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ABSTRACT

CHARACTER THEORY OF SEMISIMPLE BI-FROBENIUS ALGEBRAS

Yukio DOI

Bi-Frobenius algebras were introduced by the author and Takeuchi. They are Frobenius algebras with some coalgebra structures, and generalize both finite-dimensional Hopf algebras and scheme rings of association schemes. We study a character theory of bi-Frobenius algebras and its application to association schemes.

We work over a field k; unadorned Hom and $\otimes$ are assumed to be taken over k. Recall that a **Frobenius algebra** (A, ϕ) is a finite-dimensional k-algebra A together with a linear function $\phi \in A^* = \mathrm{Hom}(A, k)$ such that the map $\theta : A \to A^*,\ a \mapsto (\phi \leftharpoonup a)$, is a right A-module isomorphism. Here A^* is a two-sided A-module via $(a \rightharpoonup f)(b) := f(ba)$ and $(f \leftharpoonup a)(b) := f(ab)$. Let $\sum_i x_i \otimes y_i \in A \otimes A$ be the element corresponding to id_A under the isomorphism: $A \otimes A \xrightarrow{id_A \otimes \theta} A \otimes A^* = \mathrm{End}(A)$. Then the inverse of θ is given by $\theta^{-1}(f) = \sum_i f(x_i) y_i$ for all $f \in A^*$ and we see

$$a = \sum x_i \phi(y_i a) = \sum \phi(a x_i) y_i, \quad \sum a x_i \otimes y_i = \sum x_i \otimes y_i a$$

for all $a \in A$. The pair $(\phi,\ \sum_i x_i \otimes y_i)$ is called a **Frobenius system** for A. The element $v(A) := \sum_i x_i y_i$ is in $Z(A)$, the center of A. If $v(A)$ is an invertible element, then A is a separable algebra, since $\sum_i v(A)^{-1} x_i \otimes y_i$ is a separable idempotent. The element $v(A)$ is called the **volume** of A. Note that $A \times A \to k,\ (a, b) \mapsto \phi(ab)$, is an associative, non-degenerate bilinear form. If we take $\{x_i\}$ to be linearly independent, then it satisfies $\phi(y_i x_j) = \delta_{ij}$ (dual basis property).

Dually, let C be a k-coalgebra with comultiplication $\Delta : C \to C \otimes C$ and counit $\varepsilon : C \to k$. We write $\Delta(c) = \sum_{(c)} c_{(1)} \otimes c_{(2)} = \sum c_1 \otimes c_2$ for $c \in C$. The dual C^* is an algebra by convolution, that is, $(f * g)(c) = \sum f(c_1) g(c_2)$, and C has a two-sided C^*-module structure via $f \rightharpoonup c := \sum c_1 f(c_2)$ and $c \leftharpoonup f := \sum f(c_1) c_2$ for $f \in C^*,\ c \in C$. A **Frobenius coalgebra** (C, t) is a finite-dimensional k-coalgebra C together with an element $t \in C$ such that the map $\kappa : C^* \to C,\ f \mapsto (t \leftharpoonup f)$ is a right C^*-module isomorphism.

Definition. Let H be a finite-dimensional algebra with coalgebra structure $\Delta : H \to H \otimes H,\ \varepsilon : H \to k$. Given $\phi \in H^*$ and $t \in H$ and define a map $S : H \to H$ by $S(h) := t \leftharpoonup (h \rightharpoonup \phi) = \sum \phi(t_1 h) t_2$. Then the 4-tuple (H, ϕ, t, S) is called a **bi-Frobenius algebra** (or **bF algebra**) if it satisfies the following (BF1)–(BF6): (BF1) $\varepsilon \in \mathrm{Alg}_k(H, k)$; (BF2) $1 \in G(H)$; i.e. $\Delta(1) = 1 \otimes 1,\ \varepsilon(1) = 1$; (BF3) (H, ϕ) is a Frobenius algebra; (BF4) (H, t) is a Frobenius coalgebra; (BF5) S is an anti-algebra map (i.e. $S(hh') = S(h')S(h),\ S(1) = 1$); (BF6) S is an anti-coalgebra map (i.e. $\Delta(S(h)) = \sum S(h_2) \otimes S(h_1),\ \varepsilon(S(h)) = \varepsilon(h)$).

