The 36th Symposium On Ring and Representation Theory (2003)

ABSTRACT

Hirosaki University
Hirosaki-shi Aomori, JAPAN October 11(Sat.) - 13(Mon.), 2003

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Derivations of polynomial rings over a field of characteristic zero

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Foundation of the representation theory of finite dimensional algebras III

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(On Frobenius algebras)

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Factorizations of enveloping algebras of three dimensional Lie algebras and their applications

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EXAMPLES OF BROUÉ'S ABELIAN DEFECT GROUP CONJECTURE IN REPRESENTATION THEORY OF FINITE GROUPS

Shigeo KOSHITANI

In representation theory of finite groups, there is a well-known and important conjecture due to M. Broué. He conjectures that, for any prime p, if a p-block A of a finite group G has an abelian defect group P, then A and its Brauer correspondent p-block B of $N_G(P)$ are derived equivalent. In this talk we discuss on Broué's abelian defect group conjecture for several cases where A has elementary abelian defect group P of order 9 .

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Foundation of the representation theory of finite dimensional algebras

CLaus Michael Ringel

The aim of the lectures will be to outline a new fundation of the representation theory of artin algebras based on quite old investigations of Roiter and Gabriel concerning the Brauer-Thrall conjectures, and which complement the usual Auslander-Reiten methods. I will focus the attention to the Gabriel-Roiter measure which was introduced (as "Roiter measure") by Gabriel in order to make explicit the combinatorial scheme of Roiter's proof of the first Brauer-Thrall conjecture. The methods which were introduced in this way were intended just for categories of bounded representation type, but in turns out that their proper value unfolds when dealing with artin algebras of infinite representation type!

Let Λ be an artin algebra. The Λ-modules to be considered will usually be finitely generated. Recall that the first Brauer-Thrall conjecture asserts the following: If there is a bound on the length of the indecomposable Λ-modules (Λ is bounded representation type), then there are only finitely many isomorphism classes of indecomposables (Λ is of finite representation type). This conjecture was solved in 1968 by Roiter, and his method was analysed by Gabriel in 1973 by introducing a combinatorial invariant for any Λ-module. Both Roiter and Gabriel have assumed from the beginning that Λ is of bounded representation type, however this assumption is really misleading: A proper reading of the papers of Roiter and Gabriel shows that the methods exhibited by Roiter and the invariant introduced by Gabriel shed light on the structure of the category of Λ-modules for an arbitrary artin algebra Λ.

The Gabriel-Roiter measure $\mu(M)$ of a Λ-module M (as we propose to call this invariant) may be considered as a rational number (say between 0 and 1) which only depends on the submodule lattice of M. In case M is indecomposable, then one looks for suitable filtrations $M_1 \subset M_2 \subset \cdots \subset M_t = M$ using indecomposable submodules M_i; we will call these filtrations Gabriel-Roiter filtrations, and the corresponding embeddings $M_{i-1} \subset M_i$ Gabriel-Roiter inclusions. Note that any Gabriel-Roiter inclusion $X \subset Y$ is mono-irreducible (this means that for any proper submodule Y' of Y which includes X, the embedding $X \subseteq Y'$ splits). In particular, it follows that for any Gabriel-Roiter filtration $M_1 \subset M_2 \subset \cdots \subset M_t = M$, all the factors M_i/M_{i-1} are indecomposable.

The main property of the Gabriel-Roiter measure is the following: the class of modules which are direct sums of modules M with $\mu(M) \leq r$ for a fixed real number r is closed under submodules. In this way, one obtains an interesting filtration of the category $\operatorname{mod}\Lambda$ of all Λ-modules by subcategories which are closed under submodules.

We say that r is a Gabriel-Roiter measure for Λ provided there are indecomposable Λ-modules M with $\mu(M) = r$. Let us assume from now on that Λ is a connected artin algebra of infinite representation type. We will show that there are Gabriel-Roiter measures r_t, r^t for Λ with $r_1 < r_2 < r_3 < \quad \cdots \quad < r^3 < r^2 < r^1$ such that any other Gabriel-Roiter measure r for Λ satisfies $r_t < r < r^t$ for all $t \in \mathbb{N}_1$. Also, for any t, there are only finitely many isomorphism classes of indecomposables with Gabriel-Roiter measure r_t or r^t. Note that any infinite set of Gabriel-Roiter measures for Λ provides arbitrarily large indecomposable modules, thus, in particular, we encounter two different proofs of the first Brauer-Thrall conjecture.

We will say that the indecomposables with Gabriel-Roiter measure of the form r_t form the *take-off part* of the category $\operatorname{mod}\Lambda$. The take-off part can be used in order to construct very interesting indecomposable Λ-modules of infinite length. For certain hereditary algebras, the take-off modules are just the preprojective ones, but in general there will be no indecomposable module which is both a take-off module and preprojective. In contrast, all the indecomposable modules with Gabriel-Roiter measure of the form r^t will be preinjective. Note that we get in this way a partition of the module category into three parts: the take-off part, the central part and the preinjective part, where both the take-off part and the preinjective part are in some sense of combinatorial nature. The lectures will provide additional information on this partition.

We also will consider the dual partition. Looking at both partitions at the same time, we obtain a rhombic picture for the category $\operatorname{mod}\Lambda$ which sheds a lot of light on the structure of $\operatorname{mod}\Lambda$ and some of its subcategories. For tame hereditary algebras, the rhombic picture corresponds quite well to the commonly used intuitive visualization using the Auslander-Reiten quiver, but in contrast to the old visualization, the new one is based on invariant data.

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Representation dimension and finitistic dimension conjecture

Osamu Iyama

30 years ago, M. Auslander introduced a concept of representation dimension of artin algebras, which measures homologically how far an artin algebra is from being of finite representation type. His methods have been effectively applied not only for the representation theory of artin algebras, but also for the theory of quasi-hereditary algebras of Cline-Parshall-Scott by Dlab-Ringel. Recently, the author proved that any artin algebra has a finite representation dimension by showing that any module is a direct summand of some module whose endomorphism ring is quasi-hereditary. Our method is to construct certain chain of subcategories of $\operatorname{mod}\Lambda$. It was also applied to solve Solomon's second conjecture on zeta functions of orders by the author. We will formulate it in terms of rejective subcategories, which was applied to study the representation theory of orders and give a characterization of their finite Auslander-Reiten quivers. Moreover, we will explain the recent result of Igusa-Todorov and that of Rouquier.

