HIGHER DIMENSIONAL AUSLANDER-REITEN THEORY ON MAXIMAL ORTHOGONAL SUBCATEGORIES[1]

Osamu Iyama

Abstract. Auslander-Reiten theory, especially the concept of almost split sequences and their existence theorem, is fundamental to study categories which appear in representation theory, for example, modules over artin algebras [ARS][GR][R], their functorially finite subcategories [AS][S], their derived categories [H], Cohen-Macaulay modules over Cohen-Macaulay rings [Y], lattices over orders [A2,3][RS], and coherent sheaves on projective curves [AR][GL]. In these Auslander-Reiten theory, the number '2' is quite symbolic. For one thing, almost split sequences give minimal projective resolutions of simple objects of projective dimension '2' in functor categories. For another, Cohen-Macaulay rings and orders of Krull-dimension '2' have fundamental sequences and provide us one of the most beautiful situation in representation theory [A4][E][RV][Y], which is closely related to McKay's observation on simple singularities [M]. In this sense, usual Auslander-Reiten theory should be '2-dimensional' theory, and it would have natural importance to search a domain of higher Auslander-Reiten theory from the viewpoint of representation theory and non-commutative algebraic geometry (e.g. [V1,2][Ar][GL]). In this paper, we introduce $(n-1)$-orthogonal subcategories as a natural domain of '$(n+1)$-dimensional' Auslander-Reiten theory. We show that higher Auslander-Reiten translation and higher Auslander-Reiten duality can be defined quite naturally for such categories. Using them, we show that our categories have *n-almost split sequences*, which are completely new generalization of usual almost split sequences and give minimal projective resolutions of simple objects of projective dimension '$n+1$' in functor categories. We also show the existence of higher dimensional analogy of fundamental sequences for Cohen-Macaulay rings and orders of Krull-dimension '$n+1$'. We show that an invariant subring (of Krull-dimension '$n+1$') corresponding to a finite subgroup G of $\mathrm{GL}_{n+1}(k)$ has a natural maximal $(n-1)$-orthogonal subcategory.

1 From Auslander-Reiten theory

1.1 Let us recall M. Auslander's classical theorem [A1] below, which introduced a completely new insight to representation theory of algebras (see 2.3 for $\mathrm{dom.dim}\,\Gamma$).

Theorem A (Auslander correspondence) *There exists a bijection between the set of Morita-equivalence classes of representation-finite finite-dimensional algebras Λ and that of finite-dimensional algebras Γ with $\mathrm{gl.dim}\,\Gamma \leq 2$ and $\mathrm{dom.dim}\,\Gamma \geq 2$. It is given by $\Lambda \mapsto \Gamma := \mathrm{End}_\Lambda(M)$ for an additive generator M of $\mathrm{mod}\,\Lambda$.*

In this really surprising theorem, the representation theory of Λ is encoded in the structure of the homologically nice algebra Γ called an *Auslander algebra*. Since the category $\mathrm{mod}\,\Gamma$ is equivalent to the functor category on $\mathrm{mod}\,\Lambda$, Auslander correspondence

[1]The detailed version [I2,3] of this paper have been submitted for publication elsewhere.

gave us a prototype of the use of functor categories in representation theory. In this sense, Auslander correspondence was a starting point of later Auslander-Reiten theory [ARS] historically. Theoretically, Auslander correspondence gives a direct connection between two completely different concepts, i.e. a representation theoretic property 'representation-finiteness' and a homological property '$\mathrm{gl.dim}\,\Gamma \leq 2$ and $\mathrm{dom.dim}\,\Gamma \geq 2$'. It is a quite interesting project to find correspondence between representation theoretic properties and homological properties (e.g. [I1]).