Since H is finite-dimensional, its k-linear dual H^* is also an algebra and coalgebra. One can easily show that (H^*, t, ϕ, S^*) becomes a bF algebra. We call it **the dual** of (H, ϕ, t, S).

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Linearity Defect of graded modules over an exterior algebra

Kohji Yanagawa

Let $S = K[x_1, \dots, x_n]$ be a polynomial ring over a field K, and $E := K\langle y_1, \dots, y_n\rangle$ an exterior algebra. We regard them as graded rings with $\deg x_i = \deg y_i = 1$.

Let $A = \bigoplus_{i \geq 0} A_i$, $A_0 = K$, be a Koszul algebra (e.g. $A = S$ or E), and $\operatorname{gr} A$ the category of finitely generated graded left A-modules. For $M \in \operatorname{gr} A$ and $i \in \mathbb{Z}$, $M_{\langle i \rangle}$ denotes the submodule of M generated by its degree i part M_i. We say M is *weakly Koszul*, if $M_{\langle i \rangle}$ has a graded free resolution of the form

$$\cdots \to A^{\oplus \beta_j}(-i-j) \to \cdots \to A^{\oplus \beta_2}(-i-2) \to A^{\oplus \beta_1}(-i-1) \to A^{\oplus \beta_0}(-i) \to M_{\langle i \rangle} \to 0$$

for all i. If M is weakly Koszul, so is its ith syzygy $\Omega_i(M)$ for all $i \geq 0$. We define the *linearity defect* $\operatorname{ld}(M)$ of M by

$$\operatorname{ld}_A(M) := \inf\{\, i \mid \Omega_i(M) \text{ is weakly Koszul}\,\}.$$

Clearly, $\operatorname{ld}_S(M) \leq \operatorname{proj.dim}_S(M) \leq n$. But even if A is commutative, $\operatorname{ld}_A(M)$ can be infinite. But we have the following.

Theorem 1. *(1)* (Eisenbud et. al. [1]) *We have* $\operatorname{ld}_E(N) < \infty$ *for all* $M \in \operatorname{gr} E$.

(2) If $n \geq 2$, *then* $\sup\{\operatorname{ld}_E(N) \mid N \in \operatorname{gr} E\} = \infty$. *But* $\operatorname{ld}_E(N) \leq w^{n!} 2^{(n-1)!}$, *where* $w := \max\{\dim_K N_i \mid i \in \mathbb{Z}\}$.

(3) (Herzog-Römer) *If* J *is a monomial ideal of* $E = K\langle y_1, \dots, y_n\rangle$, *then* $\operatorname{ld}_E(J) \leq n - 1$.

The parts (1) and (2) of the theorem are closely related to the *Bernstein-Gel'fand-Gel'fand correspondence* $D^b(\operatorname{gr} S) \cong D^b(\operatorname{gr} E)$. The main results of this talk improve/refine the part (3).

Set $[n] := \{1, \dots, n\}$, and let $\Delta \subset 2^{[n]}$ be a simplicial complex (i.e., $F \subset G \in \Delta$ implies $F \in \Delta$). To Δ, we assign monomial ideals of S and E as follows:

$$I_\Delta := (\prod_{i \in F} x_i \mid F \subset [n], F \notin \Delta) \subset S, \qquad J_\Delta := (\prod_{i \in F} y_i \mid F \subset [n], F \notin \Delta) \subset E.$$

Any monomial ideal of E is of the form J_Δ for some Δ. And S/I_Δ is called the *Stanley-Reisner ring* of Δ, and important in combinatorial commutative algebra. The following theorem follows from BGG correspondence for S/I_Δ and E/J_Δ.

Theorem 2. (Okazaki-Y [2]) *Let* $\Delta \neq \emptyset$ *be a simplicial complex on* $[n]$. *Then;*

(1) $\operatorname{ld}_S(I_\Delta) = \operatorname{ld}_E(J_\Delta)$. *(So we denote this value by* $\operatorname{ld}(\Delta)$.*)*

(2) $\operatorname{ld}(\Delta)$ *is a topological invariant of the geometric realization* $|\Delta^\vee|$ *of the dual complex* $\Delta^\vee := \{\, F \subset [n] \mid [n] - F \notin \Delta\,\}$ *of* Δ.

