbb{K}X^{n+1}$. For $0 < i < n$, define the map $\partial_{n,i} \colon \mathsf{MC}_n(X) \to \mathsf{MC}_{n-1}(X)$ by

$$\partial_{n,i}(x_0, \ldots, x_n) = \begin{cases} (x_0, \ldots, \hat{x_i}, \ldots, x_n) & \text{if } d(x_{i-1}, x_i) + d(x_i, x_{i+1}) = d(x_{i-1}, x_{i+1}) \\ 0 & \text{otherwise} \end{cases}$$

The condition $d(x_{i-1}, x_i) + d(x_i, x_{i+1}) = d(x_{i-1}, x_{i+1})$ means that the triangle inequality holds with equality, which roughly corresponds to the condition that the points x_{i-1}, x_i, x_{i+1} lie on a straight line in X. Using this, define $\partial_n = \sum_{0<i<n}(-1)^i \partial_{n,i}$, which yields a chain complex $(\mathsf{MC}_*(X), \partial_*)$. Its homology $\mathsf{MH}_n(X) = \mathrm{Ker}\partial_n / \mathrm{Im}\partial_{n+1}$ is called the *magnitude homology.* Additionally, by considering the sum of distances $\ell = \sum_i d(x_i, x_{i+1})$, we obtain a secondary grading $\mathsf{MH}_*(X) = \bigoplus_\ell \mathsf{MH}^\ell_*(X)$.

When a (connected, undirected, simple) graph G is given, its vertex set inherits a metric space structure by assigning length 1 to each edge and measuring shortest-path distances. The first instance of magnitude homology appeared in this setting in [3], and it has been actively studied as an important case.

In this context, Asao–Ivanov [1] described $\mathsf{MH}_*(X)$ in terms of Tor (assuming X is finite):

$$\mathsf{MH}_*(X) \cong \mathrm{Tor}^{\sigma X}_*(\mathbb{K}^{\#X}, \mathbb{K}^{\#X}).$$

Here, if X is a metric space arising from a graph G, then σX is isomorphic to the path algebra of a quiver for G, modulo certain relations. By *explicitly* describing a minimal projective resolution, we have:

Theorem 1 (Asao–Wakatsuki [2])**.** *Let (X, d) be a geodetic metric space (i.e., any two points are connected by a unique shortest path). Then $\mathsf{MH}^l_n(X)$ is a free module $\mathbb{K}\Theta^l_n$ spanned by the cycles $\Theta^l_n = \{(x_0, \ldots, x_n) \in X^{n+1} \mid (1)\cdots(4)\} \subset X^{n+1}$ defined by the following conditions:*

(1) $\sum_i d(x_i, x_{i+1}) = l$
(2) $1 \leq \forall i \leq n-1$, *not* $x_{i-1} \leq x_i \leq x_{i+1}$
(3) $x_0 \leq a \leq x_1 \Rightarrow a = x_0$ *or* x_1
(4) $1 \leq \forall i \leq n-1$, $((x_i \leq a \leq x_{i+1}$ *and* $a \neq x_{i+1}) \implies x_{i-1} \leq x_i \leq a)$

Here $x \leq y \leq z$ denotes the condition $d(x, y) + d(y, z) = d(x, z)$ for $x, y, z \in X$.

Consider the case where the metric space (X, d) is derived from a graph G. We say that (X, d) is *diagonal* if it satisfies $\mathsf{MH}^l_n(X) = 0$ whenever $n \neq l$. This class has been investigated as an important one in [3], and (when $\mathbb{K}$ is a field) it is equivalent to the Koszulness of the algebra σX. As an application of Theorem 1, we can prove that for a geodetic metric space (X, d), diagonality is equivalent to the absence of 4-cuts. Since the absence of 4-cuts expresses a geometric property of the metric space, this result provides a connection between algebraic and geometric properties.

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Tilting modules for algebraic groups

Noriyuki Abe

We consider the category of algebraic representations of a reductive algebraic group. This category (essentially) has the structure of a highest weight category, and in particular, standard and costandard modules are defined. A module admitting both a filtration by standard modules and a filtration by costandard modules is called a tilting module in this context. It is known that indecomposable tilting modules are parametrized by irreducible representations, but their structure is not easy to understand. In particular, the multiplicities of standard modules appearing in an indecomposable tilting module are important invariants, especially because they are related to the characters of irreducible representations in this setting.

It has been known that the combinatorics of the affine Weyl group plays an important role in the representation theory of reductive algebraic groups. Riche-Williamson [RW18] conjectured that this relationship also holds at the categorical level. In this framework, the affine Weyl group is replaced by its categorification. More precisely, one can categorify the Hecke algebra associated to the affine Weyl group, leading to a category known as the Hecke category. Riche-Williamson conjectured that tilting modules can be described using the Hecke category. This conjecture was proved in type A by Riche-Williamson themselves, and in general by Bezrukavnikov-Riche [BR22]. In particular, the multiplicities of standard modules in indecomposable tilting modules can be described via the Hecke category. There are several (equivalent) realizations of the Hecke category. The one used by Bezrukavnikov-Riche is based on Soergel bimodules [Abe21]. In this talk, I will begin with an introduction to basic terminology related to representation theory of algebraic reductive groups, and then present these results.

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Affine Yangians and W-algebras of type A

Mamoru Ueda

The Yangian $Y_\hbar(\mathfrak{g})$ associated with a finite dimensional simple Lie algebra $\mathfrak{g}$ was first introduced as a quantum group by Drinfeld. We sometimes call the Yangian of this type *the finite Yangian*. If we set $\hbar = 0$, the finite Yangian $Y_\hbar(\mathfrak{g})$ coincides with the universal enveloping algebra of the current algebra $\mathfrak{g} \otimes \mathbb{C}[z]$. Using the current presentation of the finite Yangian, we can extend the definition of the Yangian to a symmetrizable Kac-Moody Lie algebra. In the case that $\mathfrak{g}$ is of affine type, Guay-Nakajima-Wendlandt gave a coproduct for the Yangian. We call the Yangian of this type *the affine Yangian*.