1.2 Let R be a complete regular local ring of dimension d and Λ a module-finite R-algebra. We call Λ an *isolated singularity* [A3] if $\mathrm{gl.dim}\,\Lambda \otimes_R R_{\mathfrak{p}} = \mathrm{ht}\,\mathfrak{p}$ holds for any non-maximal prime ideal $\mathfrak{p}$ of R. We call a left Λ-module M *Cohen-Macaulay* if it is a projective R-module. We denote by $\mathrm{CM}\,\Lambda$ the category of Cohen-Macaulay Λ-modules. Then $D_d := \mathrm{Hom}_R(\ ,R)$ gives a duality $\mathrm{CM}\,\Lambda \leftrightarrow \mathrm{CM}\,\Lambda^{op}$. We call Λ an *R-order* (or *Cohen-Macaulay R-algebra*) if $\Lambda \in \mathrm{CM}\,\Lambda$ [A2,3]. In this case, let $\underline{\mathrm{CM}}\Lambda := (\mathrm{CM}\,\Lambda)/[\Lambda]$ be the *stable category* and $\overline{\mathrm{CM}}\Lambda := (\mathrm{CM}\,\Lambda)/[D_d\Lambda]$ the *costable category*. A typical example of an order is a commutative complete local Cohen-Macaulay ring Λ containing a field since such Λ contains a complete regular local subring R [Ma]. Let $\mathbf{E} : 0 \to R \to E_0 \to \cdots \to E_d \to 0$ be a minimal injective resolution of the R-module R. We denote by $D := \mathrm{Hom}_R(\ ,E_d)$ the Matlis dual. Let us recall the fundamental theorems [A2,3][Y] below, where we will give the definition of (pseudo) almost split sequences in 2.5 (put $n := 1$ there).

Theorem B *(1)* (Auslander-Reiten translation) *There exists an equivalence* $\tau : \underline{\mathrm{CM}}\Lambda \to \overline{\mathrm{CM}}\Lambda$.

(2) (Auslander-Reiten duality) *There exist functorial isomorphisms* $\overline{\mathrm{Hom}}_\Lambda(Y, \tau X) \simeq D\,\mathrm{Ext}^1_\Lambda(X,Y) \simeq \underline{\mathrm{Hom}}_\Lambda(\tau^{-1}Y, X)$ *for any* $X, Y \in \mathrm{CM}\,\Lambda$.

Theorem C *(1)* $\mathrm{CM}\,\Lambda$ *has almost split sequences.*

(2) If $d = 2$, then $\mathrm{CM}\,\Lambda$ *has pseudo almost split seqeces.*

Consequently, almost all simple objects in the functor category $\mathrm{mod}(\mathrm{CM}\,\Lambda)$ have projective dimension 2. If $d = 2$, then all simple objects in the functor category $\mathrm{mod}(\mathrm{CM}\,\Lambda)$ have projective dimension 2. In this sense, we can say that *Auslander-Reiten theory for the case $d = 2$ is very nice.* Using (pseudo) almost split sequences, we can define the *Auslander-Reiten quiver* $\mathfrak{A}(\Lambda)$ of Λ (see 2.6 and put $n := 1$ there).

1.3 Let us recall Auslander's contribution [A4][Y] to McKay correspondence [M]. Let k be a field of characteristic zero and G a finite subgroup of $\mathrm{GL}_d(k)$ with $d \geq 2$. Recall that the *McKay quiver* $\mathfrak{M}(G)$ of G [M] is defined as follows: The set of vertices is the set $\mathrm{irr}\,G$ of isoclasses of irreducible representations of G. Let V be the representation of G acting on k^d through $\mathrm{GL}_d(k)$. For $X, Y \in \mathrm{irr}\,G$, we denote by d_{XY} the multiplicity of X in $V \otimes_k Y$, and draw d_{XY} arrows from X to Y.