(3) *For* $n \geq 4$, $\operatorname{ld}(\Delta) = n - 3$ *(this is the largest possible value) if and only if* Δ *is an* n*-gon.*

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Compactly generated triangulated categories, Well generated triangulated categories, The homotopy category of flat R-modules

Amnon Neeman

(Talk 1) Compactly generated triangulated categories

ABSTRACT: Compactly generated triangulated categories are easy enough to define. There is a short list of main theorems about them. We will state the definitions and the theorems, and then illustrate their power by looking at simple examples from algebraic geometry.

(Talk 2) Well generated triangulated categories

ABSTRACT: These are generalizations of compactly generated categories. We will explain the need for looking at the more general picture, and then outline the theorems one has been able to prove so far.

(Talk 3) The homotopy category of flat R-modules

ABSTRACT: This is a conference on ring theory, and in this talk we will explain how the general theory applies to give a string of results on categories of representations. We will begin with a survey of recent papers by Krause, Jorgensen, and by Iyengar and Krause. These three articles explore the relationship between the categories $K(InjR)$, $K(ProjR)$ and $K(FlatR)$. It turns out that the relation becomes clearer and more transparent when one uses the techniques of well generated triangulated categories. We will explain what was proved in the three papers mentioned above, and also the recent improvements on them.

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On S-Cohn-Jordan Extensions

Jerzy Matczuk

In Memory of P.M. Cohn

Let a monoid S act on a ring R by injective endomorphisms. An over-ring $A(R;S)$ of R is called the S-Cohn-Jordan extension of R if (1) the action of S on R extends to an action of S on $A(R;S)$ by automorphisms and (2) for any $a \in A(R;S)$, there exists $s \in S$ such that $s \cdot a \in R$. A classical result of P.M. Cohn, which was originally formulated in much more general context of Ω-algebras (instead of rings), says that such an extension always exists provided the monoid S posseses a group $S^{-1}S = G$ of left quotients.

The aim of the talk is to present a series of results relating various algebraic properties of R and that of $A(R;S)$. For example primeness, Goldie conditions and other finiteness conditions will be considered.

Some possible applications to the skew semigroup rings $R\#S$ and skew polynomial rings $R[x;\sigma,\delta]$ will be also discussed.

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Local QF-rings with radical cubed zero

Isao Kikumasa, Kiyoichi Oshiro, Hiroshi Yoshimura

In 1976, in his Book "Algebra II", Faith conjectured "no" for the problem: Are semiprimary right self-injective rings quasi-Frobenius ? Since that time, this conjecture is known by the Faith conjecture. Although many ring theorists have tried to solve this conjecture, the conjecture still remains open. In our talk, first, we translate this problem into a problem on two-sided vector spaces. In some sense, the difficulty of this problem stems from the question: Can we make freely non-commutative local QF-rings ? In connection with my translation of the Faith conjecture, we present our construction of local QF-rings with radical cubed zero using a field of complex numbers, quaternion of Hamilton or number fields etc.

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DUAL OJECTIVITY OF QUASI-DISCRETE MODULES AND LIFTING MODULES

Yosuke Kuratomi

In [1], K.Oshiro and his students introduced "ojectivity (generalized injectivity)", a new concept of relative injectivity, and using this injectivity we obtained some results for direct sums of extending modules. Afterward, S.H.Mohamed and B.J.Müller [2] defined a dual concept of ojectivity as follows :

Definition. M is said to be "N-dual ojective (or generalized N-projective)" if, for any epimorphism $g : N \to X$ and any homomorphism $f : M \to X$, there exist decompositions $N = N_1 \oplus N_2$, $M = M_1 \oplus M_2$, a homomorphism $h_1 : M_1 \to N_1$ and an epimorphism $h_2 : N_2 \to M_2$, such that $gh_1 = f|_{M_1}$ and $fh_2 = g|_{N_2}$.

The concept of relative dual ojectivity is generalization of relative projectivity and this projectivity has an important meaning for the study of direct sums of lifting modules (cf.[2], [3]).

In this talk we introduce some results on "dual ojectivity" and apply it to direct sums of quasi-discrete modules.

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Perfect isometries and the Alperin-Mckay conjecture

Charles W. Eaton

In the representation theory of finite groups, one of the main themes is the relationship between representations of a group G and those of certain subgroups. When we consider also modular representations, with respect to a field k of characteristic p, we see that local subgroups are particularly strongly related to G (a local subgroup is the normaliser of a p-subgroup, although this definition may be generalised).

