The 41st Symposium on Ring Theory

and Representation Theory

ABSTRACT

Shizuoka University, Shizuoka

September 5–7, 2008

第41回 環論および表現論シンポジウム

プログラム

9月5日（金曜日)

09:30 – 10:30 西田 憲司（信州大学）
Iwasawa algebras, crossed products and filtered rings

10:45 – 11:45 Alberto Facchini（Università di Padova）
Geometric regularity of direct-sum decompositions of modules

13:15 – 14:00 伊山 修 (名古屋大学)
Tilting mutation and its application

14:15 – 14:45 宮原 大樹（信州大学）
On filtered semi-dualizing bimodules

14:55 – 15:15 酒井 洋介（筑波大学）
An elementary exact sequence of modules with application to tiled orders

15:30 – 15:50 中島 晴久（城西大学）
Some remarks on descent of divisor class groups of Krull domains

16:00 – 16:30 荒谷 督司 (奈良教育大学)
The number of complete exceptional sequences

16:40 – 17:25 平松直哉 (岡山大学), 吉野雄二（岡山大学）
Picard groups of additive full subcategories

9月6日 (土曜日)

09:30 – 10:30 Alberto Facchini（Università di Padova）
Monogeny class, epigeny class, lower part, upper part

10:45 – 11:45 Nicole Snashall（University of Leicester）
Hochschild cohomology and support varieties of modules

13:15 – 14:00 長瀬 潤　(奈良工業高等専門学校)
Hochschild cohomology of Brauer algebras

14:15 – 14:45 阿部 弘樹（筑波大学), 星野 光男 (筑波大学)
Derived equivalences for triangular matrix rings

14:55 – 15:15 竹花 靖彦（函館工業高等専門学校）
QF-3′ modules relative to torsion theories and others

15:30 – 15:50 小松 弘明（岡山県立大学）

Left differential operators of modules over rings

16:00 – 16:30 中本 和典 (山梨大学), 面田 康裕 (明石工業高等専門学校)

The moduli spaces of non-thick irreducible representations for the free group of rank 2

16:40 – 17:25 和久井 道久 (関西大学)

Polynomial invariants of representation categories of semisimple and cosemisimple Hopf algebras

9月7日（日曜日)

09:30 – 10:30 Nicole Snashall（University of Leicester）

Representation theory and the structure of the Hochschild cohomology ring modulo nilpotence

10:45 – 11:15 脇 克志 (山形大学)

About decomposition numbers of J_4

11:25 – 11:55 本瀬 香 (弘前市)

Notes on the Feit-Thompson conjecture

The 41st Symposium on Ring Theory and Representation Theory (2008)

Program

September 5 (Friday)

[09:30 – 10:30] Kenji Nishida (Shinshu University)
Iwasawa algebras, crossed products and filtered rings

[10:45 – 11:45] Alberto Facchini (Università di Padova)
Geometric regularity of direct-sum decompositions of modules

[13:15 – 14:00] Osamu Iyama (Nagoya University)
Tilting mutation and its application

[14:15 – 14:45] Hiroki Miyahara (Shinshu University)
On filtered semi-dualizing bimodules

[14:55 – 15:15] Yosuke Sakai (University of Tsukuba)
An elementary exact sequence of modules with application to tiled orders

[15:30 – 15:50] Haruhisa Nakajima (Josai University)
Some remarks on descent of divisor class groups of Krull domains

[16:00 – 16:30] Tokuji Araya (Nara University of Education)
The number of complete exceptional sequences

[16:40 – 17:25] Naoya Hiramatsu (Okayama University),
Yuji Yoshino (Okayama University)
Picard groups of additive full subcategories

September 6 (Saturday)

[09:30 – 10:30] Alberto Facchini (Università di Padova)
Monogeny class, epigeny class, lower part, upper part

[10:45 – 11:45] Nicole Snashall (University of Leicester)
Hochschild cohomology and support varieties of modules

[13:15 – 14:00] Hiroshi Nagase (Nara National College of Technology)
Hochschild cohomology of Brauer algebras

[14:15 – 14:45] Hiroki Abe (University of Tsukuba),
Mitsuo Hoshino (University of Tsukuba)
Derived equivalences for triangular matrix rings

[14:55 – 15:15] Yasuhiko Takehana (Hakodate National College of Technology)
QF-3′ modules relative to torsion theories and others

[15:30 – 15:50] Hiroaki Komatsu（Okayama Prefectural University）

Left differential operators of modules over rings

[16:00 – 16:30] Kazunori Nakamoto (University of Yamanashi),
Yasuhiro Omoda (Akashi National College of Technology)

