The 38th Symposium On Ring and Representation Theory (2005)

ABSTRACT

Aichi Institute of Technology
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Algebras arising from Schur-Weyl type dualities (I) (II) (III)

Susum Ariki (有木　進)

The talks are mostly expository as was requested by the organizers. Our goal is to study algebras which are intimately related to quantum groups and algebraic groups of classical type. Birman-Wenzl and Murakami's algebras, Hecke algebras, their affine versions and degenerate versions are examples. We also have Yangians, Hecke-Clifford algebras etc. An interesting feature of the study is that we have variety of methods to study the algebras: we sometimes have geometric realizations, relationship with Kashiwara's crystal etc.

In the first talk, I introduce these algebras. In the second talk, I select several results on these algebras and explain them. Perhaps Nazarov's and Suzuki's results will be included. Stress is on the affine versions, and truncation to finite dimensional algebras is often useful. In the third talk, I explain the quasihereditary/cellular algebra framework, which is suitable for the study of the truncated algebras. I end with a recent result of mine which is a joint work with Andrew Mathas and Hebing Rui.

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Orthogonality of subcategories
(Joint with I. Reiten and Y. Yoshino)

Osamu Iyama (伊山　修)

Let G be a finite small subgroup of $SL_d(k)$ and $S := k[[x_1, ..., x_d]]$. We will discuss (i)–(iv) below.

(i) maximal $(d-2)$-orthogonal subcategories of $CM S^G$,
(ii) non-commutative crepant resolutions of S^G,
(iii) tilting S G-modules,
(iv) Fomin-Zelevinsky mutation.

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Broué's abelian defect group conjecture

Shigeo Koshitani (越谷 (こしたに) 重夫)

I am going to give a (hopefully new) example and direct calculations for Broué's abelian defect group conjecture, which is on modular representation theory of finite groups, and which is due to Michel Broué.

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Integral group algebras and cyclotomic polynomials

Kaoru Motose

Let ZG be the group algebra of a finite abelian group G over the ring Z of rational integers. It is well known that the units of finite order in ZG have the form $\pm g$ for some g G (see [1]). However, units of infinite order in ZG are not so many found just as transcendental numbers, even if G is cyclic.

Z. Marciniak and S.K. Sehgal [3] construct many units of infinite order using cyclotomic polynomials. These units cover the alternative units, the Hoechsmann units, the Bass cyclic units [3], and Yamauchi's results [4].

I talk about the Euclidean Algorithm for cyclotomic polynomials in $Z[x]$, and easy applications to some their results in [3].

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LINKAGE AND DUALITY OF MODULES

Kenji Nishida

The theory of linkage, devised by Peskine and Szpiro[4], is recently generalized to module theoretic version by Martsinkovsky and Strooker in [3]. They generalize the theory for wide class of rings, including non-commutative semiperfect Noetherian rings. The use of composition of two sort of functors, syzygy and transpose, enables them to extend the definition of linkage. These functors are fundamental for homological theory of Noetherian rings. There appears a relation to the duality theory introduced and studied by Auslander and Bridger[1] and Iyama[4].

We shall apply this relation to Gorenstein dimension and Cohen-Macaulayness of modues, mainly over commutative Gorenstein local ring, and obtain characterization of these notions by linkage. Then we see an invariance of them under linkage. A duality studied by Iyama[4] can be applicable to the full subcategory of Cohen-Macaulay modules of codimension $k > 0$ over a Gorenstein local ring. We decide the image of this subcategory and give a characterization of a Cohen-Macaulay module of codimension $k > 0$.

Let Λ be a left and right Noetherian ring. Let modΛ (respectively, modΛ^{op}) be the category of all finitely generated left (respectively, right) Λ-modules. Throughout, all modules are finitely generated and left modules and right modules are considered as Λ^{op}-modules. We denote the stable category by $\underline{\text{mod}}\Lambda$, the syzygy functor by $\Omega : \underline{\text{mod}}\Lambda \to \underline{\text{mod}}\Lambda$, and the transpose functor by $\text{Tr} : \underline{\text{mod}}\Lambda \to \underline{\text{mod}}\Lambda^{op}$. Recall the definition of the functor Tr ([1], Chapter two, section 1). Let $P_1 \xrightarrow{f} P_0 \to M \to 0$ be a projective resolution of a module M modΛ. Then the transpose of M, TrM modΛ^{op}, is equal to Cok$f$, where $(-)$: modΛ → modΛ^{op} is defined by M := $\text{Hom}_\Lambda(M, \Lambda)$.