A generalization of Zwara's theorem on degeneration of modules

Yuji Yoshino

We generalize a result of Zwara concerning the degeneration of modules over Artinian algebras to that over general algebras. In fact, let R be any algebra over a field (maybe non-commutative, non-Noetherian, non-Artinian) and let M and N be finitely generated left R-modules. Then, we show that M degenerates to N (along a discrete valuation ring) if and only if there is a short exact sequence of finitely generated left R-modules $0 \to Z \overset{(\phi,\psi)}{\to} M \oplus Z \to N \to 0$ such that the endomorphism ψ on Z is nilpotent. We give several applications of this theorem to commutative ring theory. In particular, we show that the property for a module having G-dimension 0 is an open property.

References

[1] Y.Yoshino, *On Degenerations of modules*, Preprint (2003).

[2] ———, *Degeneration and G-dimension of modules*, Lecture at Lisbon Conference of Commutative Algebra, June, 2003.

[3] ———, *On degenerations of Cohen-Macaulay modules*, Journal of Algebra **248**, 272–290 (2002).

[4] G.Zwara, *Degenerations of finite dimensional modules are given by extensions*, Composito Mathematica **121**, 205–218 (2000).

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FINITE GENERATIONS OF RINGS OF DIFFERENTIAL OPERATORS OF SEMIGROUP ALGEBRAS

SAITO, Mutsumi

This talk is based on the papers with William N. Traves [1] and [2].

Let $A := \{ \mathbf{a}_1, \mathbf{a}_2, \dots , \mathbf{a}_n \} \subset \mathbb{Z}^d$ be a finite subset. We denote by $\mathbb{N}A$, $\mathbb{Z}A$, and $\mathbb{R}_{\geq 0}A$ the monoid, the abelian group, and the cone generated by A, respectively.

Let $R_A := \mathbb{C}[\mathbb{N}A]$ denote the semigroup algebra of $\mathbb{N}A$. We consider two rings: the ring $D(R_A)$ of differential operators of R_A and its graded ring $\mathrm{Gr}(D(R_A))$ with respect to the order filtration.

As a starting point for the study of $D(R_A)$, we consider the finite generations of $D(R_A)$ and $\mathrm{Gr}(D(R_A))$.

While considering the finite generation of $\mathrm{Gr}(D(R_A))$, we encountered the notion of a scored semigroup; a semigroup $\mathbb{N}A$ is **scored** if the difference $(\mathbb{R}_{\geq 0}A \cap \mathbb{Z}A) \setminus \mathbb{N}A$ consists of a finite union of hyperlane sections of $\mathbb{R}_{\geq 0}A \cap \mathbb{Z}A$ parallel to facets of the cone $\mathbb{R}_{\geq 0}A$.

We have proved the following:

Theorem 1. (1) $\mathrm{Gr}(D(R_A))$ *is finitely generated if and only if* R_A *is a scored semigroup algebra.*

(2) $D(R_A)$ *is finitely generated for all semigroup algebras* R_A.

The problem of the Noetherian properties of $D(R_A)$ is still open.

In the talk, I use some examples to illustrate the idea of the proof of the theorem, and to explain the scored property. The scored property implies Serre's condition (S_2). However neither the scored property nor the Cohen-Macaulay property implies the other.

References

[1] M. Saito and W.N. Traves, *Differential algebras on semigroup algebras*, AMS Contemporary Math. **286** (2001), 207–226.

[2] ——— and ———, *Finite generations of rings of differential operators of semigroup algebras*, preprint.

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ON DEGENERATIONS OF MODULES OVER GENERAL RINGS

YOSUKE OHNUKI and MASAHISA SATO

The aim of our talk is to generalize the relationship of degenerations and some tubes, developed by Riedtmann, Zwara and etc. Recall [2], [4] that for a finite dimensional algebra A over an algebraically closed field k and (finite) d-dimensional A-modules M, N, M degenerates to N (i.e., N lies in the $Gl_d(k)$-orbit closure of M) if and only if there is an exact tube (N_i, α_i, β_i)

$$\begin{array}{ccccccccc}
0 & \longleftarrow & N_1 & \xleftarrow{\beta_1} & N_2 & \xleftarrow{\beta_2} & N_3 & \xleftarrow{\beta_3} & \cdots \\
\downarrow & & {\scriptstyle\alpha_1}\downarrow & & {\scriptstyle\alpha_2}\downarrow & & {\scriptstyle\alpha_3}\downarrow & & \\
N_1 & \xleftarrow{\beta_1} & N_2 & \xleftarrow{\beta_2} & N_3 & \xleftarrow{\beta_3} & N_4 & \xleftarrow{\beta_4} & \cdots .
\end{array}$$

(each square is exact) such that N_1 is isomorphic to N and we have some positive integer h with $N_{h+m} \simeq N_h \oplus M^m$ for any $m \geq 0$.

In order to define a degeneration for modules a general ring, we take an almost exact tube (N_i, α_i, β_i) i.e., all α_i are monomorphisms, all β_i are epimorphisms and $0 \to N_i \to N_{i-1} \oplus N_{i+1} \to N_i \to 0$ are exact for any $i \geq 2$. Then it induces the inverse limit of exact sequences which is the exact sequence of the following form

$$0 \longrightarrow \varprojlim N_i \xrightarrow{\varprojlim \alpha} \varprojlim N_{i+1} \longrightarrow \mathrm{Cok}\alpha_1 \longrightarrow 0.$$

We shall define degenerations by the purely ring theoretical notion.

Definition 1. Let R be an associative ring with a unit, and M, N be R-modules. We call N is a *degeneration* of M if $\mathrm{Cok}\alpha_1$ is isomorphic to N and $\varprojlim N_{i+1}$ is isomorphic to $\varprojlim N_i \oplus M$.

Then N is a degeneration of M if and only if there is an exact sequence $0 \to Z \to Z \oplus M \to N \to 0$ for some module Z.