Recently, the Yangian has been actively studied in connection with W-algebras. A W-algebra $\mathcal{W}^k(\mathfrak{g}, f)$ is a vertex algebra associated with a finite dimensional reductive Lie algebra $\mathfrak{g}$, a nilpotent element $f \in \mathfrak{g}$ and a complex number called "level". A W-algebra $\mathcal{W}^k(\mathfrak{g}, f)$ coincides with the affine Lie algebra $\widehat{\mathfrak{g}}$ in the case $f = 0$ and the Virasoro algebra in the case $\mathfrak{g} = \mathfrak{sl}(2)$. For general $\mathfrak{g}$ and f, the relations of a W-algabera is so complicated that we can not easily write down the defining relations of a W-algebra directly. In finite setting, this difficulty has been partially resolved. The finite analogue of a W-algebra $\mathcal{W}^k(\mathfrak{g}, f)$ is a finite W-algebra $\mathcal{W}^{\rm fin}(\mathfrak{g}, f)$, which is an associative algebra associated with a finite dimensional reductive Lie algebra $\mathfrak{g}$ and a nilpotent element $f \in \mathfrak{g}$. Brundan-Kleshchev wrote down a finite W-algebra of type A as a quotient algebra of the shifted Yangian, which is a subalgebra of the finite Yangian of type A. Using the parabolic presentation of the Yangian, we find that the shifted contains a tensor product of finite Yangians of type A. De Sole, Kac, and Valeri gave a homomorphism from the tensor product of the finite Yangians of type A to a finite W-algebra of type A by using the Lax operator. This homomorphism becomes a restriction of Brundan-Kleshchev's homomorphism.

In affine setting, relationships between affine Yangians and rectangular W-algebras have been actively studied. A rectangular W-algebra $\mathcal{W}^k(\mathfrak{gl}(ln), (l^n))$ is a W-algebra associated with $\mathfrak{gl}(ln)$ and a nilpotent element of type (l^n). In the case $n = 1$, a W-algebra is called the principal W-algebra. Schiffmann-Vasserrot defined the affine Yangian associated with $\widehat{\mathfrak{gl}}(1)$ geometrically and wrote down a principal W-algebra of type A as a quotient algebra of this Yangian. Schiffmann-Vasserrot also geve a geometric representation of a principal W-algebra of type A, which corresponds to the AGT conjecture in physics.

In order to consider the general case, we take a positive integer N and its partition $N = \sum_{1\leq i\leq l} q_i$, $q_1 \geq q_2 \geq \cdots \geq q_l > q_{l+1} = 0$. We also take $f \in \mathfrak{gl}(n)$ as a nilpotent element of type $(1^{q_1-q_2}, 2^{q_2-q_3}, \cdots, l^{q_l-q_{l+1}})$. In the case $q_1 = q_2 = \cdots = q_l$, the W-algebra $\mathcal{W}^k(\mathfrak{gl}(ln), f)$ is $\mathcal{W}^k(\mathfrak{gl}(ln), (l^n))$.

Theorem 1 (rectangular case: Kodera-U. [1], non-rectangular case U. [2])**.** *Suppose that $q_i - q_{i+1} \geq 3$. Then, we can give a homomorphism from the affine Yangian associated with $\widehat{\mathfrak{sl}}(q_i - q_{i+1})$ to the universal enveloping algbera of $\mathcal{W}^k(\mathfrak{gl}(N), f)$ by using the Miura map for $\mathcal{W}^k(\mathfrak{gl}(N), f)$ and the coproduct, evaluation map and edge contractions for the affine Yangian. In rectangular setting, this homomorphism becomes surjective.*

Crutzig-Diaconescu-Ma conjectured that there exists a surjective homomorphism from the shifted affine Yangian to the iterated W-algebra of type A and we can obtain a geometric representation of the iterated W-algebra by using this homomorphism, which is a generalization of the AGT conjecture. Since the expected homomorphism is an affine analogue of Brundan-Kleshchev's one, we expect that we can obtain the Crutzig-Diaconescu-Ma's homomorphism by extending the homomorphism given in Theorem 1.

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Thick representations of the symmetric group

Kazunori Nakamoto, Shingo Okuyama, and Yasuhiro Omoda

In this talk, we deal with thick representations and dense representations of the symmetric group S_n. Let V be a finite-dimensional vector space over a field k.

Definition 1 (*cf.* [1] and [2])**.** Let $\rho : G \to \mathrm{GL}(V)$ be a representation of a group G. We say that ρ is *m-thick* if for any subspaces V_1, V_2 of V with $\dim_k V_1 = m$ and $\dim_k V_2 = \dim_k V - m$ there exists $g \in G$ such that $(\rho(g)V_1) \cap V_2 = 0$. If ρ is m-thick for any $0 < m < \dim_k V$, then we say that ρ is *thick.* We also say that ρ is *m-dense* if $\Lambda^m \rho : G \to \mathrm{GL}(\Lambda^m V)$ is irreducible. If ρ is m-dense for any $0 < m < \dim_k V$, then we say that ρ is *dense.*

Proposition 2 ([1, Corollaries 2.8 and 2.9])**.** *For a group representation $\rho : G \to \mathrm{GL}(V)$,*

$$\textit{dense} \implies \textit{thick} \implies \textit{irreducible.}$$

If $\dim_k V \leq 3$, then they are all equivalent.

The following is a more refined definition of m-thickness.

Definition 3 ([3])**.** For a group representation $\rho : G \to \mathrm{GL}(V)$, we say that ρ is *(i,j)-thick* if there exists $g \in G$ such that $(\rho(g)V_1) \cap V_2 = 0$ for any subspaces V_1, V_2 of V with $\dim_k V_1 = i$ and $\dim_k V_2 = j$. If $i + j \leq \dim V$ and ρ is not (i,j)-thick, then ρ is not thick.

Notation 4. Let S_n be the symmetric group of degree n. For a partition λ of n, we denote by V_λ the irreducible representation of S_n over $\mathbb{C}$ corresponding to λ.