Theorem D *Let G be a finite subgroup of* $\mathrm{GL}_2(\mathbb{C})$, $\Omega := \mathbb{C}[[x,y]]$ *and* $\Lambda := \Omega^G$ *the invariant subring. Assume that G does not contain pseudo-reflection except the identity. Then Λ is representation-finite with* $\mathrm{CM}\,\Lambda = \mathrm{add}_\Lambda\,\Omega$, *and the Auslander-Reiten quiver* $\mathfrak{A}(\Lambda)$ *of Λ coincides with the McKay quiver* $\mathfrak{M}(G)$ *of G.*

1.4 Aim We observed that Auslander-Reiten theory is 2-dimensional-like. Now we can state the aim of this paper. *For each $n \geq 1$, find a domain of $(n+1)$-dimensional*

Auslander-Reiten theory. Namely, *find natural categories $\mathcal{C}$ such that Theorems above replaced '2' and* $\mathrm{CM}\,\Lambda$ *by '$n+1$' and $\mathcal{C}$ respectively hold.*

2 Main results

2.1 Definition Let $\mathcal{A}$ be an abelian category, $\mathcal{B}$ a full subcategory of $\mathcal{A}$ and $n \geq 0$. For a functorially finite [AS] full subcategory $\mathcal{C}$ of $\mathcal{B}$, we put

$$\begin{aligned} \mathcal{C}^{\perp_n} &:= \{X \in \mathcal{B} \mid \mathrm{Ext}^i_{\mathcal{A}}(\mathcal{C}, X) = 0 \text{ for any } i\ (0 < i \leq n)\}, \\ {}^{\perp_n}\mathcal{C} &:= \{X \in \mathcal{B} \mid \mathrm{Ext}^i_{\mathcal{A}}(X, \mathcal{C}) = 0 \text{ for any } i\ (0 < i \leq n)\}. \end{aligned}$$

We call $\mathcal{C}$ a *maximal n-orthogonal subcategory* of $\mathcal{B}$ if

$$\mathcal{C} = \mathcal{C}^{\perp_n} = {}^{\perp_n}\mathcal{C}$$

holds. By definition, $\mathcal{B}$ is a unique maximal 0-orthogonal subcategory of $\mathcal{B}$.

2.2 Example Let Λ be a simple singularity of type Δ and dimension $d = 2$, $\mathcal{A} := \mathrm{mod}^{\mathbb{Z}}\,\Lambda$ the category of graded Λ-modules and $\mathcal{B} := \mathrm{CM}^{\mathbb{Z}}\,\Lambda$ the category of graded Cohen-Macaulay Λ-modules. Then the number of maximal 1-orthogonal subcategories of $\mathcal{B}$ is given as follows:

Δ	A_m	B_m, C_m	D_m	E_6	E_7	E_8	F_4	G_2
number	$\frac{1}{m+2}\binom{2m+2}{m+1}$	$\binom{2m}{m}$	$\frac{3m-2}{m}\binom{2m-2}{m-1}$	833	4160	25080	105	8

This is obtained by showing that maximal 1-orthogonal subcategories of $\mathcal{B}$ correspond bijectively to clusters of the cluster algebra of type Δ [I2,3]. See Fomin-Zelevinsky [FZ1,2] and Buan-Marsh-Reineke-Reiten-Todorov [BMRRT]. See also Geiss-Leclerc-Schröer [GLS].

2.3 For a finite-dimensional algebra Γ, we denote by $0 \to \Gamma \to I_0 \to I_1 \to \cdots$ a minimal injective resolution of the Γ-module Γ. Put $\mathrm{dom.dim}\,\Gamma := \inf\{i \geq 0 \mid I_i \text{ is not projective}\}$ [T]. The following theorem gives a higher dimensional version of Theorem A.