Put the functors $T_k := \text{Tr}\Omega^{k-1}$ for $k > 0$ and $\lambda := \Omega\text{Tr}$. Using the operator λ, Martsinkovsky and Strooker defined the notion of linkage of modules [3].

Definition. A finitely generated Λ-module M and a Λ^{op}-module N are said to be *horizontally linked* if $M = \lambda N$ and $N = \lambda M$, in other words, M is horizontally linked (to λM) if and only if $M = \lambda^2 M$.

In this talk, we shall study linkage of modules.

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Stable equivalences related with syzygy functors

Yosuke OHNUKI

Let Λ and Λ^0 be finite dimensional self-injective algebras over a fixed field K. We denote by $\underline{\text{mod}}\,\Lambda$ and $\underline{\text{mod}}\,\Lambda^0$ the stable categories of the categories of finite dimensional left Λ-modules and Λ^0-modules, respectively. Happel proved in [1] that the stable category of a self-injective algebra is a triangulated category whose translation is the inverse of the syzygy functor. I study whether a stable equivalence is an equivalence as triangulated categories. Let A, A^0 be triangulated categories, and T, T^0 be translations of A, A^0, respectively. An additive functor $\Phi : A \to A^0$ is called a triangle functor if it preserves any triangle, that is, $\Phi X \overset{\Phi f}{\to} \Phi Y \overset{\Phi g}{\to} \Phi Z \overset{\Phi h}{\to} \Phi T X$ ' $T^0 \Phi X$ is a triangle in A^0 for each triangle $X \overset{f}{\to} Y \overset{g}{\to} Z \overset{h}{\to} TX$ in A.

Theorem 1. *Assume that there is an equivalence $\Phi : \underline{\text{mod}}\,\Lambda \to \underline{\text{mod}}\,\Lambda^0$ for self-injective algebras Λ and Λ^0. Then the following conditions are equivalent.*

(1) *Φ is a triangle functor.*
(2) *$\Omega^0\Phi$ ' $\Phi\Omega$, where Ω and Ω^0 are the sygyzy functors of Λ and Λ^0, respectively.*

A derived equivalence induces a stable equivalence of Morita type. The symmetry is invariant under a derived equivalence for self-injective algebras. However, it is not invariant under a stable equivalence. By the Theorem 2, we can construct the example which is the stable equivalence for the symmetric algebra and the non-symmetric self-injective algebra. In this example, K is not algebraically closed. We can show that if K is an algebraically closed field, then the symmetry is the invariant under a stable equivalence constructed in Theorem 2.

Theorem 2 ([2])**.** *Let Λ and Λ^0 be socle equivalent self-injective algebras, that is, there is an algebra isomorphism $p : \Lambda/\operatorname{soc}\Lambda \to \Lambda^0/\operatorname{soc}\Lambda^0$. Assume that there are non-degenerate K-linear maps $\lambda : \Lambda \to K$ and $\lambda^0 : \Lambda^0 \to K$ such that $\lambda(ab) = \lambda^0(a^0b^0)$ for all a, b $\operatorname{rad}\Lambda$ and a^0, b^0 $\operatorname{rad}\Lambda^0$ with $\overline{a}^0 = p(\overline{a})$ and $\overline{b}^0 = p(\overline{b})$. Then the stable categories $\underline{\text{mod}}\,\Lambda$ and $\underline{\text{mod}}\,\Lambda^0$ are equivalent.*

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Hochschild Cohomology of Stratified Algebras

Hiroshi Nagase

When studying Hochschild cohomology it is natural to try relating cohomology H(B) of an algebra B to H(A) of an 'easier' or 'smaller' algebra A. In the case that B is a one-point extension of A, two long exact sequences with terms of H(B) and H(A) have been studied by Happel [2]. Happel's long exact sequences have been generalized to triangular matrix algebras, for example by Michelena and Platzeck [3].