Theorem 5 ([4])**.** *The dense representations of the symmetric group S_n over $\mathbb{C}$ are those on the following list:*

(1) *the trivial representation $V_{(n)}$ of S_n for $n \geq 1$,*
(2) *the sign representation $V_{(1^n)}$ of S_n for $n \geq 2$,*
(3) *the standard representation $V_{(n-1,1)}$ of S_n for $n \geq 3$,*
(4) *the product of the standard and sign representation $V_{(2,1^{n-2})}$ of S_n for $n \geq 4$,*
(5) *the 2-dimensional irreducible representation $V_{(2^2)}$ of S_4,*
(6) *the 5-dimensional irreducible representations $V_{(2^3)}$ and $V_{(3^2)}$ of S_6.*

We also talk about several results on thick representations of S_n.

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semibricks in super category O

Shunsuke Hirota

When seriously considering the structure of a given module in terms of its Jordan-Hölder series, it can be challenging. This leads to the question: can we gain insights by taking a coarser view, perhaps by considering a filtration by larger modules? This question can also be rephrased as: are there interesting semibricks in a concrete setting? Classical Verma modules in Lie theory and their natural habitat, Category O, are crucial objects that have profoundly influenced representation theory, with many phenomena being governed by $\mathfrak{sl}_2$. We've observed that natural semibricks can be constructed within Category O, but the description of the subabelian category they generate seemed difficult (though there might still be interesting possibilities). In this talk, we will introduce how a natural semibrick, arising from phenomena governed by $\mathfrak{gl}(1|1)$ (which can be thought of as the $\mathfrak{sl}_2$ in the super world) within Super Category O (a natural generalization of Category O with multiple highest weight category structures), generates a subabelian category where projective modules are precisely Verma supermodules and their variants, describable by quivers. This provides insights into the existence of several new natural modules in Super Category O and sheds light on the socle of Verma supermodules.

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Explicit constructions of simple comodule algebras over a pointed Hopf algebra

Daisuke Nakamura

An abelian category equipped with an action of a tensor category is called a module category, which is a categorification of the notion of a module over a ring. In [2], P. Etingof and V. Ostrik introduced a class of module categories called exact, as a generalization of the notion of semisimple module categories. An exact module category can be decomposed into a direct sum of indecomposable exact module categories, and many researchers have worked on their classification over a fixed tensor category.

In [1], N. Andruskiewitsch and M. Mombelli showed indecomposable exact module categories over the category $\mathrm{Rep}(H)$ of representations of a finite-dimensional Hopf algebra H correspond to right H-simple left H-comodule algebras. For certain Hopf algebras with tractable structure such as the small quantum group $H = u_q(\mathfrak{sl}_2)$, this result has already led to a complete classification [3].

In this talk, we focus on right simple comodule algebras over pointed Hopf algebras. Pointed Hopf algebras are a major class that includes quantum groups and cocommutative Hopf algebras. The following theorem is the main result and will play a key role in constructing such comodule algebras.

Theorem 1. *Let H be a coradically graded finite-dimensional pointed Hopf algebra and $A \neq 0$ be a graded finite-dimensional left H-comodule algebra. The following are equivalent:*

- *A is right H-simple.*
- *A is a homogeneous subcomodule algebra of the smash product $H^{\mathrm{coinv}} \#({}_{\psi}\Bbbk F)$ for some subgroup F of the group-like elements $G(H)$ of H and $\psi \in \mathrm{H}^2(F, \Bbbk^\times)$.*

In this talk, we will construct right H-simple H-comodule algebras as a starting point for classifying indecomposable exact module categories over $\mathrm{Rep}(H)$ for a concrete Hopf algebra H.

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On R-matrices constructed from certain finite-dimensional Yetter-Drinfeld modules

Ryota Wakao

In 2024, Garoufalidis and Kashaev showed that knot invariants can be constructed via R-matrices arising from braided Hopf algebras equipped with additional data [1]. In particular, they construct explicit examples of such invariants using certain Nichols algebras, including the ADO polynomials and the colored Jones polynomials.

In this talk, we will describe how to recover the knot invariants in their construction by specializing the braided Hopf algebra to a Hopf superalgebra. The reason for focusing on Hopf superalgebras is that, according to [2], low-dimensional Hopf superalgebras have been classified and provide a rich source of concrete examples.

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On the ubiquity of dominant local rings

Ryo Takahashi

In what follows, all subcategories are assumed to be strictly full. We begin with recalling the notation for generation in a triangulated category introduced by Bondal, Van den Bergh and Rouquier.

Definition 1. Let $\mathcal{T}$ be a triangulated category.

(1) For a subcategory $\mathcal{X}$ of $\mathcal{T}$, we denote by $\langle \mathcal{X} \rangle$ the smallest subcategory of $\mathcal{T}$ containing $\mathcal{X}$ and closed under finite direct sums, direct summands and shifts. When $\mathcal{X}$ is given by a single object X, we simply write $\langle X \rangle$.

(2) For two subcategories $\mathcal{X}, \mathcal{Y}$ of $\mathcal{T}$, we denote by $\mathcal{X} * \mathcal{Y}$ the subcategory of $\mathcal{T}$ consisting of objects $E \in \mathcal{T}$ which fits into an exact triangle $X \to E \to Y \to \Sigma X$ in $\mathcal{T}$ such that $X \in \mathcal{X}$ and $Y \in \mathcal{Y}$.

(3) For an object $X \in \mathcal{T}$ and a nonnegative integer n, we set

$$\langle X \rangle_n = \begin{cases} 0 & (n = 0), \\ \langle X \rangle & (n = 1), \\ \langle \langle X \rangle_{n-1} * \langle X \rangle \rangle & (n \geqslant 2). \end{cases}$$

Let R be a noetherian ring. Denote by $\mathrm{D_{sg}}(R)$ the *singularity category* of R in the sense of Buchweitz and Orlov, which is defined to be the Verdier quotient

$$\mathrm{D_{sg}}(R) = \mathrm{D^b}(\operatorname{mod} R)/\,\mathrm{K^b}(\operatorname{proj} R),$$

where $\mathrm{D^b}(\operatorname{mod} R)$ stands for the bounded derived category of finitely generated R-modules, and $\mathrm{K^b}(\operatorname{proj} R)$ for the bounded homotopy category of finitely generated projective R-modules. Note that $\mathrm{D_{sg}}(R)$ is a triangulated category. Now we can state the definition of a dominant local ring.