Theorem A′ (($n+1$)-dimensional Auslander correspondence) *For any $n \geq 1$, there exists a bijection between the set of equivalence classes of maximal $(n-1)$-orthogonal subcategories $\mathcal{C}$ of* $\mathrm{mod}\,\Lambda$ *with additive generators M and finite-dimensional algebras Λ, and the set of Morita-equivalence classes of finite-dimensional algebras Γ with* $\mathrm{gl.dim}\,\Gamma \leq n+1$ *and* $\mathrm{dom.dim}\,\Gamma \geq n+1$. *It is given by $\mathcal{C} \mapsto \Gamma := \mathrm{End}_\Lambda(M)$.*

2.4 In the rest of this section, let R be a complete regular local ring of dimension d, Λ an R-order which is an isolated singularity, $\mathcal{A} := \mathrm{mod}\,\Lambda$ and $\mathcal{B} := \mathrm{CM}\,\Lambda$. For $n \geq 1$, we define functors τ_n and τ_n^- by

$$\tau_n := \tau \circ \Omega^{n-1} : \underline{\mathrm{CM}}\Lambda \to \overline{\mathrm{CM}}\Lambda \quad \text{and} \quad \tau_n^- := \tau^- \circ \Omega^{-(n-1)} : \overline{\mathrm{CM}}\Lambda \to \underline{\mathrm{CM}}\Lambda,$$

where $\Omega : \underline{\mathrm{CM}}\Lambda \to \underline{\mathrm{CM}}\Lambda$ is the syzygy functor and $\Omega^- : \overline{\mathrm{CM}}\Lambda \to \overline{\mathrm{CM}}\Lambda$ is the cosyzygy functor. For a subcategory $\mathcal{C}$ of $\mathrm{CM}\,\Lambda$, we denote by $\underline{\mathcal{C}}$ and $\overline{\mathcal{C}}$ the corresponding subcategories of $\underline{\mathrm{CM}}\Lambda$ and $\overline{\mathrm{CM}}\Lambda$ respectively.

Theorem B′ *Let $\mathcal{C}$ be a maximal $(n-1)$-orthogonal subcategory of* $\mathrm{CM}\,\Lambda$ *($n \geq 1$).*

(1) (n-Auslander-Reiten translation) *For any $X \in \mathcal{C}$, $\tau_n X \in \mathcal{C}$ and $\tau_n^- X \in \mathcal{C}$ hold. Thus $\tau_n : \underline{\mathcal{C}} \to \overline{\mathcal{C}}$ and $\tau_n^- : \overline{\mathcal{C}} \to \underline{\mathcal{C}}$ are mutually quasi-inverse equivalences.*

(2) (n-Auslander-Reiten duality) *There exist functorial isomorphisms $\overline{\mathcal{C}}(Y, \tau_n X) \simeq D\,\mathrm{Ext}^n_\Lambda(X, Y) \simeq \underline{\mathcal{C}}(\tau_n^- Y, X)$ for any $X, Y \in \mathcal{C}$.*

2.5 Definition Let $\mathcal{C}$ be a full subcategory of $\mathrm{CM}\,\Lambda$ and $J_\mathcal{C}$ the Jacobson radical of $\mathcal{C}$. We call an exact sequence

$$0 \to Y \xrightarrow{f_n} C_{n-1} \xrightarrow{f_{n-1}} \cdots \xrightarrow{f_1} C_0 \xrightarrow{f_0} X \to 0$$
$$(\text{resp. } 0 \to Y \xrightarrow{f_n} C_{n-1} \xrightarrow{f_{n-1}} \cdots \xrightarrow{f_1} C_0 \xrightarrow{f_0} X)$$

with terms in $\mathcal{C}$ an *n-almost split sequence* (resp. *pseudo n-almost split sequence*) if $f_i \in J_\mathcal{C}$ holds for any i and the following sequences are exact.

$$0 \to \mathcal{C}(\ ,Y) \xrightarrow{\cdot f_n} \mathcal{C}(\ ,C_{n-1}) \xrightarrow{\cdot f_{n-1}} \cdots \xrightarrow{\cdot f_1} \mathcal{C}(\ ,C_0) \xrightarrow{\cdot f_0} J_\mathcal{C}(\ ,X) \to 0$$
$$0 \to \mathcal{C}(X,\ ) \xrightarrow{f_0 \cdot} \mathcal{C}(C_0,\ ) \xrightarrow{f_1 \cdot} \cdots \xrightarrow{f_{n-1} \cdot} \mathcal{C}(C_{n-1},\ ) \xrightarrow{f_n \cdot} J_\mathcal{C}(Y,\ ) \to 0$$