We would like to try generalizing these results. We consider the case that B has an ideal I which gives a fully faithful functor from derived category of B/I to that of B. Such an ideal is called a *stratifying* ideal by Cline, Parshall and Scott[1]. Heredity ideals are examples of stratifying ideals. By a characterization of stratifying ideals, it is not di cult to show that any triangular matrix algebra has a stratifying ideal.

It is known that stratifying ideals are idempotent ideals, namely any stratifying ideal of B has the form of BeB for some idempotent e in B. By using of this idempotent e, we get the following two long exact sequences, which are our main results.

$$\cdots \to \mathrm{Ext}^n_{B^e}(B, BeB) \to H^n(B) \to H^n(A) \to \mathrm{Ext}^{n+1}_{B^e}(B, BeB) \to \cdots, \text{ and}$$

$$\cdots \to \mathrm{Ext}^n_{B^e}(A, BeB) \to H^n(B) \to H^n(A) \quad H^n(eBe) \to \mathrm{Ext}^{n+1}_{B^e}(A, BeB) \to \cdots,$$

where $A = B/BeB$ and $B^e = B \quad B^{op}$.

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Broué's conjecture: methods and results (I) (II) (III)

Raphaël Rouquier

We will explain the recent approaches to the abelian defect conjecture, following work of several mathematicians. That conjecture predicts what the derived category of a block of a finite group should be, under certain assumptions.

Locals methods have provided gluing possibilities which essentially reduce the problem to lifting stable equivalences of Morita type to derived equivalences, a problem in the realm of representations of algebras.

We will discuss the use of categorification in the work with Joe Chuang on symmetric groups.

We will also survey some methods specific to finite groups of Lie type and those used for special situations.

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QF Rings and QF Associated Graded Rings

Hiroyuki Tachikawa (太刀川弘幸)

For an Artinian ring R having the Jacobson radical J with $J^{n+1} = 0$, the series $R \quad J \quad J^2 \quad \cdots \quad J^n \quad J^{n+1} = 0$ is called the upper Loewy series of ${}_RR$ (resp. R_R). If we put $A_i = J^i/J^{i+1}$, we can naturally define the multiplication of elements $a + J^{i+1} \quad A_i$ and $b + J^{j+1} \quad A_j$ to be $ab + J^{i+j+1} \quad A_{i+j}$. Then by using this multiplication we make the (formal) direct sum $A_0 \quad A_1 \quad \cdots \quad A_n$ into a ring R_G. Clearly this ring R_G is positive Z−graded and A_1 generates the radical of R_G. R_G is called the associated graded ring of R. Cf.[3]. R and R_G may be not isomorphic to each other.

By Morita equivalence [8] we can assume without loss of generality that rings are basic. Let e be a primitive idempotent of ring R. Then $e + J \quad A_0$ is a primitive idemotent of R_G which we shall denote by e_G for short. If we denote the right (resp. left) annihilator of a subset M of R by $r(M)$ (resp. $l(M)$), then $Soc(Re) = r(J)e$ (resp. $Soc(eR) = el(J)$). At first we have

Proposition 3. If ${}_{R_G}Soc(R_Ge_G)$ is simple for a primitive idempotent e_G, then the ${}_RSoc(Re)$ is simple. And if $Soc(R_Ge_G)$ ' $R_Gf_G/Rad(R_G)f_G$ for a primitive idempotent f, then $Soc(Re)$ ' Rf/Jf.

QF rings [9] are same with Noetherian selfinjective rings. It holds the duality between categories of finitely generated left modules and right modules over them [5,7,8]. From Proposition 1 and the duality over R_G we have

Theorem 4. If R_G is QF, then R is QF.

Now it seems of interest to give a characterization of QF ring R which has QF associated graded ring R_G.

We say that the series $Re = r(J^{\rho+1})e \quad r(J^{\rho})e \quad r(J^{\rho-1})e \quad \cdots \quad r(J)e \quad r(R)e = 0$ is the lower Loewy series of Re.