Definition 2. Let $0 \to Z \xrightarrow{\binom{f}{g}} Z \oplus M \to N \to 0$ be an exact sequence which gives a degeneration N of M. A degeneration is called *Fitting type* if $f: Z \to Z$ induces an automorphism $\mathrm{Im}(f^n) \to \mathrm{Im}(f^n)$ for some $n \in \mathbb{N}$.

A degeneration of Fitting type is the notion generalized for a degeneration of finitely generated modules [3]. In this case we can apply the way for construction of an (M, N)-tube, introduced in [1].

We shall give some results and properties with respect to these definitions.

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[1] R. Aehle, Ch. Riedtmann and G. Zwara *Complexity of degenerations of modules*, Comment. Math. Helv. **76** (2001), 781–803.

[2] Ch. Riedtmann, *Degenarations for Representations of Quivers with Relations*, Ann. Sci. École Normal Sup. **19** (1986), 275–301.

[3] Y. Yoshino, *On Degenerations of Modules*, Preprint.

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LIFTING MODULES OVER RIGHT PERFECT RINGS

Chang Chaehoon, Yamaguchi University
and
Yosuke Kuratomi, Kitakyushu National College of Technology

A right R-module M is said to be an *extending* module, if it satisfies the following property: For any submodule X of M, there exists a direct summand of M which contains X as an essential submodule, that is, for any submodule X of M, there exists a closure of X in M which is a direct summand of M. Dually, M is said to be a *lifting* module, if it satisfies the dual property: For any submodule X of M, there exists a direct summand of M which is a co-essential submodule of X, that is, for any submodule X, there exists a co-closure of X in M which is a direct summand of M (cf.[1]). Any submodule X of M has a closure in M, but need not have a co-closure in M. We seem that the difficulty of study of lifting modules is hiding here.

In 1984, Okado [2] has studied the decomposition of extending modules over right noetherian and obtained the following:

Theorem (Okado) A ring R is right noetherian if and only if every extending R-module is expressed as a direct sum of indecomposable modules.

A dual problem is open. Our talk is concerned with this problem and the main result is the following:

Result Any lifting module over right perfect rings is expressed as a direct sum of indecomposable modules.

References

[1] K. Oshiro, Semiperfect modules and quasi-semiperfect modules, Osaka J. Math. 20 (1983), 337-372.

[2] M. Okado, On the decomposition of extending modules, Math. Japonica 29 (1984), 939-941.

NON-COMMUTATIVE VALUATION RINGS OF $K(X;\sigma,\delta)$ OVER A DIVISION RING K

Guangming Xie*, Hidetoshi Marubayashi*
Shigeru Kobayashi*, Hiroaki Komatsu**

Let K be a division ring with a σ-derivation δ, where σ is an endomorphism of K and $K(X;\sigma,\delta)$ be the quotient division ring of the Ore extension $R = K[X;\sigma,\delta]$ over K in an indeterminate X. Let V be a total valuation ring of K and (σ,δ) be compatible with V. Then $R^{(1)} = V[X;\sigma,\delta]_{J(V)[X;\sigma,\delta]}$, the localization of $V[X;\sigma,\delta]$ at $J(V)[X;\sigma,\delta]$, is a total valuation ring of $K(X;\sigma,\delta)$ such that $R^{(1)} \cap K = V,\ X \in R^{(1)}$. The purpose of this talk is to describe non-commutative valuation rings B of $K(X;\sigma,\delta)$ such that $R^{(1)} \supseteq B,\ B \cap K = V$ and $X \in B$. First we shall denote non-commutative valuation rings of $K(X;\sigma,\delta)$ containing R. If δ is not a quasi-algebraic σ-derivation, then there are no proper non-commutative valuation rings of $K(X;\sigma,\delta)$ containing R. In the case where δ is a quasi-algebraic σ-derivation and $\sigma \in \mathrm{Aut}(K)$, we shall describe all non-commutative valuation rings of $K(X;\sigma,\delta)$ containing R by using the classification of maximal ideals of R. If $\sigma \notin \mathrm{Aut}(K)$ and δ is a quasi-algebraic σ-derivation, then there is a monic invariant polynomial $\mathrm{p}(X)$ of minimal non-zero degree such that $K[X;\sigma,\delta]\mathrm{p}(X)$ is a maximal ideal. Set $\hat{K} = \cup_{i=1}^{\infty}\mathrm{p}(X)^{-i}K\mathrm{p}(X)^{i}$, a division ring containing K. We can extend σ and δ to $\hat{\sigma}$ and $\hat{\delta}$ with $\hat{\sigma} \in \mathrm{Aut}(\hat{K})$ and obtain $\hat{R} = \cup_{i=1}^{\infty}\mathrm{p}(X)^{-i}R\mathrm{p}(X)^{i} = \hat{K}[X;\hat{\sigma},\hat{\delta}]$ with a maximal ideal $\hat{P} = \hat{R}\mathrm{p}(X)$. If $\hat{K}$ is left algebraic over K and $R\mathrm{p}(X)$ is completely prime, then $\hat{R}_{\hat{P}}$ is the only proper total valuation ring of $K(X;\sigma,\delta)$ such that $\hat{R}_{\hat{P}} \supseteq R$.

Next we assume that V is a total valuation ring of K and (σ,δ) is compatible with V. Then σ and δ naturally induce $\bar{\sigma}$ and $\bar{\delta}$ of $\bar{V} = V/J(V)$, a division ring. Let $\varphi : R^{(1)} = V[X;\sigma,\delta]_{J(V)[X;\sigma,\delta]} \to \overline{R^{(1)}} = R^{(1)}/J(R^{(1)}) \cong \bar{V}(X;\bar{\sigma},\bar{\delta})$ be the natural homomorphism. Then there is a one-to one correspondence between the set of all total valuation rings B of $K(X;\sigma,\delta)$ satisfying $B \cap K = V,\ X \in B,\ B \subseteq R^{(1)}$ and the set of all total valuation rings $\mathfrak{B}$ of $\bar{V}(X;\bar{\sigma},\bar{\delta})$ with $\mathfrak{B} \supseteq \bar{V}[X;\bar{\sigma},\bar{\delta}]$, which is given by $\varphi(B) = \mathfrak{B}$ and $\varphi^{-1}(\mathfrak{B}) = B$. By using this correspondence, we shall describe non-commutative valuation rings B of $K(X;\sigma,\delta)$ satisfying $B \cap K = V,\ X \in B,\ B \subseteq R^{(1)}$.