Examples of the sort of information about G we may wish to obtain from local subgroups are the number of non-projective simple kG-modules, the number of irreducible characters of degree not divisible by $|G|_p$, and even categorical information, such as the derived equivalence class of the module category of kG. Much is known, including for example, the fact that the derived category is determined by normalisers of p-subgroups when a Sylow p-subgroup is cyclic. However, more is still conjecture, and there are other situations where we do not even know what the conjectures should be.

Existing conjectures vary from the purely numerical, for example those of Alperin, Alperin-McKay, Dade and others, where predictions are made which apply to all groups, to the categorical, such as Broué's conjecture, which applies only in restricted cases (blocks with abelian defect groups). Perfect isometries may be regarded as lying between numerical and categorical information.

The purpose of these lectures will be to explain the range of conjectures, from the Alperin-McKay conjecture to Broué's conjecture, via perfect isometries, and the problems associated with generalising Broué's conjecture. A theme will be how numerical and categorical relationships together have an important part to play in the subject.

In the first lecture, I will introduce the necessary background material. This includes blocks of finite groups; invariants of blocks and groups; perfect isometries and a little about module categories. In the second lecture I will give an overview of the existing results and conjectures and their current status. In the third I will present a suggestion for a generalisation of the idea of a perfect isometry.

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Valuation Rings and Maximal Orders in Nonassociative Quaternion Algebras

John S. Kauta

A *nonassociative quaternion algebra* over a field F has the form $A = N \oplus NJ$, where N is a separable quadratic extension field of F with an F-involution $x \mapsto \hat{x}$, $Jx = \hat{x}J$ for $x \in N$, and $J^2 = b \in N \setminus F$. We define the notion of a valuation ring for A, and we also define a value function w on A with values from a totally ordered group. We determine the structure of the set B_w on which the value function assumes non-negative values and we prove that, given a valuation ring of A, there is a value function w associated to it if and only if the valuation ring is integral and invariant under proper F-automorphisms of A. The restriction $w|_N$ is a valuation on N with valuation ring $S = N \cap B_w$. If S is inertial over F, then B_w is a valuation ring of A if and only if $w(b) = 0$; if in addition S is a DVR, then B_w is a valuation ring of A if and only if it is a maximal order in A and $w(b) = 0$. On the other hand if S is tamely and totally ramified over F, then B_w is a valuation ring of A if and only if the image of b in $S/J(S)$ is not a square. If in addition V is a DVR, then B_w is a valuation ring of A if and only if B_w is a maximal order in A and the image of b is not a square in $S/J(S)$; further, B_w is the intersection of maximal orders if $w(b) = 0$.

Keywords: Value functions, valuation rings, maximal orders, nonassociative quaternion algebras.

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Cohen-Macaulay modules and holonomic modules over Gorenstein filtered rings

HIROKI MIYAHARA

Let Λ be a (not necessary commutative) ring. A family of additive subgroups of Λ $\{\mathcal{F}_p\Lambda \mid p \in \mathbb{N}\}$ ($\mathbb{N}$: the set of all non negative integer) is called a *filtration* of Λ, if

(1) $1 \in \mathcal{F}_0\Lambda$
(2) $\mathcal{F}_p\Lambda \subset \mathcal{F}_{p+1}\Lambda$
(3) $(\mathcal{F}_p\Lambda)(\mathcal{F}_q\Lambda) \subset \mathcal{F}_{p+q}\Lambda$
(4) $\Lambda = \bigcup_{p\in\mathbb{N}} \mathcal{F}_p\Lambda$

A pair $(\Lambda, \mathcal{F})$ is called *filtered ring* if Λ has a filtration. Let $\sigma_p : \mathcal{F}_p\Lambda \longrightarrow \mathcal{F}_p\Lambda/\mathcal{F}_{p-1}\Lambda$ $(\mathcal{F}_{-1}\Lambda = 0)$ be a natural homomorphism, then $\mathrm{gr}\Lambda := \bigoplus_{p\in\mathbb{N}} \mathcal{F}_p\Lambda/\mathcal{F}_{p-1}\Lambda$ is a graded ring with multiplication

$$\sigma_p(a)\sigma_q(b) = \sigma_{p+q}(ab), \quad a \in \mathcal{F}_p\Lambda, b \in \mathcal{F}_q\Lambda$$