In their book [2] Artin-Nesbitt-Thrall proved that subquotient modules $J^ke/j^{k+1}e$ and $r(J^{\rho+1-k}e/r(J^{\rho-k})e$ have non-zero isomorhic constituents for every $0 \quad k \quad \rho$. Which kind of rings do satisfy the coincidence of the above subquotient modules themselves ?

A positive Z-graded ring $R = A_0 \quad A_1 \quad \cdots \quad A_n$ is called to be standard if A_1 generates the radical of R. Then we have

Proposition 5. If R is a normal positive Z-graded QF ring, then the upper Loewy series of Re coincides with the lower Loewy series of Re for any primitive idempotent e.

Again by the duality for QF rings and Proposition 2 it follows

Theorem 6. The following conditions (i), (ii) and (iii) are equivalent to each other:

(i) The associated graded ring R_G is QF,

(ii) R is QF and for any primitive idempotent e the upper Loewy series of Re is coincident with the lower Loewy series of Re,

(iii) R is QF and for any primitive idempotent e_i and integer $0 \quad k \quad \rho_i$ it holds that ${}_RJ^ke_i/J^{k+1}e_i$ ' ${}_RHom_R(e_iJ^{\rho-k}/e_iJ^{\rho-k+1}{}_R, {}_RR_R)$, where $J^{\rho_i}e_i \neq 0$ but $J^{\rho_i+1}e_i = 0$.

Let π be a Nakayama permutation of QF ring R on the set of all non isomorphic primitive idempotents e_i, $i = 1, 2, \cdots, n$. Then it holds that ${}_RRe_{\pi(j)}/Je_{\pi(j)}$ ' ${}_RHom_R(e_jR/e_jJ_R, {}_RR_R)$

Corollary 7. R_G is QF if and only if R is QF and for any primitive idempotent e_i it holds that ${}_RJ^ke_i/J^{k+1}e_i$ ' $\sum_j^n n_{i,j} \times Re_{\pi(j)}/Je_{\pi(j)}$ for a direct sum decomposition

$e_i J^{\rho-k}/e_i J^{\rho-k+1}{}_R \cong \oplus_j^n n_{i,j} \times e_j R/e_j J$, where $n_{i,j} \times e_j R/e_j J$ means the direct sum of $n_{i,j}$ copies of $e_j R/e_j J$.

As indecomposable commutative algebras are local, Nakayama permutation is always identity. Corollary 5 is reported as symmetry of Hilbert function.

Theorem 4 seems to be a generalization of Iarrobino's result [4; Proposition 1.7] for a commutative local algebra. But this is strictly not because from the beginning he assumes that R is local Gorenstein.

Even for commutative algebras we can use Theorem 2 to check whether they are QF (= 0-Gorenstein). We introduce the following Example 6.

Example 8. Let Λ be a quotient ring $K[x_0, x_1, \cdots, x_n]/I$ such that the ideal I are generated by $n+1$ polynomials $x_i^t - \frac{1}{x_i}\prod_{j=0}^n x_j$, $i = 0, 1, \cdots, n$, for the pairs (n, t).

In case of $t \neq n$, for $\min\{n, t\} \le |t-n|s < \max\{n, t\}$ there is an idempotent $f \equiv \prod_{i=0}^n x_i^{|t-n|s}$ mod I and $\Gamma = (1-e)\Lambda$ is an Artinian local algebra and $\Gamma \neq \Gamma_G$.

Our problem is whether Γ is QF. Using Gröbner bases [1] of $(\prod_{i=0}^n x_i^{(t-n)s}, I)$ we can prove that Γ_G (in place of Γ) is QF. For the detail Cf. [11] and Kikumasa-Yoshimura [6].

We can extend our consideration for Λ to the case of $n = t$. Then Λ has no proper idempotent and a local Noetherian ring of Krull dim $\Lambda = 1$. As for all cases where $t \neq n$ Λ is 0-Gorenstein we expect that Λ is Gorenstein. However we are disappointed by the next example which provides a negative answer. Y.Yoshino was so kind to comment this possibility before.