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Non-semisimple Hopf algebras over a field of characteristic $p \neq 0$

Kenji Yokogawa

Let K be a field of characteristic $p \neq 0$.

Definition We call a Hopf algebra H of dimension p over K generated by a p-nilpotent element as a *p-nilpotent* Hopf algebra.

A group ring $H = K[\sigma]$, $\sigma^p = 1$, is such a Hopf algebra with a p-nilpotent generator $\sigma - 1$ (we call Hopf algebra of this type as a Hopf algebra of *automorphism type*). A Hopf algebra generated by a derivation d, $d^p = 0$, is also such a Hopf algebra (we call as a Hopf algebra of *derivation type*). Another type of p-nilpotent Hopf algebra is not known. So the question whether these two types of Hopf algebra are the all of p-nilpotent Hopf algebras arises.

これらの algebra は, non-semisimple で、分類するのに適当な tool が無い. そこで、具体的に計算する事から始めた. 計算と言っても基本的には, coassociative な algebra-homo $\Delta : H \to H \otimes H$ を決めてやれば良い訳であるが, 計算量が多くて人間業ではない.
そこでパソコンと SoftWare「Mathematica」を使って以下のような結果が得られた.
ここでは, 結果よりもパソコンと SoftWare の利用方法を中心に話を進めたいと考えています.
注: SoftWare には「Mathematica」以外に「Maple」がある. 色々な点で「Maple」の方が優れているように思われる.

主な結果

Proposition 1. *A non-semisimple Hopf algebra H of dimension p is a p-nilpotent Hopf algebra.*

Lemma 2. *There exists a p-nilpotent generator x whose cocoefficients satisfy the relations;* $c_{11} = c_{21} = c_{22} = c_{32} = c_{33} = \cdots = c_{q\,q-1} = c_{qq} = 0$ *(where $p = 2q+1$, $c_{i,j}$ denotes the coefficient of $x^i \otimes x^j$ in $\Delta(x)$).*

Definition If a p-nilpotent generator x satisfies the relations of Lemma 2, we call x as a *normal generator*.

Theorem 3. *The cocoefficients of a normal generator x of a p-nilpotent Hopf algebra satisfy the relations;* $c_{10} = c_{01} = 1, c_{p-1\,1} = c$, $c_{p-k\,k} = (-1)^{k-1}k^{-1}c$, $(2 \leq k \leq p-1)$, $c \in K$, *the other terms are equal to 0 and $S(x) = -x$ (S is an antipode).*

Conversely, an algebra generated by a p-nilpotent element whose cocoefficients and antipode satisfy the above relations becomes a p-nilpotent Hopf algebra.

Corollary 4. *If K is a prime field then there exists exactly p non-isomorphic p-nilpotent Hopf algebras, and if K is an algebraically closed field then there exists exactly 2 non-isomorphic p-nilpotent Hopf algebras.*

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Valuations on coproducts of skew fields and free fields

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The universal field of fractions of free algebras are called free fields. By using free fields P.M.Cohn [1,2] constructed skew field extensions of arbitrary finite right degree and infinite left degree. Free fields are sometimes difficult to manipulate owing to the lack of a convenient normal form for their elements. Valuations form a useful tool in the analysis of free fields. In this lecture we give several properties of valuations on coproducts of skew fields and free fields. Our main results are the following:

Let D be a skew field with a valuation ν and K a bicentral subfield of D. Let $\{D_i \mid i \in I\}$ be a family of subfields of D which contain the common subfield K of D and ν_i $(i \in I)$ the valuation on D_i $(i \in I)$ which is a restriction of ν to D_i. Then the ring coproduct $*_K D_i$ of D_i over K has a valuation ω extending all the valuation ν_i. Moreover, if the subfields D_i $(i \in I)$ satisfies certain conditions, then the field coproduct $\circ_K D_i$ has a valuation ω extending all the ν_i.

As an application of the result, we have:

Let $\kappa(X)$ be a free field over a commutative field κ on a set $X = \{x_i \mid i \in I\}$ and ν_i a κ-valuation of $\kappa(x_i)$. T hen there is a κ-valuation ν on $\kappa(X)$ which extends the all ν_i.

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BGG correspondence and Aramova-Herzog's theory on exterior algebra

Kohji Yanagawa

Recently, some commutative algebraists are studying graded modules over an exterior algebra. One of the motivations is to understand/refine results on Combinatorial Commutative Algebra (e.g. Kalai's theory on "algebraic shifting").

Let V be an n-dimensional K-vector space, and $E := \bigwedge V$ the exterior algebra. We regard E as a negatively graded ring with $E_{-i} = \bigwedge^i V$. Let N be a finitely generated graded E-module. Then, for each $i \in \mathbb{Z}$, there is a polynomial $P_i(t) \in \mathbb{Q}[t]$ satisfying $P_i(j) = \dim_K \operatorname{Ext}^j_E(K, N)_{j-i}$ for $j \gg 0$. If $P_i(t) \neq 0$, $P_i(t)$ is of the form $\dfrac{e}{m!} \cdot t^m +$ (terms with lower degree) for some positive integer e. In this case, we set $d_i(N) := m + 1$ and $e_i(N) := e$. To extract essential information from an injective resolution, Aramova-Herzog [1] and Römer [4] introduced the following notion.

Definition 1 ([1, 4])**.** We say $(d, i) \in \mathbb{N} \times \mathbb{Z}$ is a *distinguished pair* for a finitely generated graded E-module N if $d_i(N) = d$ and $d_j(N) < d + i - j$ for all $j < i$.

Note that $N^* := \operatorname{Hom}_E(N, E)$ is a graded E-module again. Next result induces Bayer-Charalambous-Popescu [2, Theorem 2.8], which is a well-known result on Stanley-Reisner rings.