The moduli spaces of non-thick irreducible representations for the free group of rank 2

[16:40 – 17:25] Michihisa Wakui (Kansai University)

Polynomial invariants of representation categories of semisimple and cosemisimple Hopf algebras

September 7 (Sunday)

[09:30 – 10:30] Nicole Snashall（University of Leicester）

Representation theory and the structure of the Hochschild cohomology ring modulo nilpotence

[10:45 – 11:15] Katsushi Waki (Yamagata University)

About decomposition numbers of J_4

[11:25 – 11:55] Kaoru Motose (Hirosaki)

Notes on the Feit-Thompson conjecture

Iwasawa algebras, crossed products and filtered rings

Kenji Nishida

Let p be a prime integer and $\mathbb{Z}_p$ denote the ring of p-adic integers. A topological group G is a compact p-adic analytic group if and only if G has an open normal uniform pro-p subgroup H of finite index [4]. The *Iwasawa algebra* of G is defined by

$$\Lambda(G) := \varprojlim \mathbb{Z}_p[G/N],$$

where N ranges over the open normal subgroups of G.

The ring theoretical survey of Iwasawa algebras is given by K. Ardakov and K.A. Brown [1]. In this talk, I address crossed products and filtered rings arising from Iwasawa algebras. Therefore, we direct our attention to the fact that a ring $\Lambda(G)$ is a crossed product of a finite group G/H over a ring $\Lambda(H)$ (Iwasawa algebra of H): $\Lambda(G) \cong \Lambda(H) * (G/H)$. This remarkable isomorphism comes from the fact that

$$\Lambda(G) = \varprojlim \mathbb{Z}_p[G/U],$$

where U ranges over the open normal subgroups of G such that $U \subset H$.

Since the topological group H has good conditions, a ring $\Lambda(H)$ has good properties among them, we need:

(1) local with the radical $J := \mathrm{rad}\Lambda(H)$ and $\Lambda(H)/J \cong \mathbb{F}_p$, a field of p-elements,

(2) a filtered ring with the J-adic filtration whose associated graded ring is isomorphic to a polynomial ring $\mathbb{F}_p[x_0, \cdots, x_d]$, where $d = \mathrm{dim}G$ is a minimal number of generators of G as a topological group.

(3) a left and right Noetherian domain,

(4) Auslander regular with $\mathrm{gldim}\Lambda(H) = d + 1$.

(cf. [1], [2], [3], [4], [5])

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Geometric regularity of direct-sum decompositions of modules

Alberto Facchini

In this talk, rings R will be associative rings with identity and modules M_R will be unital right R-modules. Our aim will be to describe the direct-sum decompositions $M_R = M_1 \oplus \cdots \oplus M_n$ of a module M_R as a direct sum of finitely many direct summands, and present some tools to study direct-sum decompositions. The algebraic structure that describes direct-sum decompositions in a class $\mathcal{C}$ of right R-modules closed under isomorphism, direct summands and finite direct sums is the commutative monoid $V(\mathcal{C})$ of isomorphism classes of modules in $\mathcal{C}$. It is a reduced commutative monoid. Conversely, let k be a field and A a reduced commutative monoid. Then there exist a right and left hereditary k-algebra R and a class $\mathcal{C}$ of projective right R-modules, closed under isomorphism, direct summands and finite direct sums, such that $V(\mathcal{C}) \cong A$ [1, 2].

If we want to study the direct-sum decompositions of a fixed module M_R, we consider the monoid $V(M_R) := V(\mathrm{add}(M_R))$, where $\mathrm{add}(M_R) := \{\, N_R \mid N_R$ is isomorphic to a direct summand of M_R^n for some $n \geq 0\}$. This is a commutative monoid with order-unit. For any ring R, $V(R)$ will denote the monoid $V(R_R)$. We will define the category of commutative monoids with order-unit. Then V turns out to be a functor of the category **Rings** into the category of commutative monoids with order-unit.

As an example, we shall consider the case of artinian modules, for which we solved a problem posed by W. Krull in 1933: for artinian modules, Krull-Schmidt fails [4]. Nevertheless, in this case, decompositions are still very regular, as we will see. Further tools and related topics we will consider are semilocal rings and local morphisms (in noncommutative algebra), and modules whose endomorphism ring is semilocal, because their direct-sum decompositions have very regular geometric patterns described by Krull monoids. Krull monoids are the analogue for commutative monoids of what Krull domains are for commutative integral domains. The main results we will present are contained in [3, 6, 5, 7].