Definition 2. Suppose that the ring R is commutative and local. We define the *dominant index* $\mathrm{dx}(R)$ by the infimum of integers $n \geqslant -1$ such that for every nonzero object $X \in \mathrm{D_{sg}}(R)$ the unique simple R-module k belongs to $\langle X \rangle_{n+1}$. We say that R is *(uniformly) dominant* if $\mathrm{dx}(R) < \infty$.

In this talk, we will roughly review what is known about dominant local rings and why they are important. Then we will consider in what situations a given local ring turns out to be dominant. All the new things which will be presented in this talk come from the speaker's ongoing joint work with Toshinori Kobayashi [4].

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Silting correspondences and Calabi-Yau dg algebras

Osamu Iyama

This talk is based on a joint work with Norihiro Hanihara [5].

I will discuss the correspondences between two important classes of objects, that is, *silting objects* [9, 1] and *d-cluster tilting objects* [3, 7] in derived categories and cluster categories.

First, for a smooth dg algebra A and its $(d+1)$-Calabi-Yau completion Π with $d \geq 0$ [8], we study the relationship between silting objects of A and Π respectively. More explicitly, we introduce the notion of *d-silting objects*, and show that the functor

$$- \otimes_A^{\mathrm{L}} \Pi : \operatorname{per} A \to \operatorname{per} \Pi$$

gives an embedding

$$- \otimes_A^{\mathrm{L}} \Pi : \operatorname{silt}^d A \to \operatorname{silt} \Pi$$

of posets, where $\operatorname{silt}^d A$ is the poset of d-silting objects in $\operatorname{per} A$, and $\operatorname{silt} \Pi$ is the poset of silting objects in $\operatorname{per} \Pi$. For example, if A is an e-Calabi-Yau dg algebra with $e \leq d$, then we obtain a bijection $- \otimes_A^{\mathrm{L}} \Pi : \operatorname{silt} A = \operatorname{silt}^d A \simeq \operatorname{silt} \Pi$.

Secondly, for a smooth connective $(d+1)$-Calabi-Yau dg algebra Π with $d \geq 1$ and its cluster category $\mathcal{C}(\Pi)$, we study the map

$$\pi : \operatorname{silt} \Pi \to d\text{-ctilt}\, \mathcal{C}(\Pi)$$

given by the canonical functor $\pi : \operatorname{per} \Pi \to \mathcal{C}(\Pi)$, where $d\text{-ctilt}\,(\Pi)$ is the set of d-cluster tilting objects in $\mathcal{C}(\Pi)$ [2, 4, 6]. For the fundamental domain $\mathcal{F}$ in $\operatorname{per} \Pi$, the restriction $\pi : \mathcal{F} \to \mathcal{C}(\Pi)$ of π is an equivalence. We call Π *$\mathcal{F}$-liftable* if the induced map

$$\pi : \operatorname{silt} \Pi \cap \mathcal{F} \to d\text{-ctilt}\, \mathcal{C}(\Pi)$$

is bijective. If $d = 1$ or 2, then Π is always $\mathcal{F}$-liftable by a result of Keller-Nicholas [6]. It was asked in [6] if Π is always $\mathcal{F}$-liftable also for $d \geq 3$. We give a negative answer by giving a classification of $\mathcal{F}$-liftable Calabi-Yau dg algebras in some special cases. More explicitly, we prove that $\mathcal{F}$-liftable Calabi-Yau dg algebras Π such that $H^0(\Pi)$ is hereditary are precisely the Calabi-Yau completions of hereditary algebras.

We explain our results by polynomial dg algebras.

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Classifying KE-closed subcategories over a commutative noetherian ring

Shunya Saito and Toshinori Kobayashi

The classification of subcategories of module categories is one of the long-studied topics in the representation theory of algebras. Especially, several classes of subcategories of the module category $\mathsf{mod}\,R$ over a commutative noetherian ring R have been classified so far. See [1, 2, 3, 4, 5, 6, 7] for example.

In this talk, I will talk about the classification result of KE-closed subcategories (additive subcategories closed under kernels and extensions) of $\mathsf{mod}\,R$. For this, I will introduce a class of functions on $\operatorname{Spec} R$ called *n-Bass functions* and prove the following:

Theorem 1. *Suppose that R is (S_2)-excellent (for example, finitely generated algebras over a field, complete local rings, commutative noetherian rings admitting a dualizing complex). Then there is an explicit bijection between the following two sets:*

- *The set of KE-closed subcategories of* $\mathsf{mod}\,R$*.*
- *The set of* 2*-Bass functions on* $\operatorname{Spec} R$*.*

This talk is based on joint work with Toshinori Kobayashi (Meiji University).

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On the extension-closed subcategories of the Cohen-Macaulay module category over a hypersurface of finite or countable representation type

Kei-ichiro Iima

This talk is based on joint work with Ryo Takahashi. Throughout this talk, let k be an algebraically closed uncountable field of characteristic zero, and let R be a complete local hypersurface over k with finite or countable representation type. We denote by $\mathrm{mod} R$ the category of finitely generated R-modules, by $\mathrm{CM}(R)$ the full subcategory of $\mathrm{mod} R$ consisting of all maximal Cohen-Macaulay R-modules, by $\underline{\mathrm{CM}}(R)$ the stable category of $\mathrm{CM}(R)$, by $\mathrm{CM}_0(R)$ the full subcategory of $\mathrm{CM}(R)$ consisting of maximal Cohen-Macaulay modules which are locally free on the punctured spectrum of R.

By Takahashi [2, 3], there are one-to-one correspondences between the following:

- the resolving subcategories of $\mathrm{mod} R$ contained in $\mathrm{CM}(R)$,
- the thick subcategories of $\underline{\mathrm{CM}}(R)$,
- the specialization-closed subsets of the singular locus of R.

The bijections are explicitly given, which leads complete classifications of the resolving subcategories and thick subcategories mentioned above.