Theorem 2 (Römer [4])**.** *(d, i) is distinguished for N if and only if $(d, 2n - d - i)$ is distinguished for N^*. If this is the case, we have $e_i(N) = e_{2n-d-i}(N^*)$.*

Let W be the dual vector space of V, and $S := \operatorname{Sym}_K W$ the symmetric algebra. The Bernstein-Gel'fand-Gel'fand correspondence (BGG correspondence) states that the derived category $D^b(\mathbf{mod}_S)$ of finitely generated graded S-modules is equivalent to the similar category $D^b(\mathbf{mod}_E)$ for E (see for example [3]). In this talk, we show that, under the BGG correspondence, Theorem 2 is translated into a statement on $D^b(\mathbf{mod}_S)$ which is a natural consequence of the local duality (Serre duality).

We also refine Theorem 2 in the $\mathbb{Z}^n$-graded context, while the arguments in [1] and [4] does not work in this context (since they use a generic base change of V). And we give a derived category version of [2, Theorem 2.8].

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Factorizations of enveloping algebras of three dimensional Lie algebras and their applications

Jun Morita

Let F be a field of characteristic zero, and L be a Lie algebra over F. The universal enveloping algebra of L is denoted by $U(L)$. Let V be an L-module. Then we will reach the following fact (cf. [1]).

(**Fact**) *Let L be a three dimensional Lie algebra generated by two elements x, y. Then $U(L) = \sum_{i,j,k} Fx^i y^j x^k$. Furthermore, if both of x and y act on V as locally nilpotent operators, then V is locally finite as an L-module.*

For example, we take

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

as a set of generators for $L = s\ell_2$. Then we have $U(s\ell_2) = \sum_{i,j,k} Fe^i f^j e^k$. This immediately shows that an $s\ell_2$-module V is **integrable** if e and f act on V locally nilpotently.

The key formulas to establish the above (Fact) are as follows:

$$\begin{array}{ll} (\mathbf{A}_k) & yxy^k \equiv \frac{k}{k+1}xy^{k+1} + \frac{1}{k+1}y^{k+1}x \mod U_k, \\ (\mathbf{B}_k) & y^k xy \equiv \frac{1}{k+1}xy^{k+1} + \frac{k}{k+1}y^{k+1}x \mod U_k, \\ (\mathbf{C}_k) & yU_k \subset U_{k+1}, \quad U_k y \subset U_{k+1}, \end{array}$$

where $U_k = \sum_{0 \le m \le k}(Fxy^m + Fy^m x + Fy^m)$. Since we can also apply these to the three dimensional **Heisenberg Lie algebra**, we have a new factorization of the universal enveloping algebras for **Kac-Moody Lie algebras**, more generally for **Borcherds Lie algebras**. Namely, if $\mathfrak{g}$ is a Kac-Moody Lie algebra (or a Borcherds Lie algebra) with a standard triangular decomposition $\mathfrak{g} = \mathfrak{g}_- \oplus \mathfrak{h} \oplus \mathfrak{g}_+$ and $\mathfrak{g}' = [\mathfrak{g}, \mathfrak{g}]$ is the derived subalgebra of $\mathfrak{g}$, then we obtain

$$U(\mathfrak{g}') = U(\mathfrak{g}_\pm)U(\mathfrak{g}_\mp)U(\mathfrak{g}_\pm),$$

which is called an **additive Gauss decomposition**. Similar decompositions exist for many **extended affine Lie algebras**. There is also a **quantum group** version, which implies a condition for **integrability** as an application (cf. [3]).

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Some Applications of Koszul Duality

Izuru Mori

Let $A = \oplus_{i=0}^{\infty} A_i$ be a connected graded algebra over a field k and $M = \oplus_{i=-\infty}^{\infty} M_i$ a graded left A-module. For each $n \in \mathbb{Z}$, we define a graded left A-module $M(n)$ by $M(n) = M$ as an ungraded left A-module but $M(n)_i = M_{n+i}$ for all $i \in \mathbb{Z}$. A linear resolution of M is a resolution of the form

$$\cdots \to \bigoplus A(-2) \to \bigoplus A(-1) \to \bigoplus A \to M \to 0,$$

that is, a free resolution in which each differential is given by right multiplication of a matrix whose entries are all degree 1 elements (linear elements) of A. We say that A is Koszul if k has a linear resolution as a graded left A-module.

If A is a Koszul algebra, then we can define the Koszul dual of A by

$$A^! = \bigoplus_{i=0}^{\infty} \mathrm{Ext}_A^i(k, k),$$

which has a structure of a connected graded algebra over k by the Yoneda product. An important classical result is that there is a duality, known as the Koszul duality, between the categories of graded left modules having linear resolutions over A and $A^!$, respectively. Recently, it has been shown that the Koszul duality can be extended to a duality between the derived categories of finitely generated graded left modules over A and $A^!$, respectively. Through this extended Koszul daulity, we are more able to translate results on A to those on $A^!$, and vice versa.

It is known that A is an Artin-Schelter regular Koszul algebra, which is an important class of algebras in Noncommutative Algebraic Geometry, if and only if $A^!$ is a Frobenius Koszul algebra. Further, the extended Koszul duality induces an equivalence between the derived category of a quantum projective space, which is a projective scheme associated to an Artin-Schelter regular Koszul algebra, and the stable category of the correnponding Frobenius Koszul algebra. So there is potential interaction between two research areas, Noncommutative Algebraic Geometry and the study of Frobenius algebras.

In this talk, I will give some examples of such interaction, namely, rationality of the Poincare series, Gorenstein condition, Grothedieck group, Serre duality, e.t.c.

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ガロア被覆を用いたフロベニウス多元環の構造と表現について

Kunio Yamagata

体上の有限次元フロベニウス多元環が，ある多元環の反復多元環（repetitive algebra）と，ある無限次元巡回群をガロア群にもつガロア被覆をもつための判定法を紹介する．また有限次元加群の圏への応用も紹介の予定．

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ON MODULES OF G-DIMENSION ZERO OVER NON-GORENSTEIN LOCAL RINGS

Ryo Takahashi

Let R be a commutative noetherian local ring with residue class field k. Put $(-)^* = \mathrm{Hom}_R(-, R)$, and denote by $\Omega_R^n M$ the nth syzygy module of a finitely generated R-module M.

