INVARIANTS OF COMPLEX REDUCTIVE ALGEBRAIC GROUPS WITH SIMPLE COMMUTATOR SUBGROUPS

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Abstract. Let G be a complex connected reductive algebraic group with simple commutator subgroup G'. Consider a finite dimensional representation $\rho : G \to GL(V)$ and denote by $\mathbb{C}[V]^G$ the $\mathbb{C}$-algebra consisting of polynomial functions on V which are invariant under the action of G. The purpose of this paper is to discuss on our partial classification of ρ's such that $\mathbb{C}[V]^G$ are polynomial rings over $\mathbb{C}$. Our method is based on the property of PLH defined and studied in [9] and our study on relative equidimensionality of representations under the assumption that G' is simple.

Key Words: coregular representations; relative invariants; reductive algebraic groups; algebraic tori; simple commutator subgroups; equidimensional actions

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1. Introduction

This is a worked out version of the author's talk in the 39-th Symposium on Ring Theory and Representation Theory held at Hiroshima University in 2006.

Let $\mathbb{C}$ be the complex number field (or an algebraically closed field of characteristic zero) and suppose that algebraic groups are defined over $\mathbb{C}$. We denote by G a reductive algbraic group with its identity connnected component G^0 whose commutator subgoup is denoted to G'. and, without specifying, we may assume that $G = G^0$. We use the following notations:

$\mathfrak{X}(G)$: $\mathfrak{X}(G)$ denote the group of rational characters of G whose composition is represented as an addition.

(V, G) : for a finite-dimensional representation $\rho : G \to GL(V)$, we denote ρ by (V, G).

R_χ : $R_\chi = \{f \in R \mid \sigma(f) = \chi(\sigma)f \ (\forall \sigma \in G)\}$ for a rational G-module R, which is called χ-invariants of G in R.

$\mathbb{C}[Y]$: the affine $\mathbb{C}$-algebra of polynomial functions on an affine variety Y over $\mathbb{C}$.

$Y//G$: the affine variety defined by $\mathbb{C}[Y]^G$ for a regular action (Y, G) of G on Y.

In this paper, we will study on the following classical problem:

Problem 1. For a (not necessarily connected) G, determine all finite dimensional representations (V, G) such that $\mathbb{C}[V]^G$ are polynomial rings.

This paper is based on the author's talk and the detailed proof of some results in this paper will be published elsewhere.

The following case for this problem are studied:
(1) (G. C. Shephard-J. A. Todd [14, 3]). Suppose that G is finite. Then

$$\mathbb{C}[V]^G \text{ is a polynomial ring} \iff G|_V \text{ is generated by pseudo-reflections.}$$

(2) (G.W. Schwarz-O.M. Adamovich-E.O. Golovina [2, 12]). Suppose that $G = G^0 = G'$ is simple. Then

$$\begin{aligned} &\mathbb{C}[V]^G \text{ is a polynomial ring} \iff \\ &(V, G) \text{ is listed in the Tables given by [2, 12].} \end{aligned}$$

(3) (P. Littelmann [5]). For $G = G^0 = G'$, the problem has been solved for irreducible representations.
(4) (D. A. Shmel'kin [11]). Suppose that $G^0 = G'$ is simple. Then

$$\begin{aligned} &\mathbb{C}[V]^G \text{ is a polynomial ring} \iff \\ &(V, G) \text{ is listed in the Tables given by [11].} \end{aligned}$$

(5) (well known?). Suppose that $G = G^0$ and $G' = \{1\}$. Then

$$\mathbb{C}[V]^G \text{ is a polynomial ring} \iff \mathbb{C}[V]^G \text{ is factorial.}$$

We will study on Problem 1 in the restricted case as follows:

Problem 2 (The Small Problem). Suppose that G' is simple. Determine all representations (V, G) of a connected non-semisimple G such that $\mathbb{C}[V]^G$ are polynomial rings. Precisely, we raise the following *problem:*

$$\begin{aligned} \mathbb{C}[V]^G \text{ is a polynomial ring (+some unknown conditions)} \\ \Longrightarrow \mathbb{C}[V]^{G'} \text{ is a polynomial ring ?} \end{aligned}$$

By Theorem 5 & 12, we obtain this implication for some groups *without conditions*. Then, in this case, the Small Problem can be reduced to Schwarz-Adamovich-Golovina classification (cf. (2)) and coregular toric representation (cf. (5)),

2. Blowing-up representations of codimension one

Recall that

$$\begin{aligned} &(X, G) \text{ is a stable action of } G \text{ on an affine variety } X \\ \iff &\exists \Omega \subseteq X : \text{a non-empty open subset consisiting of closed } G\text{-orbits} \\ \iff &(X, G^0) : \text{stable} \\ \iff &\text{both } (X, (G^0)'), (X//(G^0)', G^0/(G^0)') \text{ are stable (cf. [10]).} \end{aligned}$$

Definition 1. We define a generalization of stability as follows:

$$(X, G) \text{ is relatively stable} \iff (X//(G^0)', G^0/(G^0)') \text{ is stable.}$$

Hereafter we suppose that G is connected reductive, and then we have an epimorphism $Z \times G' \to G$ with a finite kernel, where Z is a connected torus. So, we may assume that $G = Z \times G'$.

Remark 2. For a representation (V, G),

$$\text{it is relatively stable} \iff \mathbb{C}[V]_\chi \neq \{0\} \text{ implies } \mathbb{C}[V]_{-\chi} \neq \{0\} \text{ for } \chi \in \mathfrak{X}(Z).$$

Especially in the case where $\mathbb{C}[V_i]^{G'} \neq \mathbb{C}$ for arbitrary irreducible component V_i of V, we see that

$$\exists \text{ a relatively stable subrepresentation } (V', G) \text{ of } (V, G)$$
$$\text{such that } \mathbb{C}[V]^G = \mathbb{C}[U]^G.$$

2.1. **Definition and properties of PLH.** Let Y be an affine variety such that $\mathbb{C}[Y]$ is a positively graded factorial domain defined over $\mathbb{C}$ with a grade preseving rational action of Z as $\mathbb{C}$-automorphisms. For an element $\sigma \in \mathrm{Aut}_{\mathbb{C}-alg}(\mathbb{C}[Y])$ is said to be a *generalized reflection*, if $\mathrm{ht}((\sigma - I_Y)(\mathbb{C}[Y]) \cdot \mathbb{C}[Y]) = 1$. Let U be a minimal finite-dimensional ratinal module of Z admitting an homogeneous Z-equivariant epimorphism $\mathbb{C}[U] \to \mathbb{C}[Y]$. In this circumstance, we consider;

$$(W, w) \text{ a pair of a } Z\text{-submodule } W \text{ of } U \text{ and a nonzero vector } w \in U$$
$$\text{such that } U \cap < Z \cdot w >_{\mathbb{C}} = \{0\}$$

and the morphism

$$(\bullet + w) : W \ni x \to x + w \in U.$$

Definition 3. Suppose that (Y, Z) is stable. We say that (W, w) is a PLH of (Y, Z) or (U, Z), if $(\bullet + w)$ induces the $\mathbb{C}$-isomorphism $\mathbb{C}[U]^Z \cong \mathbb{C}[W]^{Z_w}$, i.e.,

$$\mathbb{C}[U]^Z \hookrightarrow \mathbb{C}[U]^{Z_w} \xrightarrow{(\bullet + w)^*} \mathbb{C}[W]^{Z_w}.$$

Lemma 4. *Suppose that (Y, Z) is stable ane let (W, w) be a PLH of (Y, Z) (or (U, Z)). Then:*

1. *If (W', w') is a PLH of (W, w), then (W', w') is also a PLH of (U, Z).*
2. $\mathrm{Cl}(\mathbb{C}[Y]^Z) \cong \mathrm{Cl}(\mathbb{C}[Y]^{Z_w})$.