By definition, both resolving subcategory and thick subcategory are extension-closed. The main results in this talk is the following theorem.

Theorem 1. [1] *If R is a complete local hypersurface over k, then the following hold.*

(1) *If either R is artinian hypersurface or R has simple singularity and* $\dim R = 2$*, then extension-closed subcategories of* $\mathrm{CM}(R)$ *are only trivial.*

(2) *If R is 1-dimensional simple singularity and domain, then extension-closed subcategories of* $\mathrm{CM}(R)$ *are only trivial.*

(3) *If R is not domain and 1-dimensional simple singularity,* $\mathrm{CM}(R)$ *has non-trivial extension-closed subcategories.*

(4) *If R is either 1-dimensional (A_∞) type or 2-dimensional (D_∞) type, then extension-closed subcategories of* $\mathrm{CM}_0(R)$ *are only trivial.*

(5) *If R is either 1-dimensional (D_∞) type or 2-dimensional (A_∞) type, then* $\mathrm{CM}_0(R)$ *has non-trivial extension-closed subcategories.*

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On the Rouquier dimensions of singularity categories and their annihilators

Yuki Mifune

Let R be a commutative noetherian ring. Denote by $\operatorname{mod} R$ the category of finitely generated R-modules, by $\mathrm{D}^{\mathrm{b}}(R)$ the bounded derived category of $\operatorname{mod} R$. The singularity category of R, introduced by Buchweitz [1], is defined as the Verdier quotient of $\mathrm{D}^{\mathrm{b}}(R)$ by the category of perfect complexes over R; that is,

$$\mathrm{D}_{\mathrm{sg}}(R) = \mathrm{D}^{\mathrm{b}}(R)/\operatorname{thick} R.$$

This category reflects the singularity of R is the sense that $\mathrm{D}_{\mathrm{sg}}(R)$ is trivial if and only if R is regular.

For an essentially small triangulated category $\mathcal{T}$, we can define the Rouquier dimension of $\mathcal{T}$, denote by $\dim \mathcal{T}$ [6]. This invariant measures how many times one needs to take mapping cones starting from a single object to generate the entire category $\mathcal{T}$. Concerning upper bounds for the Rouquier dimension of $\mathrm{D}_{\mathrm{sg}}(R)$, Liu [4] obtained the following result.

Theorem 1 (Liu)**.** *Let $(R, \mathfrak{m}, k)$ be a noetherian local ring with an isolated singularity and I an $\mathfrak{m}$-primary ideal of R contained in the annihilator of $\mathrm{D}_{\mathrm{sg}}(R)$. Then one has*

$$\mathrm{D}_{\mathrm{sg}}(R) = \langle k \rangle_{\ell\ell(R/I)(\mu(I)-\operatorname{depth} R+1)}.$$

In particular, we have

$$\dim \mathrm{D}_{\mathrm{sg}}(R) \leq \ell\ell(R/I)(\mu(I) - \operatorname{depth} R + 1) - 1.$$

Here, the annihilator of $\mathrm{D}_{\mathrm{sg}}(R)$, denoted by $\operatorname{ann}_R \mathrm{D}_{\mathrm{sg}}(R)$, is defined as the set of elements $r \in R$ such that r annihilates the endomorphism rings of all objects in $\mathrm{D}_{\mathrm{sg}}(R)$. We denote by $\mu(I)$ the minimal number of generators of the ideal I, and by $\ell\ell(R/I)$ the Loewy length of R/I. In the case where R is Cohen–Macaulay, the theorem was proved by Dao and Takahashi [2].

Our main result generalizes a theorem of Liu to arbitrary commutative noetherian rings.

Theorem 2. *Let R be a commutative noetherian ring and I an ideal of R contained in $\operatorname{ann}_R \mathrm{D}_{\mathrm{sg}}(R)$. Then one has*

$$\mathrm{D}_{\mathrm{sg}}(R) = \langle \operatorname{mod} R/I \rangle_{\mu(I)-\operatorname{grade} I+1}.$$

Combining the results of Iyengar–Takahashi [3] and Liu [4], we can take the Jacobian ideal as the ideal I in the above theorem in the case where R is close to being Cohen–Macaulay and is a quotient of a formal power series ring or a polynomial ring over a field. Using this result, we give an example of computing an upper bound for the Rouquier dimension of the singularity category in the case where R is a specific ring.

This talk is based on a preprint [5].

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Cohomology annihilators and strong generation of syzygy subcategories

Kaito Kimura

Cohomology annihilators were introduced by Iyengar and Takahashi [2], and they studied the relationship between them and the strong generation of module categories (and of derived categories). One of the main motivations for this talk is the following result of theirs:

Theorem (Iyengar–Takahashi) *Let R be a commutative Noetherian ring of Krull dimension d. If for any prime ideal $\mathfrak{p}$ of R, $\mathsf{ca}^{\dim R/\mathfrak{p}+1}(R/\mathfrak{p}) \neq 0$, then the subcategory of d-th syzygies is strongly generated.*

For each integer $n \geq 0$, we denote by $\mathsf{ca}^n(R)$ the ideal consisting of elements a that annihilate $\operatorname{Ext}_R^n(M,N)$ for all finitely generated R-modules M, N. The union $\bigcup_{n\geq 0} \mathsf{ca}^n(R)$ is called cohomology annihilator of R, which is denoted by $\mathsf{ca}(R)$. In this abstract, the subcategory of s-th syzygies is said to be strongly generated if there exist a finitely generated module G and integer n such that the s-th syzygy of any finitely generated module can be built from G using at most n extensions, up to direct summands and finite direct sums. (Although this notation differs from that used in [2], we adopt it here in order to emphasize the role of the index s.)

Iyengar and Takahashi [2] proved that if a d-dimensional ring is either a localization of a finitely generated algebra over a field or an equicharacteristic complete local ring, then the subcategory of d-th syzygies is strongly generated. They also showed in [3] that such rings satisfy the assumptions of the above theorem by focusing on the Jacobian ideal. In those results, the fact that the cohomology annihilator is a defining ideal of the singular locus plays an essential role. It was shown by Dey, Lank, and Takahashi [1] that the cohomology annihilator defines the singular locus if the ring is quasi-excellent. This talk provides results independent of the characteristic of the ring, which generalize their results in light of recent developments on quasi-excellent rings. If time permits, we will also highlight some interesting results on singular and non-Gorenstein loci obtained in the process. This talk is based on a preprint [4].