Definition 1. Let M be a finitely generated R-module.
(1) If the natural homomorphism $M \to M^{**}$ is isomorphic and $\mathrm{Ext}_R^i(M, R) = \mathrm{Ext}_R^i(M^*, R) = 0$ for every $i > 0$, then we say that M has *G-dimension zero*, and write $\text{G-dim}_R M = 0$.
(2) If n is the smallest integer such that $\text{G-dim}_R(\Omega_R^n M) = 0$, then we say that M has *G-dimension* n, and write $\text{G-dim}_R M = n$. If such an integer n does not exist, then we say that M has *infinite G-dimension*, and write $\text{G-dim}_R M = \infty$.

Conjecture 2. Let R be a non-Gorenstein local ring. Suppose that there exists a non-free R-module of G-dimension zero. Then there exist infinitely many isomorphism classes of indecomposable R-modules of G-dimension zero.

Denote by $\mathrm{mod}R$ the category of finitely generated R-modules.

Definition 3. Let $\mathcal{X}$ be a full subcategory of $\mathrm{mod}R$.
(1) Let $\phi : X \to M$ be a homomorphism from $X \in \mathcal{X}$ to $M \in \mathrm{mod}R$. We call ϕ an *$\mathcal{X}$-precover* (or a *right $\mathcal{X}$-approximation*) of M if for any homomorphism $\phi' : X' \to M$ with $X' \in \mathcal{X}$ there exists a homomorphism $f : X' \to X$ such that $\phi' = \phi f$. We call ϕ an *$\mathcal{X}$-cover* (or a *right minimal $\mathcal{X}$-approximation*) of M if ϕ is an $\mathcal{X}$-precover and any endomorphism f of X with $\phi = \phi f$ is an automorphism.
(2) The category $\mathcal{X}$ is said to be *contravariantly finite* in $\mathrm{mod}R$ if every $M \in \mathrm{mod}R$ has an $\mathcal{X}$-precover.

Let $\mathcal{G}$ denote the full subcategory of $\mathrm{mod}R$ consisting of all R-modules of G-dimension zero.

Theorem 4. *Let R be a henselian non-Gorenstein local ring of depth zero. Suppose that there exists a non-free R-module in $\mathcal{G}$. Then the residue class field k of R does not admit a $\mathcal{G}$-precover as an R-module. In particular, the category $\mathcal{G}$ is not contravariantly finite in* $\mathrm{mod}R$.

Theorem 5. *Conjecture 2 is true if R is a henselian local ring of depth zero, especially if R is an artinian local ring.*

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Tilting complexes associated with a sequence of idempotents

Mitsuo Hoshino, Yoshiaki Kato

Rickard [3] showed that the Brauer tree algebras with the same numerical invariants are derived equivalent to each other. Let A be a Brauer tree algebra corresponding to a Brauer tree whose edges are labelled $1, 2, \ldots, n$. Note that there exists a partition of the edges $\{1, \ldots, n\} = E_0 \cup \cdots \cup E_l$, where E_s consists of the edges i for which there exists a sequence of edges $i_0, i_1, \ldots, i_s = i$ such that i_0 is adjacent to the exceptional vertex and for any $0 \leq r < l$, $i_r \neq i_{r+1}$ and i_r, i_{r+1} have a vertex in common. He constructed a tilting complex $P^\bullet \in \mathsf{K}^{\mathrm{b}}(\mathcal{P}_A)$ such that $P^j = 0$ for $j > 0$ and $j < -l$, $P^{-j} \in \mathrm{add}(\oplus_{i \in E_{l-j}} e_i A)$, where $e_i \in A$ is a local idempotent corresponding to the edge i, for $0 \leq j \leq l$ and $\mathrm{End}_{\mathsf{K}(\mathrm{Mod}\text{-}A)}(P^\bullet)$ is a Brauer "star" algebra with the same numerical invariants as A. On the other hand, Okuyama [2] pointed out recently that for Brauer tree algebras A, B with the same numerical invariants there exists a sequence of Brauer tree algebras $B_0 = A, B_1, \ldots, B_l = B$ such that B_{r+1} is the endomorphism algebra of a tilting complex for B_r of term length two defined by an idempotent. See König and Zimmermann [1] for another example of derived equivalences which are iterations of derived equivalences induced by tilting complexes of term length two. We will formulate these results.

Let A be a noetherian ring and $e_0, e_1, \ldots, e_l \in A$ a sequence of idempotents such that $\mathrm{add}(e_0 A_A) = \mathcal{P}_A$, $e_{i+1} \in e_i A e_i$ for $0 \leq i < l$ and $\mathrm{Ext}^j_A(A/Ae_iA, e_iA) = 0$ for $0 \leq j < i \leq l$. First, we will show that there exists a tilting complex $P^\bullet \in \mathsf{K}^{\mathrm{b}}(\mathcal{P}_A)$ such that $P^i = 0$ for $i > 0$ and $i < -l$, $P^{-i} \in \mathrm{add}(e_iA)$ for $0 \leq i \leq l$ and $\mathrm{H}^{-j}(P^\bullet) \in \mathrm{Mod}\text{-}(A/Ae_iA)$ for $0 \leq j < i \leq l$, and that such a tilting complex $P^\bullet$ is essentially unique. Next, we will show that there exists a sequence of rings $B_0 = A, B_1, \ldots, B_l = \mathrm{End}_{\mathsf{K}(\mathrm{Mod}\text{-}A)}(P^\bullet)$ such that for any $0 \leq i < l$, B_{i+1} is the endomorphism ring of a tilting complex for B_i of term length two defined by an idempotent. Furthermore, in case A is a selfinjective artin algebra over a commutative artin ring R and $\mathrm{add}(e_iA_A) = \mathrm{add}(D({}_AAe_i))$ for $1 \leq i \leq l$, where $D = \mathrm{Hom}_R(-, E(R/\mathrm{rad}\, R))$, we will show that $\mathrm{End}_{\mathsf{K}(\mathrm{Mod}\text{-}A)}(P^{\bullet}\bullet)$ is a selfinjective artin R-algebra whose Nakayama permutation coincides with that of A. Finally, we deal with the case where A is a finite dimensional algebra over a field k and $\mathrm{add}(e_iA_A) = \mathrm{add}(D({}_AAe_i))$ for $1 \leq i \leq l$, where $D = \mathrm{Hom}_k(-, k)$. We will construct a two-sided tilting complex which corresponds to $P^\bullet$. Simultaneously, we will provide a sufficient condition for an algebra B containing A as a subalgebra to be derived equivalent to A.