For a ring monomorphism $S \to R$ of Krull domains such that $S = \mathcal{Q}(S) \cap R$ and $\mathfrak{q} \in \mathrm{Ht}_1(S)$, let $X_{\mathfrak{q}}(R) = \{\mathfrak{P} \in \mathrm{Ht}_1(R) \mid P \cap S = \mathfrak{q}\}$, where $\mathrm{Ht}_1(\circ)$ stands for the set consisting of all prime ideals of $\circ$ of height one. In the case where $S = R^L$ for a subgroup L of $\mathrm{Aut}(R)$, the sets $X_{\mathfrak{q}}(R)$ $(\forall \mathfrak{q} \in (S))$ are not empty (cf. [7]).

2.2. **Notations for representations.** Suppose that (V, G) is relatively stable. Let $V_{G'}$ denote the G-submodule of V satisfying

$$V = V_{G'} \oplus V^{G'}.$$

We express

$$\mathbb{C}[V]^{G'} = \mathbb{C}[V^{G'}] \otimes_{\mathbb{C}} \mathbb{C}[f_i, g_j, h_k],$$

where a homogeneous generating system

$$\{f_1, \dots, f_l, g_1, \dots, g_m, h_1, \dots, h_n\}$$

of $\mathbb{C}[V_{G'}]^{G'}$ as a $\mathbb{C}$-algebra can be choosed in such a way that the following conditions are satisfied:

$$\begin{aligned}&\mathrm{ht}(f\mathbb{C}[V] \cap \mathbb{C}[V]^G) = \mathrm{ht}(g\mathbb{C}[V] \cap \mathbb{C}[V]^G) = 1,\\&\mathrm{ht}(h\mathbb{C}[V] \cap \mathbb{C}[V]^G) \geq 2,\\&|X_{f\mathbb{C}[V]\cap\mathbb{C}[V]^G}(\mathbb{C}[V]^{G'})| = 1,\\&|X_{g\mathbb{C}[V]\cap\mathbb{C}[V]^G}(\mathbb{C}[V]^{G'})| \geq 2.\end{aligned}$$

We apply the concept of PHL for the pair of Y and U to the affine variety Y defined by $\mathbb{C}[V]^{G'}$ and

$$U = V^{G'} \oplus < f_i, g_j, h_k >_{\mathbb{C}}^{\vee}$$

with the natural $\mathbb{C}$-epimorphism

$$\mathbb{C}[U] \to \mathbb{C}[Y] = \mathbb{C}[V]^{G'}$$

having grade preserving Z-actions.

2.3. **The Main Therem for blowing-up representations.** The main result of this Section is

Theorem 5. *Suppose that G' is simple and (V, G) is relatively stable. Suppose that*

$$\{\mathfrak{q} \in \mathrm{Ht}_1(\mathbb{C}[V_{G'}]^{G'}) \ \mid \ |X_{\mathfrak{q}}(\mathbb{C}[V]^{G'})| \geq 2\}$$

is nonempty and the action of Z on the $\mathbb{C}$-subalgebra $\mathbb{C}[V_{G'}]^{G'}$ is non-trivial. Then the following conditions are equivalent:

(1) *$\mathbb{C}[V]^G$ is factorial.*
(2) *The stabilizer $Z_w|_{\mathbb{C}[V]^{G'}}$ is a finite group generated by generalized reflections in $\mathrm{Aut}_{\mathbb{C}-alg}(\mathbb{C}[V]^{G'})$, where w is a point in U such that (W, w) is a minimal PLH of (Y, Z) for some $W \subseteq U$.*
(3) *$\mathbb{C}[V]$ is a free $\mathbb{C}[V]^{G'}$-module and the condition (2) holds.*
(4) *$\mathbb{C}[V]^G$ is a polynomial ring.*