We call $f_0 : C_0 \to X$ a *sink map* and $f_n : Y \to C_{n-1}$ a *source map*. We say that $\mathcal{C}$ *has n-almost split sequences* if, for any non-projective $X \in \mathrm{ind}\,\mathcal{C}$ (resp. non-injective $Y \in \mathrm{ind}\,\mathcal{C}$), there exists an n-almost split sequence $0 \to Y \to C_{n-1} \to \cdots \to C_0 \to X \to 0$. Similarly, we say that $\mathcal{C}$ *has pseudo n-almost split sequences* if, for any projective $X \in \mathcal{C}$ (resp. injective $Y \in \mathcal{C}$), there exists a pseudo n-almost split sequence $0 \to Y \to C_{n-1} \to \cdots \to C_0 \to X$.

Theorem C′ *Let $\mathcal{C}$ be a maximal $(n-1)$-orthogonal subcategory of $\mathrm{CM}\,\Lambda$ $(n \geq 1)$.*
(1) $\mathcal{C}$ has n-almost split sequences.
(2) If $d = n+1$, then $\mathcal{C}$ has pseudo n-almost split sequeces.

Consequently, almost all simple objects in the functor category $\mathrm{mod}\,\mathcal{C}$ have projective dimension $n+1$. If $d = n+1$, then all simple objects in the functor category $\mathrm{mod}\,\mathcal{C}$ have projective dimension $n+1$. In this sense, we can say that *$(n+1)$-dimensional Auslander-Reiten theory for the case $d = n+1$ is very nice.*

2.6 We will define the *Auslander-Reiten quiver* $\mathfrak{A}(\mathcal{C})$ of $\mathcal{C}$. For simplicity, we assume that the residue field k of R is algebraically closed. The set of vertices of $\mathfrak{A}(\mathcal{C})$ is $\mathrm{ind}\,\mathcal{C}$. For $X, Y \in \mathrm{ind}\,\mathcal{C}$, we denote by d_{XY} be the multiplicity of X in C for the sink map $C \to Y$, which equals to the multiplicity of Y in C' for the source map $X \to C'$. Draw d_{XY} arrows from X to Y.

Theorem D′ *Let G be a finite subgroup of $\mathrm{GL}_d(\mathbb{C})$, $\Omega := \mathbb{C}[[x_1, \cdots, x_d]]$ and $\Lambda := \Omega^G$ the invariant subring. Assume that G does not contain pseudo-reflection except the identity, and that Λ is an isolated singularity. Then $\mathcal{C} := \mathrm{add}_\Lambda \Omega$ is a maximal $(d-2)$-orthogonal subcategory of $\mathrm{CM}\,\Lambda$. Moreover, the Auslander-Reiten quiver $\mathfrak{A}(\mathcal{C})$ of $\mathcal{C}$ coincides with the McKay quiver $\mathfrak{M}(G)$ of G, i.e. there exists a bijection $\mathbb{H} : \mathrm{irr}\,G \to \mathrm{ind}\,\mathcal{C}$ such that $d_{XY} = d_{\mathbb{H}(X),\mathbb{H}(Y)}$ for any $X, Y \in \mathrm{irr}\,G$.*

3 Non-commutative crepant resolution and representation dimension

3.1 Let us generalize the concept of Van den Bergh's non-commutative crepant resolution [V1,2] of commutative normal Gorenstein domains to our situation.