Example 9. Let $t = n = 2$. Then $\{f_2 = x_0x_1 - x_2^2, f_0 = x_1x_2 - x_0^2, f_1 = x_2x_0 - x_1^2\}$ generates I which defines an intersection of quadratic cones. By using the Buchberger's algorithm we obtain the reduced Gröbner bases $\{f_0, f_1, f_2, f_3 = S(f_0, f_1) = -x_0^3 - x_1^3\}$ of I with respect to the degree- lexicographical order $x_0 < x_1 < x_2$. Cf. [1]. Thus the Poincaré series $F(\Lambda, \lambda) = 1 + \sum_{n=1}^{\infty} 3\lambda^n = \frac{3}{1-\lambda} - 2 = \frac{2\lambda+1}{1-\lambda}$. Hence there is no ρ which satisfies $(-1)^1\lambda^\rho F(\Lambda, \lambda) = (-1)\lambda^\rho \frac{2\lambda+1}{(1-\lambda)} == \frac{\lambda+2}{(\lambda-1)} = \frac{2\frac{1}{\lambda}+1}{1+\frac{1}{\lambda}} = F(\Lambda, \frac{1}{\lambda})$.

Therefore by Macauley's Theorem Λ is not Gorenstein. Cf.[10]

Further we want to remark in case of $t = n = 3$, Λ is neither Cohen-Macauley nor toric [11], because it contains a polynomial ring $K[\bar{x}_0]$ and there exists $\gamma(\neq 0) \in \Lambda$ such that $\gamma\bar{x}_0 = 0$. Of course Λ is not Gorenstein for this case too.

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A generalization of n-torsionfree modules

Ryo Takahashi

Throughout this talk, we assume that R is a commutative noetherian ring, and that all R-modules are finitely generated. Recall that an R-module M is called n-torsionfree if $\mathrm{Ext}^i_R(\mathrm{Tr}M, R) = 0$ for any $1 \quad i \quad n$. Auslander and Bridger [1] proved the following theorem.

Theorem A (Auslander-Bridger). The following are equivalent for an R-module M:

(1) $\Omega^n M$ is n-torsionfree;
(2) There exists an exact sequence $0 \to Y \to X \to M \to 0$ of R-modules such that $\mathrm{Ext}^i_R(X, R) = 0$ for any $1 \quad i \quad n$ and $\mathrm{pd}_R Y < n$.

On the other hand, Auslander and Buchweitz [2] proved the following theorem.

Theorem B (Auslander-Buchweitz). Let R be a Cohen-Macaulay local ring with a canonical module. Then for every R-module M there exists an exact sequence $0 \to Y \to X \to M \to 0$ of R-modules such that X is maximal Cohen-Macaulay and $\mathrm{id}_R Y < \infty$.

Such an exact sequence as in Theorem B is called a (maximal) Cohen-Macaulay approximation of M.

Definition. Let C be an R-module and set $(-)^\dagger = \mathrm{Hom}_R(-, C)$. For an R-module M, let λ_M denote the natural homomorphism $M \to M^{\dagger\dagger}$.
(1) We say that M is 1-C-torsionfree if λ_M is injective. We say that M is n-C-torsionfree, where $n \geq 2$, if λ_M is bijective and $\mathrm{Ext}^i_R(M^\dagger, C) = 0$ for all $1 \quad i \quad n - 2$.
(2) We say that C is 1-semidualizing if λ_R is injective and $\mathrm{Ext}^1_R(C, C) = 0$. We say that C is n-semidualizing, where $n \geq 2$, if λ_R is bijective and $\mathrm{Ext}^i_R(C, C) = 0$ for all $1 \quad i \quad n$.

For an R-module M, we denote by $C\mathrm{dim}_R M$ the infimum of nonnegative integers n such that there exists an exact sequence $0 \to C_n \to C_{n-1} \to \cdots \to C_0 \to M \to 0$ with each C_i being in $\mathrm{add}_R C$. The main result of this talk is the following.

Theorem. Let C be an n-semidualizing R-module. Then the following are equivalent for an R-module M:

(1) $\Omega^n M$ is n-C-torsionfree;
(2) There exists an exact sequence $0 \to Y \to X \to M \to 0$ of R-modules such that $\mathrm{Ext}^i_R(X, C) = 0$ for any $1 \quad i \quad n$ and $C\mathrm{dim}_R Y < n$.