The assumption of this theorem is characterized by

Proposition 6. *$m > 0$ if and only if the following condition is satisfied:*

$$\{\mathfrak{q} \in \mathrm{Ht}_1(\mathbb{C}[V_{G'}]^{G'}) \ \mid \ |X_{\mathfrak{q}}(\mathbb{C}[V]^{G'})| \geq 2\} \text{ is nonempty.}$$

Remark 7. Since the condition (2) in Theorem 5 implies that $\mathbb{C}[V]^{G'}$ is a polynomial ring. Hence, under the condition that $m > 0$, we see that

$$\mathbb{C}[V]^G \text{ is a polynomial ring} \Longrightarrow \mathbb{C}[V]^{G'} \text{ is a polynomial ring,}$$

i.e., the Small Problem is affirmative in this case.

2.4. **A Sketch of the Proof of Theorem 5.** By Lemma 4, we see that

$$\mathbb{C}[V]^G \text{ is factorial} \Longrightarrow \mathbb{C}[Y]^{Z_w} \text{ is factorial}$$
$$\Longrightarrow (Y, Z_w) : \text{cofree}$$
$$\Downarrow$$
$$(Y, Z_w) : \text{equidimensional}$$
$$\Longrightarrow (Y, (Z_w)^0) : \text{equidimensional}$$
$$\Downarrow$$

Theorem 5 is reduced to the next theorem (cf. Theorem 8). □

Theorem 8. *Suppose that G' is simple and (V, G) is relatively stable. Suppose that the action of Z on $\mathbb{C}[V_{G'}]^{G'}$ is non-trivial and on Y is equidimensional. Then*

(1) $\mathbb{C}[V]$ *is a free* $\mathbb{C}[V]^G$*-module.*
(2) $\mathbb{C}[V]$ *is a free* $\mathbb{C}[V]^{G'}$*-module.*

The proof of this theorem, which is regarded as a generalization of a part of [1] and [13], is not ring theoretical but is a consequence of case-by-case arguments on invariants of representatons of simple algebraic groups.

3. No-blowing-up representations of codimension one

We will, now, treat the case where $m = 0$ and $n > 0$ (we suppose that (V, G) is relatively stable).

Lemma 9. *If $\mathbb{C}[V]^G$ a polynomial ring, then the localization $((\mathbb{C}[V_{G'}]^{G'})^{Z_w})_{h_1 \cdots h_n}$ is a regular ring. Here w is a nonzero point of U such that (W, w) is a minimal PLH of (Y, Z).*

Express $V_{G'} = \bigoplus_i V_i$, where each V_i is irreducible component of $V_{G'}$ as a representation of G. For any polynomial $h \in \mathbb{C}[V]$, put

$$\operatorname{supp}(h) = \{i \mid (\mathbb{C}[V] \xrightarrow{\text{can.}} \mathbb{C}[V/V_i])(h) \neq 0\}.$$

Consider the following condition:

$$\mathbb{C}[V_i]^{G'} \neq \mathbb{C} \quad (\forall i \text{ such that} V_i \subseteq V_{G'}). \tag{3.1}$$

Lemma 10. *Suppose that the condition (3.1) holds. For any $1 \leq k \leq n$, we see that* $\operatorname{supp}(h_k)$ *is a set of singleton.*

Lemma 11. *Suppose that the condition (3.1) holds. Put $J := \cup_{k=1}^n \operatorname{supp}(h_k)$. Then*

$$\mathbb{C}[\bigoplus_{i \in J} V_i]^{G'} \cong \bigotimes_{i \in J} \mathbb{C}[V_i]^{G'}$$

and this is a $|J|$-dimensional polynomial ring.

3.1. **The Main Theorem for no-blowing-up representaitons.** We use the following notation: Let $\Phi_1, \dots, \Phi_r$ be the fundamental irreducible representations of a simple group of rank r whose numbering are standard (e.g, [15]) and $\phi\psi$ denote the highest weight irreducible representation in $\phi \otimes \psi$ of irreducible ϕ and ψ.