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The detailed version of this note has been published in the Journal of Pure and Applied Algebra.

Let's use cyclotomic polynomials in your lecture for your students

Kaoru MOTOSE

We will present some results on cyclotomic polynomials. We will talk about a factorization of a number, some fundamental theorems on algebra, and applications to a code and a cipher (cryptography). Moreover, we will present some relations between cyclotomic polynomials and Gauss sums.

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On calculations of modular irreducible characters with the help of computers

Katsushi Waki

Let p be a prime. We denote p-modular system by (K, R, F) where F is an algebraically closed field of characteristic p. Let G be a finite group. It is not so easy to calculate of irreducible modular characters correspondint to simple FG-module from ordinary characters correspondint to simple KG-module. I will introduce a standard way for the calculation of modular irreducible characters without a concrete construction of simple FG-modules.

I also show some short demonstrations of the calculations of irreducibel modular characters by GAP[1]

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Self-duality of quasi-Harada rings and locally distributive rings

Kazutoshi Koike

Recently Y. Baba [1] proved that for a QH ring (quasi-Harada ring) R, if gRg is a local serial ring with some condition, then R has a self-duality, where g is an idempotent of R with gR a minimal faithful right module. (Note that (two-sided) QH rings are QF-3.) Motivated by this result, we also investigate self-duality of QH rings and obtain several results including an improvement of the result of Y. Baba and applications to self-duality of locally distributive rings.

Y. Baba and K. Iwase [2] called a left artinian ring R *left QH* in case every indecomposable projective *right* R-module is quasi-injective. In our study, the following easy lemma is very important.

Lemma 1. *Let R be a left QH ring and let $S = \mathrm{End}_R(E(R_R))$. Then there exist a diagonally complete subring S' of S and an ideal I of S' such that $R \cong S'/I$.*

Here we say that a subring R' of a ring R is *diagonally complete* in case there exists a complete set $\{e_1, \ldots, e_n\}$ of orthogonal idempotents for R such that $e_iRe_i \subset R'$ $(1 \leq i \leq n)$. Diagonally complete subrings R' inherit many properties from R. Particularly we have

Theorem 2. *Diagonally complete subrings inherit Morita duality and good self-duality.*

Good self-duality is a special type of self-duality. Every serial ring has a good self-duality and factor rings inherit good self-duality. From the results above, we can obtain an improvement of the result of Baba. We can also prove the following. (Compare with properties of left H-rings.)

Theorem 3. (1) *Every left QH ring R has a right Morita duality. In particular R is right artinian.*

(2) *Every left QH ring R can be constructed from a QF ring.*

An artinian ring R is said to be *locally distributive* in case each of its left and right indecomposable projective R-module is distributive. Clearly serial rings are locally distributive. Azumaya conjectured that every exact artinian ring has a self-duality (see [4]). While locally distributive rings are exact, even the self-duality for locally distributive rings is still open. Since locally distributive right QF-2 rings are left QH, we can apply results of left QH rings to these rings and obtain the following partial results. Here almost self-duality is a generalization of self-duality (see [3]).

Theorem 4. *Let R be a locally distributive right QF-2 ring with basic set $\{e_1, \ldots, e_n\}$ of orthogonal primitive idempotents and let $m = \#\{e_i \mid \mathrm{soc}(R_R)e_i \neq 0\}$. Then*

(1) *R has an almost self-duality.*

(2) *If R is right serial or $m \leq 2$, then R has a good self-duality.*

(3) *If $n \leq 3$, then R has a self-duality.*

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On $S_R(H)$-blocks II

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Let G be a finite group, p a prime divisor of the order of G and (K, R, k) a p-modular system, i.e., R is a complete discrete valuation ring with maximal ideal (π), K is the quotient field of R of characteristic 0 and $k(:= R/(\pi))$ is the residue class field of R of characteristic p. Moreover, we assume that K contains the $|G|$th roots of unity.

For a subset X of G, $\widehat{X}$ denotes the sum of all elements of X in the group algebra $\mathfrak{o}G$, where $\mathfrak{o}$ is R, K or k. We consider the *Hecke algebra* $S_{\mathfrak{o}}(H) := \mathrm{End}_{\mathfrak{o}G}(\widehat{H}\mathfrak{o}G)$ for a subgroup H of G.

In [7] G.R.Robinson has proved that $Z(S_R(H)) \simeq A_R(H)$ as R-algebras, where $A_R(H)$ denotes the endomorpism ring $\mathrm{End}_{R[G\times G]}(RG\widehat{H}RG)$. So he has defined $A_R(H)$*-blocks* (we call them $S_R(H)$*-blocks*) and their *defect groups* in $G \times G$ for the (central) primitive idempotents of $A_R(H)$ $(S_R(H))$. (We mention that the set of $S_R(H)$-blocks corresponds bijectively to the set of $S_k(H)$-blocks as $\widehat{H}RG$ is a permutation module.)

In this talk we show some results on $S_R(H)$-blocks, which are almost proved in [4] and [7] and some new results on *the principal* $S_R(H)$*-blocks* and their defect groups.

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A pair of rings with common ideals

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Abstract

A study of pairs of commutative rings with the same set of prime ideals appears in the literature. In this talk, we consider some generalizations of the study and investigate pairs of rings, not necessarily commutative, with certain ideals in common. Several examples of such pairs will be given and conditions that pass through such rings will be studied.

Finitely Cogenerated Distributive Modules

Yasutaka SHINDOH

Throughout this talk, "A" will always denote an associative ring with a non-zero identity and all modules will be unitary left A-modules. A module is said to be *distributive* if the lattice of its submodules satisfies the distributive law. Two modules M_1 and M_2 are said to form an *unrelated pair* when the following condition holds:

If L_1/N_1 is isomorphic to L_2/N_2 for some submodules $N_i \leq L_i \leq M_i$ $(i = 1, 2)$, then $L_1 = N_1$ and $L_2 = N_2$.