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Tilting mutation and its application

Osamu Iyama

We shall give an introduction to mutation theory of classical Brenner-Butler tilting modules due to Riedtmann-Schofield [RS] and Happel-Unger [HU]. Let Λ be a finite dimensional algebra. We call a Λ-module T *tilting* if

- proj.dim $T \leq 1$,
- $\mathrm{Ext}^1_\Lambda(T,T)=0$,
- there exists an exact sequence $0 \to \Lambda \to T_0 \to T_1 \to 0$ with $T_i \in \mathrm{add}T$.

In this case we have a derived equivalence between Λ and $\mathrm{End}_\Lambda(T)$. We call T *basic* if it is isomorphic to a direct sum of mutually non-isomorphic indecomposable Λ-modules.

The set of isoclasses of basic tilting Λ-modules forms a partially ordered set $\mathcal{T}(\Lambda)$, where we define

$$T \geq U \overset{\mathrm{def}}{\Longleftrightarrow} \mathrm{Ext}^1_\Lambda(T,U)=0.$$

We define the *Hasse quiver* of the partially ordered set $\mathcal{T}(\Lambda)$ by drawing an arrow $T \to U$ if $T > U$ and there is no $V \in \mathcal{T}(\Lambda)$ satisfying $T > V > U$. A basic result due to Riedtmann-Schofield and Happel-Unger is the following.

Theorem *For $T, U \in \mathcal{T}(\Lambda)$, the following conditions are equivalent.*
(a) T and U have same indecomposable direct summands except one.
(b) There is an arrow between T and U in the Hasse quiver.

In this case we say that U is a *mutation* of T. The following result due to Happel-Unger is quite useful.

Theorem *If $\mathcal{T}(\Lambda)$ is a finite set, then the Hasse quiver is connected.*

We apply tilting mutation theory to classification of cluster tilting objects over one-dimensional hypersurface singularities along a joint work with Burban, Keller and Reiten [BIKR].

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On Filtered Semi-dualizing Bimodules

Hiroki Miyahara

T. Araya, R. Takahashi and Y. Yoshino ([1]) introduced semi-dualizing bimodule which generalized semi-dualizing module in commutative ring theory. For a semi-dualizing bimodule C and a finitely generated module M, they also introduced G_C-dim M which generalized Gorenstein dimension of M, and extended the notion of Cohen-Macaulay dimension for modules over commutative Noetherian local rings to that for bounded complexes over non-commutative Noetherian rings.

In this talk, we will study filtered semi-dualizing bimodules. Let $(\Lambda, \mathcal{F})$ be a filtered ring with positive filtration, and $(M, \mathcal{F}')$ be a (discrete) filtered Λ-module. We denote by $\operatorname{gr}\Lambda$ (resp. $\operatorname{gr} M$) the associated graded ring of Λ (resp. module of M). We assume that $\operatorname{gr}\Lambda$ is left and right Noetherian and $\operatorname{gr} M$ is finitely generated $\operatorname{gr}\Lambda$-module. Then, Λ is left and right Noetherian, and M is finitely generated Λ-module (see [2]).

The following result is very important for us.

Theorem 1. Let $(C, \mathcal{F})$ be a filtered (Λ, Λ')-bimodule. If $\operatorname{gr} C$ is semi-dualizing $(\operatorname{gr}\Lambda, \operatorname{gr}\Lambda')$-bimodule, then C is semi-dualizing.

Let C be a filtered (Λ, Λ')-bimodule such that $\operatorname{gr} C$ is semi-dualizing, and M be a filtered left Λ-module. Thanks to theorem 1, we can define G_C-dim M and compare G_C-dim M with $G_{\operatorname{gr} C}$-dim $\operatorname{gr} M$. So we can show the following result.

Theorem 2. Let C be a (Λ, Λ')-bimodule such that $\operatorname{gr} C$ is semidualizing $(\operatorname{gr}\Lambda, \operatorname{gr}\Lambda')$-bimodule and let M be a filtered left Λ-module. Then G_C-dim M is less than or equal to $G_{\operatorname{gr} C}$-dim $\operatorname{gr} M$.

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An elementary exact sequence of modules with application to tiled orders

Yosuke Sakai

Let R be a ring with an identity, and let M be a right R-module. For R-submodules X, Y of M, there is an elementary short exact sequence

$$0 \longrightarrow X \cap Y \xrightarrow{\eta} X \oplus Y \xrightarrow{\varphi} X + Y \longrightarrow 0$$

where $\eta(t) = (t, -t)$ for $t \in X \cap Y$ and $\varphi(x, y) = x + y$ for $(x, y) \in X \oplus Y$.