Again let Λ be an R-order which is an isolated singularity. We call $M \in \mathrm{CM}\,\Lambda$ a *NCC resolution* of Λ if $\Lambda \oplus D_d\Lambda \in \mathrm{add}\,M$ and $\Gamma := \mathrm{End}_\Lambda(M)$ is an R-order with $\mathrm{gl.dim}\,\Gamma = d$. Our definition is slightly stronger than original non-commutative crepant resolutions in [V2] where M is assumed to be reflexive (not Cohen-Macaulay) and $\Lambda \oplus D_d\Lambda \in \mathrm{add}\,M$ is not assumed. But all examples of non-commutative crepant resolutions in [V1,2] satisfy our condition. For the case $d \geq 2$, we have the remarkable relationship below between NCC resolutions and maximal $(d-2)$-orthogonal subcategories.

Theorem *Let $d \geq 2$. Then $M \in \mathrm{CM}\,\Lambda$ is a NCC resolution of Λ if and only if $\mathrm{add}\,M$ is maximal $(d-2)$-orthogonal subcategory of $\mathrm{CM}\,\Lambda$.*

3.2 Conjecture It is interesting to study relationship among all maximal $(n-1)$-orthogonal subcategories of $\mathrm{CM}\,\Lambda$. Especially, we conjecture that *their endomorphism rings are derived equivalent*. It is suggestive to relate this conjecture to Van den Bergh's generalization [V2] of Bondal-Orlov conjecture [BO], which asserts that *all (commutative or non-commutative) crepant resolutions of a normal Gorenstein domain have the same derived category*. Since maximal $(n-1)$-orthogonal subcategories are analogy of non-commutative crepant resolutions from the viewpoint of 3.1, our conjecture is an analogy of Bondal-Orlov-Van den Bergh conjecture. We have the following partial solution.

Theorem *(1) Let $\mathcal{C}_i = \mathrm{add}\,M_i$ be a maximal 1-orthogonal subcategory of $\mathrm{CM}\,\Lambda$ and $\Gamma_i := \mathrm{End}_\Lambda(M_i)$ $(i = 1, 2)$. Then Γ_1 and Γ_2 are derived equivalent. In particular, $\#\,\mathrm{ind}\,\mathcal{C}_1 = \#\,\mathrm{ind}\,\mathcal{C}_2$ holds.*

(2) If $d \leq 3$, then all NCC resolutions of Λ have the same derived category.

3.3 Let us generalize the concept of Auslander's representation dimension [A1] to relate it to non-commutative crepant resolutios. For $n \geq 1$, define the *n-th representation dimension* $\mathrm{rep.dim}_n\,\Lambda$ of an R-order Λ which is an isolated singularity by

$$\mathrm{rep.dim}_n\,\Lambda := \inf\{\mathrm{gl.dim}\,\mathrm{End}_\Lambda(M) \mid M \in \mathrm{CM}\,\Lambda,\ \Lambda \oplus D_d\Lambda \in \mathrm{add}\,M,\ M \perp_{n-1} M\}.$$

Obviously $d \leq \mathrm{rep.dim}_n\,\Lambda \leq \mathrm{rep.dim}_{n'}\,\Lambda$ holds for any $n \leq n'$. For the case $d = 0$, $\mathrm{rep.dim}_1\,\Lambda$ coincides with the representation dimension defined in [A1]. We call Λ *representation-finite* if $\#\,\mathrm{ind}(\mathrm{CM}\,\Lambda) < \infty$. In the sense of (2) below, $\mathrm{rep.dim}_1\,\Lambda$ measures how far Λ is from being representation-finite.

Theorem *(1) Assume $d \leq n+1$. Then $\mathrm{CM}\,\Lambda$ has a maximal $(n-1)$-orthogonal subcategory $\mathcal{C}$ with $\#\,\mathrm{ind}\,\mathcal{C} < \infty$ if and only if $\mathrm{rep.dim}_n\,\Lambda \leq n+1$.*

(2) Assume $d \leq 2$. Then Λ is representation-finite if and only if $\mathrm{rep.dim}_1\,\Lambda \leq 2$.

(3) Λ has a NCC resolution if and only if $\mathrm{rep.dim}_{\max\{1,d-1\}}\,\Lambda = d$.