Let Λ be a filtered ring, and M be a (left) Λ-module. A family of additive subgroups of M $\{\mathcal{F}_p\Lambda \mid p \in \mathbb{Z}\}$ is called a *filtration* of M, if

(1) $\mathcal{F}_pM \subset \mathcal{F}_{p+1}M$
(2) $\mathcal{F}_pM = 0$ for $p \ll 0$
(3) $(\mathcal{F}_p\Lambda)(\mathcal{F}_qM) \subset \mathcal{F}_{p+q}M$
(4) $M = \bigcup_{p\in\mathbb{Z}} \mathcal{F}_pM$

A pair $(M\,\mathcal{F})$ is called *filtered Λ-module* if M has a filtration. Let $\tau_p : \mathcal{F}_pM \longrightarrow \mathcal{F}_pM/\mathcal{F}_{p-1}M$ be a natural homomorphism, then grΛ の $\mathrm{gr}M := \bigoplus_{p\in\mathbb{Z}} \mathcal{F}_pM/\mathcal{F}_{p-1}M$ is a graded grΛ-module by

$$\sigma_p(a)\tau_q(x) = \tau_{p+q}(ax), \quad a \in \mathcal{F}_p\Lambda, x \in \mathcal{F}_qM$$

We assume that grΛ is a commutative Gorenstein *local ring (with unique $*$maximal ideal $\mathcal{M}$) satisfying the following condition (P):

(P) : There exists an element of positive degree in $\mathrm{gr}\Lambda - \mathfrak{p}$
for any graded prime ideal $\mathfrak{p} \neq \mathcal{M}$

Then, $\mathrm{id}_\Lambda\Lambda = \mathrm{id}_{\Lambda^{op}}\Lambda < \infty$. so, We call such a filtered ring Λ a "*Gorenstein filtered ring*" (the well known example is *Weyl algebra*)

Let Λ be a filtered Gorenstein ring, M be a finitely generated filtered (left) Λ-module, and $\mathrm{gr}M$ be a finite grΛ-module. We put $^*\dim \mathrm{gr}M = \mathrm{ht}\mathcal{M}/I$ ($I = [0 :_{\mathrm{gr}\Lambda} \mathrm{gr}M]$), $n = {}^*\dim \mathrm{gr}\Lambda$, and $d = \mathrm{id}\Lambda$. then, $^*\dim \mathrm{gr}M \leq n - d$. so (like Weyl algebra) We call M a *holonomic Λ-module*, if $^*\dim \mathrm{gr}M = n - d$. Also, we call M a *CM Λ*-module, if $\mathrm{gr}M$ is CMgrΛ-module.

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Symmetry in the Vanishing of Ext-groups

Izuru Mori

In this talk, we will find a class of rings R satisfying the following property: for every pair of finitely generated right R-modules M and N, $Ext^i_R(M,N) = 0$ for all $i \gg 0$ if and only if $Ext^i_R(N,M) = 0$ for all $i \gg 0$. In particular, we will show that such a class of rings includes a group algebra of a finite group and the exterior algebra of odd degree.

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On derived equivalences in selfinjective algebras

Hiroki Abe and Mitsuo Hoshino

Let A be an artin algebra. Rickard [5, Proposition 9.3] showed that for any tilting complex $P^\bullet \in \mathsf{K}^{\mathrm{b}}(\mathcal{P}_A)$ the number of nonisomorphic indecomposabele direct summands of $P^\bullet$ coincides with the rank of $K_0(A)$, the Grothendieck group of A, which generalizes earlier results [2, Proposition 3.2] and [4, Theorem 1.19]. He raised a question whether a complex $P^\bullet \in \mathsf{K}^{\mathrm{b}}(\mathcal{P}_A)$ with $\mathrm{Hom}_{\mathsf{K}(\text{Mod-}A)}(P^\bullet, P^\bullet[i]) = 0$ for $i \neq 0$ is a tilting complex or not if the number of nonisomorphic indecomposable direct summands of $P^\bullet$ coincides with the rank of $K_0(A)$ (see also [4]). In case $P^\bullet$ is a projective resolution of a module $T \in \text{mod-}A$ with proj dim $T_A \leq 1$, Bongartz [1, Lemma of 2.1] has settled the question affirmatively. More precisely, he showed that every $T \in \text{mod-}A$ with proj dim $T_A \leq 1$ and $\mathrm{Ext}^1_A(T, T) = 0$ is a direct summand of a classical tilting module, i.e., a tilting module of projective dimension ≤ 1. Unfortunetely, this is not true in general (see [5, Section 8]). Our first aim of this note is to show that if A is a representation-finite selfinjective artin algebra then every $P^\bullet \in \mathsf{K}^{\mathrm{b}}(\mathcal{P}_A)$ with $\mathrm{Hom}_{\mathsf{K}(\text{Mod-}A)}(P^\bullet, P^\bullet[i]) = 0$ for $i \neq 0$ and $\mathrm{add}(P^\bullet) = \mathrm{add}(\nu P^\bullet)$, where ν is the Nakayama functor, is a direct summand of a tilting complex.