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On relative projective covers for the principal blocks of finite groups with metacyclic Sylow subgroups

Naoko Kunugi, Hiroaki Taguchi

Let p be an odd prime, k an algebraically closed field of characteristic p, and $\widetilde{G}$ a finite group with non-abelian metacyclic Sylow p-subgroup

$$C_{p^a} \rtimes C_p \cong \langle x, y \mid x^p = y^{p^a} = 1, xyx^{-1} = y^{1+p^{a-1}} \rangle.$$

The group $\widetilde{G}$ has a semidirect product structure $\widetilde{G} = G \rtimes Q$, where G is a normal subgroup of $\widetilde{G}$ and Q is a p-subgroup of order p. Moreover, a Sylow p-subgroup of $\widetilde{G}$ can be written as $P \rtimes Q$, where P is a Sylow p-subgroup of G. In this setting, P is a cyclic Sylow p-subgroup of G. Therefore, the principal block $B_0(kG)$ of G is derived equivalent to the principal block $B_0(kN_G(P))$ of the normalizer $N_G(P)$ of P in G; this is a known case where Broué's conjecture [1, 2] holds.

Now, we have decompositions of $\widetilde{G}$ and $N_{\widetilde{G}}(P)$:

$$\widetilde{G} = G \rtimes Q, \quad N_{\widetilde{G}}(P) = N_G(P) \rtimes Q.$$

These decompositions lead one to expect that the principal block $B_0(k\widetilde{G})$ of $\widetilde{G}$ is derived equivalent to the principal block $B_0(kN_{\widetilde{G}}(P))$ of $N_{\widetilde{G}}(P)$. Okuyama [3] constructed a stable equivalence of Morita type between these blocks. To lift this stable equivalence to a derived equivalence, it is necessary to determine the structure of the relative Q-projective covers of simple modules in the principal block $B_0(k\widetilde{G})$. In this talk, we will report our results on these relative Q-projective covers, which hold under some assumptions.

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Higher representation infinite algebras arising from geometry

Ryu Tomonaga

For $d \geq 1$, the class of d-representation infinite algebras, introduced by [1], is a natural generalization of non-Dynkin path algebras to the case where global dimension is d. They not only behave beautifully but also arise in many areas of mathematics including geometry. For example, if a (non-commutative) d-dimensional smooth projective variety has a d-tilting sheaf (that is, tilting sheaf whose endomorphism algebra has global dimension d or less), then this endomorphism algebra automatically becomes d-representation infinite. In this talk, we introduce two results on the connection between d-representation infinite algebras and geometry.

First, we show that d-representation infinite algebras of type $\widetilde{A}$, which is a generalization of path algebras of type $\widetilde{A}$ introduced by [1], are derived equivalent to toric Deligne-Mumford stacks of Picard rank one and vice versa. This gives a new combinatorial description to the d-preprojective/preinjective components and d-APR tilting mutations of these algebras even when $d = 1$.

Second, we show that a smooth projective surface has a 2-tilting bundle if and only if it is a weak del Pezzo surface. This gives many new examples of 2-representation infinite algebras and a partial answer to the conjecture: a smooth projective surface has a tilting object if and only if it is rational. As an application, we see that singular del Pezzo cones have non-commutative crepant resolutions, which is a generalization of [2].

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Symmetric cohomology of triangular bialgebras

Shota Inoue and Ayako Itaba

Let k be a commutative ring. A (k-)*bialgebra* is a triple $A := (A, \Delta, \varepsilon)$ where A is a (k-)algebra, and $\Delta\colon A \to A \otimes_k A$ and $\varepsilon\colon A \to k$ are algebra homomorphisms subject to certain conditions. Given a bialgebra A, it is called a *Hopf algebra* if it is equipped with a suitable endomorphism called the antipode, and it is said to be cocommutative if the equality $\Delta = \tau \circ \Delta$ holds, where τ denotes the flip morphism $A \otimes_k A \to A \otimes_k A, a \otimes a' \mapsto a' \otimes a$. The structure of a (quasi-)bialgebra on an algebra A corresponds to a nice monoidal structure on the category $A\,\mathrm{Mod}$ of A-modules, and for a bialgebra A, the symmetry on $A\,\mathrm{Mod}$ corresponds to an *R-matrix* $R \in A \otimes_k A$ of A, and such a pair (A, R) is referred to as a *triangular bialgebra*. The category $A\,\mathrm{Mod}$ for a cocommutative Hopf (or bi-)algebra A is endowed with a structure of the symmetric monoidal category in a standard way, and its symmetry corresponds to the R-matrix $1 \otimes 1$ of A. Therefore, cocommutative Hopf algebras are special cases of triangular bialgebras.

Staic [4] defined the symmetric cohomology $\mathrm{HS}^\bullet(G, M)$ of a group G with coefficients in a (left) G-module M based on the following observation; for any integer $n \geq 0$, the symmetric group S_{n+1} acts on the abelian group $\mathrm{C}^n(G, M)$ of n-cochains, where $\mathrm{C}^\bullet(G, M)$ denotes the standard complex that is used to compute the classical group cohomology $\mathrm{H}^\bullet(G, M)$ of G with coefficients in M. Shiba–Sanada–Itaba [3] defined the symmetric cohomology $\mathrm{HS}^\bullet(A, M)$ of a cocommutative Hopf algebra A with coefficients in an A-module M, generalizing the previous notion in the sense that the equality $\mathrm{HS}^\bullet(G, M) = \mathrm{HS}^\bullet(\mathbb{Z}[G], M)$ holds for a group G and a G-module M, where we write $\mathbb{Z}[G]$ for the group ring of G.