3.4 Conjecture It seems that no example of a maximal $(n-1)$-orthogonal subcategory $\mathcal{C}$ of $\mathrm{CM}\,\Lambda$ with $\#\,\mathrm{ind}\,\mathcal{C} = \infty$ is known. This suggests us to study

$$o(\mathrm{CM}\,\Lambda) := \sup_{\mathcal{C} \subseteq \mathrm{CM}\,\Lambda,\ \mathcal{C} \perp_1 \mathcal{C}} \#\,\mathrm{ind}\,\mathcal{C}\,.$$

We conjecture that $o(\mathrm{CM}\,\Lambda)$ *is always finite*. If Λ is a preprojective algebra of Dynkin type Δ, then Geiss-Schröer [GS] proved that $o(\mathrm{mod}\,\Lambda)$ equals to the number of positive roots of Δ. It would be interesting to find a geometric interpretation of $o(\mathrm{CM}\,\Lambda)$ for more general CM Λ. For some classes of CM Λ, one can calculate $o(\mathrm{CM}\,\Lambda)$ by using the theorem below. Especially, (1) seems to be interesting in the connection with known results for algebras with representation dimension at most 3 [IT][EHIS].

Theorem *(1)* $\mathrm{rep.dim}_1 \Lambda \leq 3$ *implies* $o(\mathrm{CM}\,\Lambda) < \infty$.
(2) If CM Λ *has a maximal* 1*-orthogonal subcategory* $\mathcal{C}$*, then* $o(\mathrm{CM}\,\Lambda) = \#\,\mathrm{ind}\,\mathcal{C}$.

3.5 Concerning our conjecture, let us recall the well-known proposition below which follows by a geometric argument due to Voigt's lemma ([P;4.2]). It is interesting to ask whether it is true without the restriction on R. If it is true, then any 1-orthogonal subcategory of CM Λ is 'discrete', and our conjecture asserts that it is finite. It is interesting to study the discrete structure of 1-orthogonal objects in CM Λ and the relationship to whole structure of CM Λ.

Proposition *Assume that R is an algebraically closed field. For any $n > 0$, there are only finitely many isoclasses of* 1*-orthogonal* Λ*-modules* X *with* $\dim_R X = n$.

References

[Ar] M. Artin: Maximal orders of global dimension and Krull dimension two. Invent. Math. 84 (1986), no. 1, 195–222.

[A1] M. Auslander: Representation dimension of Artin algebras. Lecture notes, Queen Mary College, London, 1971.

[A2] M. Auslander: Functors and morphisms determined by objects. Representation theory of algebras (Proc. Conf., Temple Univ., Philadelphia, Pa., 1976), pp. 1–244. Lecture Notes in Pure Appl. Math., Vol. 37, Dekker, New York, 1978.

[A3] M. Auslander: Isolated singularities and existence of almost split sequences. Representation theory, II (Ottawa, Ont., 1984), 194–242, Lecture Notes in Math., 1178, Springer, Berlin, 1986.

[A4] M. Auslander: Rational singularities and almost split sequences. Trans. Amer. Math. Soc. 293 (1986), no. 2, 511–531.

[AR] M. Auslander, I. Reiten: Almost split sequences in dimension two. Adv. in Math. 66 (1987), no. 1, 88–118.

[ARS] M. Auslander, I. Reiten, S. O. Smalo: Representation theory of Artin algebras. Cambridge Studies in Advanced Mathematics, 36. Cambridge University Press, Cambridge, 1995.

[AS] M. Auslander, S. O. Smalo: Almost split sequences in subcategories. J. Algebra 69 (1981), no. 2, 426–454.

[BO] A. Bondal, D. Orlov: Semiorthogonal decomposition for algebraic varieties, preprint.

[BMRRT] A. Buan, R. Marsh, M. Reineke, I. Reiten, G. Todorov: Tilting theory and cluster combinatorics, preprint.

[E] H. Esnault: Reflexive modules on quotient surface singularities. J. Reine Angew. Math. 362 (1985), 63–71.