Rickard [6, Theorem 4.2] showed that the Brauer tree algebras over a field with the same numerical invariants are derived equivalent to each other. Subsequently, Okuyama pointed out that for any Brauer tree algebras A, B with the same numerical invariants there exists a sequence of Brauer tree algebras $A = B_0, B_1, \cdots, B_m = B$ such that, for any $0 \leq i < m$, B_{i+1} is the endomorphism algebra of a tilting complex for B_i of length ≤ 1. These facts can be formulated as follows. For any tilting complex $P^\bullet \in \mathsf{K}^{\mathrm{b}}(\mathcal{P}_A)$ associated with a certain sequence of idempotents in a ring A, there exists a sequence of rings $A = B_0, B_1, \cdots, B_m = \mathrm{End}_{\mathsf{K}(\text{Mod-}A)}(P^\bullet)$ such that, for any $0 \leq i < m$, B_{i+1} is the endomorphism ring of a tilting complex for B_i of length ≤ 1 determined by an idempotent (see [3, Proposition 3.2]). Our second aim of this note is to show that for any derived equivalent representation-finite selfinjective artin algebras A, B there exists a sequence of selfinjective artin algebras $A = B_0, B_1, \cdots, B_m = B$ such that, for any $0 \leq i < m$, B_{i+1} is the endomorphism algebra of a tilting complex for B_i of length ≤ 1.

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Azumaya's conjecture and Harada rings

Kazutoshi Koike

The aim of this talk is to present several results concerning Azumaya's conjecture as an application of study of Harada and quasi-Harada rings. An artinian ring R is said to be *exact* in case R has a composition series of two-sided ideals such that each composition factor is simple and balanced as an (R, R)-bimodule. Azumaya [1] introduced the concept of exact rings and conjectured

Azumaya's conjecture. Every exact ring has a self-duality.

Here an artinian ring R is said to have a *self-duality* in case R has a self Morita duality, i.e., there exists a duality (contravariant equivalence) between the category of finitely generated left R-modules and the category of finitely generated right R-modules. We call an artinian ring *serial* (resp. *locally distributive*) in case the lattice of submodules of each left and right indecomposable projective module is linearly ordered (resp. distributive). As is well-known, every serial ring has a self-duality. Clearly serial rings are locally distributive. It is shown that locally distributive rings are exact. Therefore if Azumaya's conjecture is true, the following problem is affirmative.

Problem. Does every locally distributive ring have a self-duality?

However the problem is still open. In this talk, we shall present several partial results (e.g., the self-duality for locally distributive right serial rings) about the problem. These results are obtained from study of quasi-Harada rings. Oshiro call certain QF-3 QF-2 artinian rings *Harada rings* in [5] and investigate the structure of Harada rings deeply. Baba and Iwase [2] generalize the concept of Harada rings as *quasi-Harada rings*, which are certain QF-2 rings. Locally distributive right QF-2 rings are left quasi-Harada rings. Thus we can use results for quasi-Harada rings to study self-duality of locally distributive (right QF-2) rings.

We shall also apply the observations of Kado and Oshiro [3] about the self-duality for Harada rings to exact and locally distributive rings and reduce Azumaya's conjecture and Problem above to other problems. For example, we show the equivalence of Azumaya's conjecture and the statement that every exact ring has a *weakly symmetric* self-duality.