We say that, for example, "(V, G') contains the irreducible representation *quasi-equivalent to* $(\Phi_4, \mathbf{D}_5)$" if $(V, \tilde{G'})$ contains $(\tilde{\Phi}_4, \tilde{G'})$ for a universal covering group $\tilde{G'}$ of G'. Precisely, consider an isomorphism $\nu : \tilde{G'} \xrightarrow{\sim} \mathbf{D}_5$ and regard

$$\tilde{G'} \xrightarrow{\nu} \mathbf{D}_5 \xrightarrow{\Phi_4} GL(\Phi_4)$$

as $(\tilde{\Phi}_4, \tilde{G'})$.

Theorem 12. *Suppose that G' is simple and that (V, G') contains none of the irreducible representations quasi-equivalent to the following list:*

$$(\Phi_1, \mathbf{A}_r), (\Phi_2, \mathbf{A}_r), (\Phi_1, \mathbf{C}_r), (\Phi_4, \mathbf{D}_5).$$

Then if $\mathbb{C}[V]^G$ is a polynomial ring, so is $\mathbb{C}[V]^{G'}$.

3.2. **Some auxiliary results.** The following criterion on stability of semisimple group actions on factorial varieties is well known:

Proposition 13 (V.L. Popov [10]). *Let (V, G') be any representation of G'. The following conditions (1) and (2) are equivalent and in the case where (V, G') is irreducible and G' is simple, the following four conditions are equivalent:*

(1) *The generic stabilizer of (V, G) is reductive.*
(2) *(V, G') is a stable action.*
(3) *$\mathbb{C}[V]^{G'} \neq \mathbb{C}$.*
(4) *V contains non-trivial closed orbit.*

For a torus T_1 of rank one and a representation R of T_1, put

$$q_{T_1}(R) := \min\{\dim R_-, \dim R_+\},$$

where $R_- = \oplus_{j\in\mathbb{N}} R_{-j\chi}$ and $R_+ = \oplus_{j\in\mathbb{N}} R_{j\chi}$ and $\mathfrak{X}(T_1) =< \chi >$. The number $q_{T_1}(R)$ does not depend on χ.

Theorem 14 (V.G. Kac-V.L. Popov-E.B. Vinberg [4]). *Suppose that G' is simple and (R, G') is any irreducible representation which is quasi-equivalent to neighter $(\Phi_1\Phi_2, \mathbf{A}_3)$ nor $(\Phi_1^3, \mathbf{A}_3)$. If $\mathbb{C}[R]^{G'}$ is not a polynomial ring, then there exists a subtorus T_1 of rank one of G' such that the following conditions hold:*

(1) *$(R^{T_1}, N_{G'}(T_1)/T_1)$ is stable and its generic stabilizer is finite.*
(2) *$q_{T_1}(R) - q_{T_1}((\mathrm{Ad}, G')) \geq 3$.*

In order to study the Small Problem, we need a refinement of Theorem 14 as follows (some results related to this can be found in [8]) :

Lemma 15. *Let N be a reductive algebraic group with its commutator subgroup N' and φ ane ψ be non-trivial finite dimensional representations of N. Suppose that the action $(\varphi//N', N/N')$ is stable (i.e., (φ, N) is relatively stable) and the generic stabilizer is finite. Then $(\varphi \oplus \psi//N', N/N')$ is stable.*

Proposition 16. *Let N be a reductive algebraic group and φ and ψ be non-trivial finite dimensional representations of N. Suppose that the action $(\varphi//N', N)$ is stable (i.e., (φ, N) is relatively stable) and the generic stabilizer is finite. Then $(\varphi \oplus \psi, N)$ is stable.*