[EHIS] K. Erdmann, T. Holm, O. Iyama, J. Schröer: Radical embeddings and representation dimension. Adv. Math. 185 (2004), no. 1, 159–177.

[FZ1] S. Fomin, A. Zelevinsky: Cluster algebras. I. Foundations. J. Amer. Math. Soc. 15 (2002), no. 2, 497–529.

[FZ2] S. Fomin, A. Zelevinsky: Cluster algebras. II. Finite type classification. Invent. Math. 154 (2003), no. 1, 63–121.

[GLS] C. Geiss, B. Leclerc, J. Schröer: Exceptional modules over preprojective algebras, preprint.

[GS] C. Geiss, J. Schröer: Extension-orthogonal components of preprojective varieties, to appear in Transactions of the American Mathematical Society.

[GL] W. Geigle, H. Lenzing: A class of weighted projective curves arising in representation theory of finite-dimensional algebras. Singularities, representation of algebras, and vector bundles (Lambrecht, 1985), 265–297, Lecture Notes in Math., 1273, Springer, Berlin, 1987.

[GR] P. Gabriel, P, A. V. Roiter: Representations of finite-dimensional algebras. Springer-Verlag, Berlin, 1997.

[H] D. Happel: Triangulated categories in the representation theory of finite-dimensional algebras. London Mathematical Society Lecture Note Series, 119. Cambridge University Press, Cambridge, 1988.

[I1] O. Iyama: The relationship between homological properties and representation theoretic realization of artin algebras, Trans. Amer. Math. Soc. 357 (2005), no. 2, 709–734.

[I2] O. Iyama: Higher dimensional Auslander-Reiten theory on maximal orthogonal subcategories, preprint.

[I3] O. Iyama: Higher dimensional Auslander correspondence, preprint.

[IT] K. Igusa, G. Todorov: On the finitistic global dimension conjecture for artin algebras, preprint.

[Ma] H. Matsumura: Commutative ring theory. Cambridge Studies in Advanced Mathematics, 8. Cambridge University Press, Cambridge, 1989.

[M] J. McKay: Graphs, singularities, and finite groups. The Santa Cruz Conference on Finite Groups (Univ. California, Santa Cruz, Calif., 1979), pp. 183–186, Proc. Sympos. Pure Math., 37, Amer. Math. Soc., Providence, R.I., 1980.

[P] J. A. de la Pena: On the dimension of the module-varieties of tame and wild algebras. Comm. Algebra 19 (1991), no. 6, 1795–1807.

[R] C. M. Ringel: Tame algebras and integral quadratic forms. Lecture Notes in Mathematics, 1099. Springer-Verlag, Berlin, 1984.

[RS] K. W. Roggenkamp, J. W. Schmidt: Almost split sequences for integral group rings and orders. Comm. Algebra 4 (1976), no. 10, 893–917.

[RV] I. Reiten, M. Van den Bergh: Two-dimensional tame and maximal orders of finite representation type. Mem. Amer. Math. Soc. 80 (1989).

[S] D. Simson: Linear representations of partially ordered sets and vector space categories. Algebra, Logic and Applications, 4. Gordon and Breach Science Publishers, Montreux, 1992.

[T] H. Tachikawa: Quasi-Frobenius rings and generalizations. Lecture Notes in Mathematics, Vol. 351. Springer-Verlag, Berlin-New York, 1973.

[V1] M. Van den Bergh: Three-dimensional flops and noncommutative rings. Duke Math. J. 122 (2004), no. 3, 423–455.

[V2] M. Van den Bergh: Non-commutative crepant resolutions. The legacy of Niels Henrik Abel, 749–770, Springer, Berlin, 2004.

[Y] Y. Yoshino: Cohen-Macaulay modules over Cohen-Macaulay rings. London Mathematical Society Lecture Note Series, 146. Cambridge University Press, Cambridge, 1990.

Department of Mathematics, University of Hyogo, Himeji, 671-2201, Japan
iyama@sci.u-hyogo.ac.jp