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Derived Equivalences of Symmetric Algebras defined by Generalized Tilting Modules

Takayoshi Wakamatsu

Let ${}_BT_A$ be a generalized tilting module with $\mathrm{Cok}(\mathcal{C}(T_A), \mathcal{PC}(T_A)) = \mathrm{mod}A$ and $\mathrm{Ker}(\mathcal{ID}(\mathrm{D}T_B), \mathcal{D}(\mathrm{D}T_B)) = \mathrm{mod}B$. Then, for a symmetric algebra $\Lambda = A \oplus M \oplus \mathrm{D}A$ with multiplication

$$(a, m, f) \cdot (a', m', f') = (aa', am' + ma' + \varphi(m \otimes m'), af' + fa' + \psi(m \otimes m'))$$

where $\varphi : {}_AM \otimes_A M_A \to {}_AM_A$ is nilpotent and $\psi : {}_AM \otimes_A M_A \to {}_A\mathrm{D}A_A$ is non-degenerate, the corresponding symmetric algebra $\Lambda^T = B \oplus M^T \oplus \mathrm{D}B$ is defined and there is an equivalence of stable module categories $\mathcal{Q} : \underline{\mathrm{mod}}\Lambda \to \underline{\mathrm{mod}}\Lambda^T$ if the modules M_A and $T \otimes_A M_A$ are in the class $\mathcal{C}(T_A)$ and the homomorphism $\theta : T \otimes_A \mathrm{Hom}_A(T, M) \to \mathrm{Hom}_A(T, T \otimes_A M)$, $\theta(t \otimes h)(t') = t \otimes h(t')$ is bijective. On the other hand, it is known by D. Happel that the stable module category $\underline{mod}\Lambda$ of any self-injective algebra Λ has a structure of triangulated category. In this talk, we prove that the functor $\mathcal{Q}$ is in fact an equivalence of triangulated categories.

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On a tensor product of square matrices in Jordan canonical form

Ryo Iwamatsu

To construct graded local Frobenius algebras over an algebraically closed field K, it is importatnt to find out a Joradan canonical form of tensor product of square matrices. In fact, it is known that any local graded Frobenius algebra is of the form of $\Lambda(\varphi, \gamma) = T(V)/R(\varphi, \gamma)$, where V is a finite dimensional vector space, γ is an element of $GL(V)$ and $\varphi : V^{\otimes n} \to K$ is a linear map satisfying several conditions. Further, if we decompose as $(V, \gamma) = \bigoplus_i (V_i, \gamma_i)$, then the conditions of φ can be described in terms of each $\varphi_{i_1 \cdots i_r} : V_{i_1} \otimes \cdots \otimes V_{i_r} \to K$. And then, we have to consider a Jordan canonical form of $\gamma_{i_1} \otimes \cdots \otimes \gamma_{i_r}$ as an element $GL(V_{i_1} \otimes \cdots \otimes V_{i_r})$. (For detail, see Wakamatsu [3]).

Let K be an algebraically closed field of characteristic $p \geqslant 0$. And let $A = J(a, s)$ and $B = J(b, t)$ be Jordan blocks, where $ab \neq 0$ and $0 < s \leqslant t$. Our problem is to find out a Jordan canonical form $A \otimes B$. In the case of $p = 0$, Martsinkovsky and Vlassov [2] gave the solution completely. In this talk, we shall introduce an algorithm for finding out a Jordan canonical form of $A \otimes B$. This problem is reduced to the problem of finding the indecomposable decomposition $R = K[X, Y]/(X^s, Y^t)$ as a $K[\theta] = K[X + Y]$-module. Our algorithm gives the generators of indecomposable summands of R concretely. And we show that there exists some homogeneous elements $\omega_0, \omega_1, \ldots, \omega_{s-1}$ such that

$$R \cong \bigoplus_{i=0}^{s-1} K[\theta]\omega_i$$

as $K[\theta]$-modules, where the degree of each ω_i is i. Independently, Iima and Yoshino gave a similary result of us. That is announced in [1].

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Finite groups having exactly one non-linear irreducible character

Kaoru MOTOSE

I talk about finite groups with exactly one non-linear irreducible character, in detail, characterizations, real representations of these groups and relations on Hurwitz theorem.

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Integrality of eigenvalues of Cartan matrices in finite groups

Tomoyuki Wada

Let G be a finite group and F be an algebraically closed field of characteristic $p > 0$. Let B be a block of the group algebra FG with defect group D and C_B be the Catan matrix of B. We are concerned with eigenvalues of C_B, especially with when all eigenvalues are rational integers and at that time with what each eigenvalue and its eigenvector represent. In this talk, we show, in some blocks, that if the Fobenius-Perron eigenvalue $\rho(B)$ of C_B is an integer, then all eigenvalues are integers and those coincide with elementary divisors of C_B. Furthermore, we can take an eigenvector matrix for them as a Brauer character table Φ_β for some block β which is related to B.