Let (A, R) be a triangular bialgebra, and M an A-module. In this talk, we introduce the symmetric cohomology $\mathrm{HS}^\bullet(A, R, M)$ of the triangular bialgebra (A, R), thereby generalizing the constructions in the previous works [4] and [3]. Let $(\mathbf{\Delta S})_+$ denote the category obtained from $\mathbf{\Delta S}$ by formally adjoining an initial object. Here, $\mathbf{\Delta S}$ is the *symmetric category*, an example of the crossed simplicial groups introduced independently by Krasauskas [2] and Fiedorowicz–Loday [1].

We defined the symmetric cohomology $\mathrm{HS}^\bullet(A, R, M)$ of (A, R) via the following steps:

Definition 1. Let (A, R) be a triangular bialgebra, and M an A-module. We can associate to (A, R) a functor $\widetilde{\mathrm{T}}(A, R)\colon (\mathbf{\Delta S})_+^{\mathrm{op}} \to A\,\mathrm{Mod}$. The functor $\widetilde{\mathrm{T}}(A, R)$ gives rise to an augmented chain complex $C_\bullet$ of A-modules, which is equipped with degreewise actions of the symmetric groups. We define $\mathrm{HS}^\bullet(A, R, M)$ to be the cohomology of a subcomplex of $\mathrm{Hom}_A(C_\bullet, M)$ whose cochains are invariant under this action.

If time allows, we will also talk about the Morita invariance of the symmetric cohomology.

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Point modules of a family consisting of the enveloping algebras of color Lie algebras

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Let k be an algebraically closed field with char $k = 0$. For a connected graded k-algebra A generated in degree 1, a graded left A-module $P = \bigoplus_{i \geq 0} P_i$ is called a point module if P is generated by an element of degree 0 and $\dim_k P_i = 1$ for $i \geq 0$. This is one of the important objects in Noncommutative algebraic geometry. Artin–Tate–Van den Bergh [1] studied point modules of 3-dimensional AS-regular algebras. Also, Belmans–De Laet–Le Bruyn [2] studied point modules of skew polynomial algebras.

Color Lie algebras were introduced by Ree [4]. Let Γ be an abelian group and $\varepsilon : \Gamma \times \Gamma \to k^\times$ be a map satisfying $\varepsilon(\alpha,\beta)\varepsilon(\beta,\alpha) = 1$, $\varepsilon(\alpha+\beta,\gamma) = \varepsilon(\alpha,\gamma)\varepsilon(\beta,\gamma)$, and $\varepsilon(\alpha,\beta+\gamma) = \varepsilon(\alpha,\beta)\varepsilon(\alpha,\gamma)$ for any $\alpha,\beta,\gamma \in \Gamma$. Let $L = \bigoplus_{\gamma\in\Gamma} L_\gamma$ be a Γ-graded k-vector space and Γ-graded bilinear form $\langle\ ,\ \rangle : L \times L \to L$ satisfying $\langle a, b\rangle = -\varepsilon(\alpha,\beta)\langle b, a\rangle$ and $\varepsilon(\gamma,\alpha)\langle a,\langle b,c\rangle\rangle + \varepsilon(\alpha,\beta)\langle b,\langle c,a\rangle\rangle + \varepsilon(\beta,\gamma)\langle c,\langle a,b\rangle\rangle = 0$ for any $a \in L_\alpha, b \in L_\beta, c \in L_\gamma,\ \alpha,\beta,\gamma \in \Gamma$. Then $L = (L,\langle\ ,\ \rangle)$ is called a (Γ,ε)-color Lie algebra. For a color Lie algebra L, we set $\Gamma_- := \{\gamma \in \Gamma \mid \varepsilon(\gamma,\gamma) = -1\}$ and $L_- := \bigoplus_{\gamma\in\Gamma_-} L_\gamma$. Let $T(L)$ denote the tensor algebra of L and $J(L)$ the two-sided ideal of $T(L)$ generated by $a\otimes b - \varepsilon(\alpha,\beta) b\otimes a - \langle a,b\rangle$ for any $a \in L_\alpha, b \in L_\beta, \alpha,\beta \in \Gamma$. The (universal) enveloping algebra of L is defined by $U(L) := T(L)/J(L)$. Price [3] proved that, if $\dim_k L < \infty$ and $L_- = 0$, then $U(L)$ is Auslander-regular and $\mathrm{gl.dim} U(L) = \dim_k L$. For any skew polynomial algebra S, there exists a color Lie algebra L such that $U(L) \cong S$.

Belmans–De Laet–Le Bruyn [2] inspire us to study algebras which have explicit normal elements. We define the following algebras $\mathcal{E}_u^{(n)}$ for studying the point modules. Let u be in $k^\times$. For $\Gamma = \mathbb{Z}\times\mathbb{Z}$, the map ε is defined by $\varepsilon : \Gamma\times\Gamma \to k^\times$, $((\alpha,\beta),(\alpha',\beta')) \mapsto u^{(\alpha\beta'-\alpha'\beta)}$. For a positive integer n, we define a (Γ,ε)-color Lie algebra $\mathcal{L}_n$ as follows:

$$\mathcal{L}_n = \langle x, e_1, e_2, \dots e_n\rangle_k \text{ with } x \in (\mathcal{L}_n)_{(1,0)}, e_i \in (\mathcal{L}_n)_{(i-1,1)} \text{ for } 1 \leq i,j \leq n.$$

For any $1 \leq i,j \leq n$, $\langle x, e_i\rangle = \begin{cases} e_{i+1} & i < n \\ 0 & i = n \end{cases}$, $\langle e_i, e_j\rangle = 0$. We define $\mathcal{E}_u^{(n)} := U(\mathcal{L}_n)$.

Note that $\mathcal{E}_u^{(n)}$ is $(n+1)$-dimensional Neotherian AS-regular algebra and e_n is a degree n normal element in $\mathcal{E}_u^{(n)}$, that is, $e_n\mathcal{E}_u^{(n)} = \mathcal{E}_u^{(n)} e_n$. We prove the following theorem about $\mathcal{E}_n$ for $n \geq 2$.