This theorem implies both Theorems A and Theorem B as corollaries. We will consider in this talk what module M satisfies the condition that $\Omega^n M$ is n-C-torsionfree.

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Castelnuovo-Mumford regularity for complexes and weakly Koszul modules

Kohji Yanagawa (柳川浩二)

Let $A := \bigoplus_{i \geq 0} A_i$ be a noetherian AS regular Koszul quiver algebra over a field k (if A is commutative, it is essentially a polynomial ring), $\mathrm{Gr}\, A$ (resp. $\mathrm{Gr}\, A^{op}$) the category of graded left (resp. right) A-modules, and $\mathrm{gr}\, A$ (resp. $\mathrm{gr}\, A^{op}$) its full subcategory consisting of finitely generated modules. Set $\mathfrak{m} := \bigoplus_{i \geq 1} A_i$. We have the left exact functor $\Gamma_{\mathfrak{m}} : \mathrm{Gr}\, A \to \mathrm{Gr}\, A$ defined by $\Gamma_{\mathfrak{m}}(M) = \{x \in M \mid \mathfrak{m}^n x = 0 \text{ for } n \gg 0\}$. We denote the i^{th} right derived functor of $\Gamma_{\mathfrak{m}}$ by $H^i_{\mathfrak{m}}(-)$.

We have a dualizing complex $D^\bullet \in D^b(\mathrm{Gr}\, A)$ (and $\in D^b(\mathrm{Gr}\, A^{op})$) which gives a duality between $D^b(\mathrm{gr}\, A)$ and $D^b(\mathrm{gr}\, A^{op})$ and admits "local duality theorem"

$$R\underline{\mathrm{Hom}}_A(-, D^\bullet) = R\Gamma_{\mathfrak{m}}(-) .$$

This is a quiver algebra version of [6].

By virtue of this duality, we can generalize a well-know result in [2] concerning graded modules over a polynomial ring to our $D^b(\mathrm{gr}\, A)$ (see also [3] for another "non-commutative + complex" version). Set $\beta^{i,j}(M^\bullet) := \dim_k \underline{\mathrm{Ext}}^{-i}_A(M^\bullet, A/\mathfrak{m})_{-j}$ for $M^\bullet \in D^b(\mathrm{gr}\, A)$. $\beta^{i,j}(-)$ measures the "size" of a minimal projective resolution.

Definition-Theorem. For $M^\bullet \in D^b(\mathrm{gr}\, A)$, we have

$$\sup\{i + j \mid H^i_{\mathfrak{m}}(M^\bullet)_j \neq 0\} = \sup\{i + j \mid \beta^{i,j}(M^\bullet) \neq 0\} < \infty.$$

We call this value the "Castelnuovo-Mumford regularity" of $M^\bullet$, and denote it by $\mathrm{reg}(M^\bullet)$.

Let $A^!$ be the quadratic dual ring of A. Then $A^!$ is finite dimensional and self-injective. It is well-known that the Koszul duality functor $DG : D^b(\mathrm{gr}\, A) \to D^b(\mathrm{gr}\, A^!)$ gives an equivalence $D^b(\mathrm{gr}\, A) \cong D^b(\mathrm{gr}\, A^!)$ (c.f. [1]). Then we have

$$\mathrm{reg}(M^\bullet) = \max\{i \mid H^i(DG(M^\bullet)) \neq 0\}.$$

Using these concepts, we interpret/refine results in [4] on weakly Koszul modules over $A^!$. We say $M \in \mathrm{gr}\, A^!$ is weakly Koszul, if the submodule $M_{\langle n \rangle}$ of M generated by its degree n component M_n satisfies "$\beta^{i,j}(M_{\langle n \rangle}) \neq 0 \Rightarrow i + j = n$" for each n. It is known that, for any $N \in \mathrm{gr}\, A^{op}$, the i^{th} syzygy of N is weakly Koszul for $i \gg 0$. As an application, we show that if J is a monomial ideal of an exterior algebra $E = \bigwedge \langle y_1, \ldots, y_d \rangle$, $d \geq 3$, then the $(d-2)^{nd}$ syzygy of E/J is weakly Koszul.

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