A module M said to be X-*cyclic* for some module X, when the following condition holds:

If $\sum_{\alpha \in \mathrm{Hom}_A(X,M)} \mathrm{Im}\alpha = M$, then there exists an epimorphism $\alpha \in \mathrm{Hom}_A(X, M)$.

A module M said to be X-*cocyclic* for some module X, when the following condition holds:

If $\bigcap_{\beta \in \mathrm{Hom}_A(M,X)} \mathrm{Ker}\beta = 0$, then there exists a monomorphism $\beta \in \mathrm{Hom}_A(M, X)$.

Using the above concepts, we obtain the following theorem as a generalization of the main result of [Vam78] and its duality.

Theorem 1. Let M be a module. Then the following conditions hold.

(1) If M is finitely generated semilocal and distributive, then M is P-cyclic for any projective module P.
(2) If M is finitely cogenerated and distributive, then M is Q-cocyclic for any injective module Q.

The aim of this talk is to prove and establish the above theorem.

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Global dimension in left serial algebras

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Let A be a finite dimentional basic conected algebra over an algebraically closed field k, and n is the number of the non isomorphic simple left modules of A. If A is quasi-hereditary, then the global dimension $gl.dim.A$ is less than or equal to $2n-2$, and the Loewy length $l(A)$ of A is less than or equal to 2^n-1 [3]. If A is serial and $gl.dim.A$ is finite, then $gl.dim.A \leq 2n-2$ and $l(A) \leq 2n-1$ [2]. On the other hand, Yamagata constructed the algebras of large global dimension with few simple modules [4], and Deng constructed the algebras of arbitrary finite global dimension with $n \geq 2$ arbitrary [1]. So, if we consider the relationship between $gl.dim.A$ and n, we need some conditions on algebras. In this talk we show that left serial quasi-hereditary algebras have global dimension less than or equal to n. Next, we show that left serial algebras have global dimension less than or qual to $2n-2$ when they have finite global dimension.

Lemma 1. *If A is an algebra whose quiver has no oriented cycles then the global dimension of A is less than or equal to $n-1$.*

Example 2. In the above lemma, the bound of global dimension is sharp. Let A be an algebra defined by the following quiver with relations $\{\alpha_{i+1}\alpha_i = 0(1 \leq i \leq n-2)\}$.

$$1 \xrightarrow{\alpha_1} 2 \xrightarrow{\alpha_2} \cdots \xrightarrow{\alpha_{n-1}} n$$

Then the global dimension of A is $n-1$.

Now we recall the definition of heredity idempotents and quasi-hereditary algebras. Let N be the Jacobson radical of A. An idempotent e is said to be a heredity idempotent of A if $eNe = 0$ and AeA is projective as a left A-module. An algebra A is said to be quasi-hereditary if there is a sequence $\{e_1, e_2, \cdots, e_m\}$ of (not necessarily primitive) idempotents of A such that, for any $1 \leq t \leq m$, $\overline{\varepsilon_t}$ is a heredity idempotent of $A/A\varepsilon_{t+1}A$ and $\varepsilon_1 = 1$, where $\varepsilon_j = e_j + e_{j+1} + \cdots + e_m$ for $1 \leq j \leq m$ and $\varepsilon_{n+1} = 0$. Such a sequence called a heredity sequence.

Theorem 3. *Let A be a serial quasi-hereditary algebra. Then $gl.dim.A \leq n$ and $l(A) \leq 2n-1$, and these bounds are sharp.*

We can show the same result when A is a right or left (not necessarily both) serial algebra, and more over when A has unique oriented cycle in its quiver.

Theorem 4. *Let A be a left serial quasi-hereditary algebra. Then $gl.dim.A \leq n$ and $l(A) \leq 2n-1$, and these bounds are sharp.*

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Derivations of polynomial rings over a field of characteristic zero

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We present some old and new results concerning several famous algebraic problems formulated using derivations of polynomial algebras. The main role in this lecture play rings of constants and locally nilpotent derivations. We present also some observations and remarks for simple and semisimple derivations.

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フロベニュース多元環について

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QF 多元環は数学のいろいろな場面に現れる重要な研究対象である。最近の話題としては Sklyanin A1gebra や AS-Regu1ar Ring の Koszul dual として現れることが知られている。講演ではフロベニウス多元環がかなり具体的に構成され分類されるものであることを説明したい。具体的には、体 K 上の分離多元環 A、A 上の両側加群 ${}_AM_A$、根基の巾零指数 $n \geq 2$ が与えられたとき、$\Lambda/rad\Lambda \cong A$、$rad\Lambda/rad^2\Lambda \cong M$、$rad^n\Lambda = 0$ となるようなフロベニウス多元環 Λ の K 同型類全体の記述について考える。 更に graded 多元環の場合には、一般の場合に比べて話が単純になるので、可能な中山同型や多元環の分類例を与えることが出来る。 QF 多元環全体の様子はそれで一応分かったことになるが、他の問題に関してその記述がそのまま使えるわけではない。そのため、幾つかの別の構成法も考える。 このうちの一つとして、多元環 A 上の巾零対称多元環 (φ, ψ) : ${}_AM \otimes_A M_A \xrightarrow{\varphi} {}_AM_A$, ${}_AM \otimes_A M_A \xrightarrow{\psi} D(A)$, から対称多元環 $A(\varphi, \psi) = A \oplus M \oplus D(A)$ が構成されること、傾斜加群 ${}_BT_A$ が与えられたとき、T_A と ${}_AM_A$ の間で一定の条件が満たされるとき、B 上の巾零対称多元環 (φ^T, ψ^T) が自然に得られ、 $A(\varphi, \psi)$ と $A(\varphi^T, \psi^T)$ が安定同値になることを示す。この二つの対称多元環は、$pd({}_BT), pd(T_A) < \infty$ の場合には自明拡大多元環の場合と同様の方法で、導来同値になるということが示される。