Theorem 1. *For each $n \geq 2$, $\mathcal{E}_u^{(n)}$ has no e_n-torsionfree point module.*

Remark that, for a connected graded k-algebra A generated in degree 1 and a normal element g in A, the point module P of A is g-torsionfree or $gP = 0$. We prove the following corollary of Theorem 1.

Corollary 2. *Let u be in $k^\times$. For each $n \geq 2$, there exists the one-to-one correspondance between the isomorphism classes of point modules of $\mathcal{E}_u^{(n)}$ and those of $\mathcal{E}_u^{(1)}$:*

$$\{P \mid P \text{ is a point module of } \mathcal{E}_u^{(n)}\}/\cong \ \overset{1:1}{\longleftrightarrow} \ \{P \mid P \text{ is a point module of } \mathcal{E}_u^{(1)}\}/\cong$$

So, the point variety structure of $\mathcal{E}_u^{(n)}$ is same as that of $\mathcal{E}_u^{(1)} \cong k\langle x,y\rangle/\langle xy - uyx\rangle$.

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The Lie structure on the first Hochschild cohomology groups of Beilinson algebras of graded down-up algebras with weights (n, m)

Ayako Itaba and Shu Minaki

Let k be an algebraically closed field with $\operatorname{char} k = 0$. Noncommutative projective schemes associated to AS-regular algebras were formulated by Artin–Zhang [1]. Let $\mathsf{mod}\,\nabla A$ denote the category of finitely generated right ∇A-modules, $\mathsf{tails}\,A$ denote the noncommutative projective scheme. The bounded derived categories of $\mathsf{tails}\,A$ and $\mathsf{mod}\,\nabla A$ are denoted by $\mathsf{D^b}(\mathsf{tails}\,A)$ and $\mathsf{D^b}(\mathsf{mod}\,\nabla A)$, respectively. Regarding the relationship between a noetherian AS-regular algebra A and its Beilinson algebra ∇A, an important result was proved by Minamoto–Mori [6]; if A is a coherent AS-regular algebra of dimension d and ∇A is its Beilinson algebra, then ∇A is extremely Fano of global dimension $d-1$, and there exists an equivalence of triangulated categories $\mathsf{D^b}(\mathsf{tails}\,A) \simeq \mathsf{D^b}(\mathsf{mod}\,\nabla A)$.

The graded algebra $A(\alpha, \beta) := k\langle x, y\rangle/(x^2y - \beta yx^2 - \alpha xyx,\ xy^2 - \beta y^2x - \alpha yxy)$ is called a (graded) down-up algebra with weights (n, m), where $\alpha, \beta \in k$ and $\deg x = n, \deg y = m \in \mathbb{N}^+$. This down-up algebra was introduced by Benkart–Roby [3] for their study of posets. Kirkman–Musson–Passman [5] showed that a graded down-up algebra $A = (\alpha, \beta)$ is a 3-dimensional noetherian AS-regular algebra if and only if $\beta \neq 0$.

The aim of our study is to investigate the Hochschild cohomology groups of ∇A of a down-up algebra $A = A(\alpha, \beta)$ with $\beta \neq 0$ and weights (n, m). If $n = 1$ and $m = 1$, then the description of the Hochschild cohomology group of ∇A is already known due to Belmans [2]. Also, if $n = 1$ and $m \geq 2$, the dimension formula for the Hochschild cohomology group of ∇A was given by the first author and Ueyama [4]. In this study, we provide the dimension formula for the Hochschild cohomology group of ∇A in the case that $n \geq 2$ and $m \geq 2$. Using this dimension formula, we show that if $m > n > 1$, then $\mathsf{D^b}(\mathsf{tails}\,A)$ is not equivalent to the derived category of any smooth projective surface. Furthermore, in the case $n \geq 2$ and $m \geq 2$, we construct a basis of the first Hochschild cohomology group of ∇A via the Bar resolution. Moreover, we compute the Lie algebra structure on the first Hochschild cohomology group induced by the Gerstenhaber bracket in the case $m \geq n \geq 1$, $m \neq 1$ and $\gcd(n, m) = 1$.

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Noncommutative Hirzebruch surfaces

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A noncommutative $\mathbb{P}^1$-bundle over a commutative scheme defined in [2] is one of the major objects of study in noncommutative algebraic geometry. Last year in this symposium [1], we classified locally free sheaf bimodules of rank 2 over $\mathbb{P}^1$ in order to classify noncommutative Hirzebruch surfaces, which are defined to be noncommutative $\mathbb{P}^1$-bundles over $\mathbb{P}^1$, however, we did not give a definition of a noncommutative Hirzebruch surface. In this talk, we give some ideas on how to define a noncommutative $\mathbb{P}^1$-bundle over a commutative scheme from the view point of representation theory of algebras, extending the construction of the preprojective algebra of the 2-Kronecker quiver. Then we will show that every noncommutative $\mathbb{P}^1$-bundle over a commutative smooth projective scheme has a suitable semi-orthogonal decomposition. Moreover, we will show that every noncommutative Hirzebruch surface even has a full strong exceptional sequence so that its derived category is equivalent to the derived category of a finite dimensional algebra. This talk is based on a joint work with Shinnosuke Okawa and Kazushi Ueda.

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Examples of tilting-discrete symmetric algebras

Takuma Aihara

The tilting-discreteness of a symmetric algebra is a finiteness condition on tilting complexes; that is, a tilting-discrete symmetric algebra has only finitely many tilting complexes of each length. In the case, we can 'essentially' obtain all tilting complexes via tilting mutation, and grasp the whole picture of tilting complexes. Typical examples of tilting-discrete symmetric algebras are representation-finite symmetric algebras [A], Brauer graph algebras whose Brauer graphs admit at most one cycle of odd length and none of even length [AAC], symmetric algebras of dihedral, semidihedral and quaternion type [EJR] and symmetric algebras of tubular type with nonsingular Cartan matrix [AHMW]. A big problem is to classify all tilting-discrete symmetric algebras.

In this talk, we give several examples of tilting-discrete symmetric algebras; one also discusses the tilting-disconnectedness of symmetric algebras. Moreover, we negatively solve a conjecture asking if a τ-tilting finite symmetric algebra is tilting-discrete.

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