Lemma 17. *Let φ and ψ be non-trivial finite dimensional representations of G'. Suppose that (φ, G') has a closed G'-orbit whose stabilizer is denoted to H_1. Let H_2 be a reductive closed subgroup of H_1 such that $(\psi^{H_2}, N_{G'}(H_2))$ is stable and its generic stabilizer is finite. Then, for any nonzero homogeneous $h \in \mathbb{C}[\varphi]^{G'}$, there exists a nonzero $x \in (\varphi \oplus \psi)^{H_2}$ such that*

(1) *Gx is closed in $\varphi \oplus \psi$.*
(2) *$h(x) \neq 0$.*
(3) *The stabilizer $N_{G'}(H_2)_x$ is finite.*

Lemma 18. *Let (φ, G') be an irreducible representattion of a simple G' such that $\dim(\mathbb{C}[\varphi]^{G'}) = 1$. For arbitrary irreducible representation ψ of G' and arbitrary nonzero homogeneous polynomial function $h \in \mathbb{C}[\psi]^{G'}$, if $\mathbb{C}[\psi]^{G'}$ is not a polynomial ring, then there exist a subtorus T_1 of rank one of G' and a nonzero vector $x \in \varphi \oplus \psi$ such that*

(1) *$h(x) \neq 0$.*
(2) *$q_{T_1}(\varphi \oplus \psi) - q_{T_1}((\mathrm{Ad}, G')) \geq 3$.*
(3) *$N_{G'}(T)_x$ is finite.*

Lemma 19. *Suppose that G' is simple, simply connected and let ψ_i be irreducible representations of G' such that $\dim \mathbb{C}[\psi_i]^{G'} = 1$ $(1 \leq i \leq u)$. Suppose that $u \geq 2$. Then $\mathbb{C}[\oplus_{i=1}^{u} \psi_i]^{G'} = \otimes_{i=1}^{u} \mathbb{C}[\psi_i]^{G'}$ if and only if $u = 2$ and $(\oplus_{i=1}^{2} \psi_i, G')$ is quasi-equivalent to $(\Phi_1 \oplus \Phi_3, \mathbf{B}_3)$ or $(\Phi_1 \oplus \Phi_3, \mathbf{D}_4)$.*

Lemma 20. *Suppose that G' is simple, simply connected and of type $\mathbf{B}_3$ or $\mathbf{D_3}$. Let ψ be an irreducible representation of G' such that $\mathbb{C}[\psi]^{G'}$ is not a polynomial ring. Suppose that $(V, G') = (\psi \oplus \Phi_1 \oplus \Phi_3, \mathbf{B}_3)$ or $(\psi \oplus \Phi_1 \oplus \Phi_3, \mathbf{D}_3)$. Moreover, let h_1 and h_2 be homogeneous polynomial fuctions satisfying $\mathbb{C}[\Phi_1]^{G'} = \mathbb{C}[h_1]$, $\mathbb{C}[\Phi_3]^{G'} = \mathbb{C}[h_2]$. Then there exists a subtorus T_1 of rank one of G' and a nonzero vector $x \in V^{T_1}$ such that*

(1) *$h_i(x) \neq 0$ $(i = 1, 2)$.*
(2) *$q_{T_1}(\varphi \oplus \psi) - q_{T_1}((\mathrm{Ad}, G')) \geq 3$.*
(3) *The stabilizer $N_{G'}(T)_x$ is finite.*

3.3. **A Sketch of the Proof of Theorem 12.** By Lemma 9, 18, 19, 20 and the Slice Étale [6], we see

$$\mathbb{C}[V]^G \text{ is a polynomial ring} \Longrightarrow$$
$$(V, G') \text{ does not contain non-coregular irreducible components.}$$
$$\text{computation} \Downarrow \text{ C.I.T.}$$
$$\mathbb{C}[V]^{G'} \text{ is a polynomial ring.} \quad \square$$

Consequently we conclude that the Small Problem is affirmative also in this case.

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