We also show that when G is a fnite group with an abelian 2-Sylow subgroup D, or when G is a finite group with an elementary abelian Sylow 3-subgroup D of order 9, $\rho(B)$ is an integer for the principal 2-block B in the former case, or the principal 3-block B in the latter case of FG if and only if B and its Brauer correspondent block b of $FN_G(D)$ are Morita equivalent (even stronger Puig equivalent).

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Mackey functor and cohomology of finite groups

Akihiko Hida

Let G be a finite group and k a field of characteristic $p > 0$. In [1], G. Mislin proved the following.

Theorem 1 ([1, Theorem])**.** *Let H be a subgroup of G containing a Sylow p-subgroup of G. Then the following are equivalent.*
(1) *The restriction*

$$\mathrm{res}_{G,H} : H^*(G,k) \longrightarrow H^*(H,k)$$

is an isomorphism.
(2) *If Q is a p-subgroup of H, $x \in G$ and ${}^xQ \subseteq H$, then $x \in HC_G(Q)$.*

That the conditon (2) implies (1) is well known. The remarkable part is the implication (1) $\Rightarrow$ (2). The proof of Theorem by Mislin uses deep results from topology. We will give an algebraic proof. We follow the approach of P. Symonds using Mackey functor for G. We consider the structure of the cohomology $H^*(-,k)$ as a Mackey functor for G.

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Invariants of reductive algebraic groups with simple commutator subgroups

Haruhisa Nakajima

Let G be a reductive algebraic group over the complex number field $\mathbf{C}$ whose semisimple part is denoted to G'. We will use any of the notation ϱ, (ϱ, G) or (V, G) to denote a finite dimensional representation $\varrho : G \to GL(V)$ over $\mathbf{C}$. An algebraic action of G on an affine variety X (abbr. (X, G)) is said to be coregular (resp. cofree, equidimensional), if $\mathbf{C}[X]^G$ is a polynomial ring (resp. if $\mathbf{C}[X]$ is a free $\mathbf{C}[X]^G$-module, if the quotient morphism $X \to X/G = \mathrm{m\,Spec}(\mathbf{C}[X]^G)$ is equidimensional).

For simple connected G's, their coregular, cofree and equidimensional representations are, respectively, classified by [2, 3], [4], and [1].

In this short communication, we will give some remarks on coregular representations of G with certain simple G' which are based on relative invariants of algebraic tori on factorial rings and some equidimensional or cofree actions. Our remarks seem to be partially related to the results on quasi-coregular representations of simple algebraic groups and equidimensional toric extensions of their representatitions (e.g., [5]).

For a convenience sake, we omit the detailed statements in this abstract.

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Von Neumann regular rings with comparability

Mamoru Kutami

A ring R is said to be (*von Neumann*) *regular* if for each $x \in R$ there exists an element y of R such that $xyx = x$, and R is said to be *unit-regular* if for each $x \in R$ there exists a unit (i.e., an invertible) element u of R such that $xux = x$. A ring R is *directly finite* provided that $xy = 1$ $(x, y \in R)$ implies $yx = 1$.

Unit-regularity and direct finiteness are important finiteness conditions for regular rings. For these relations, M. Henriksen (1973) first showed that unit-regular rings are always directly finite. But, unfortunately directly finite regular rings are not unit-regular in general, as G. Bergman (1977) showed. Therefore, the following interesting problems between unit-regularity and direct finiteness for regular rings arised:

Problem 1. Is every directly finite simple regular ring always unit-regular?

Problem 2. Which directly finite regular rings are unit-regular?

For Problem 1, O'Meara [4] introduced the notion of weak comparability and showed that directly finite simple regular rings with weak comparability must be unit-regular. Here we recall that a regular ring R is said to *satisfy weak comparability* if, for each nonzero $x \in R$, there exists a positive integer $n = n(x)$ such that $n(yR) \lesssim R_R$ implies $yR \lesssim xR$ for all $y \in R$. Thereafter, simple regular rings with weak comparability were studied in [1], [2], [3] etc..

In this talk, we mainly treat regular rings with weak comparability, and give some results on finitely generated projective modules over these rings, concerning with Problem 2.

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