In this talk, we extend the elementary short exact sequence to the case of more than two R-submodules of a given right R-module, and as an application, we compute directly a minimal projective resolution of Jacobson radical of a tiled order given by Fujita and Oshima [3], which provides a tiled order of finite global dimension without neat primitive idempotent (see [1, 2] for neat primitive idempotents).

The following proposition is a sample result of the case of three submodules.

Proposition. *Let X, Y, Z be R-submodules of a right R-module M. Let*

$$0 \to X \cap Y \cap Z \xrightarrow{\eta} (X \cap Z) \oplus (Y \cap X) \oplus (Z \cap Y) \xrightarrow{\psi} X \oplus Y \oplus Z \xrightarrow{\varphi} X + Y + Z \to 0$$

be a sequence of R-modules and R-homomorphisms defined by

$$\varphi(x, y, z) = x + y + z,\ \psi(x_0, y_0, z_0) = (x_0 - y_0,\ y_0 - z_0,\ z_0 - x_0),\ \eta(t) = (t, t, t)$$

for all $(x, y, z) \in X \oplus Y \oplus Z$, $(x_0, y_0, z_0) \in (X \cap Z) \oplus (Y \cap X) \oplus (Z \cap Y)$ and $t \in X \cap Y \cap Z$. Then the above sequence is exact if and only if $(X + Y) \cap Z \subset X + (Y \cap Z)$.

Remark. For any nonzero ideals X, Y, Z of a PID, $(X + Y) \cap Z \subset X + (Y \cap Z)$ holds, while it does not hold for any ideals X, Y, Z of $\mathbb{Z}[t]$.

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Some remarks on descent of class groups of Krull domains

Haruhisa Nakajima

Let R be a Krull domain with its quotient field $\mathcal{Q}(R)$. We will study on a certain subfield L of $\mathcal{Q}(R)$ such that a canonical homomorphism $E^*(R, R \cap L) \to \mathrm{Div}(R \cap L)$, which is essentially defined by A. Magid, induces the isomorphism $\mathrm{Cl}(R) \to \mathrm{Cl}(R \cap L)$ and give some elementary remarks on this pair $(R, R \cap L)$ related to invariant theory.

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The number of complete exceptional sequences

Tokuji Araya

Let Λ be the path algebra of the Dynkin quiver of type (A_n) over an algebraically closed field. We denote by $\operatorname{mod}\Lambda$ the category of finitely generated left Λ-modules and by $\mathfrak{D} = \mathfrak{D}^{\mathrm{b}}(\operatorname{mod}\Lambda)$ the bounded derived category.

The concept of an exceptional sequence was developed by A. L. Gorodentsev and A. N. Rudakov [2] and generalized by A. I. Bondal [1]. A complex $E \in \mathfrak{D}$ is called *exceptional* if $\operatorname{End}_{\mathfrak{D}}(E, E) \cong k$ and $\operatorname{Hom}_{\mathfrak{D}}(E, E[i]) = 0$ for $i \neq 0$. A sequence of exceptional complexes $\epsilon = (E_1, E_2, \cdots, E_r)$ is called an *exceptional sequence of length* r if $\operatorname{Hom}_{\mathfrak{D}}(E_i, E_j[s]) = 0$ for all $i < j$ and all $s \in \mathbb{Z}$ and called *complete* if $r = n$. We put $\mathcal{E}_n$ the set of complete exceptional sequences. The aim of this talk is to count how many complete exceptional sequences exist essentially.

For $\epsilon = (E_1, E_2, \cdots, E_n), \epsilon' = (E'_1, E'_2, \cdots, E'_n) \in \mathcal{E}_n$, we define $\epsilon \sim \epsilon'$ if $\bigoplus_{i=1}^{n} E_i \cong \bigoplus_{i=1}^{n} E'_i[\ell_i]$ for some $\ell_1, \ell_2, \cdots, \ell_n \in \mathbb{Z}$ and put $\widetilde{\mathcal{E}_n} = \mathcal{E}_n / \sim$.

Theorem 1. *The cardinality of* $\widetilde{\mathcal{E}_n}$ *is* $\dfrac{1}{2n+1}\dbinom{3n}{n}$.

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Picard groups of additive full subcategories

Naoya Hiramatsu and Yuji Yoshino
(Okayama University)

Let k be a commutative ring and A be a commutative k-algebra. We denote by A-Mod the module category over A. Assume that $\mathfrak{C}$ is an additive full subcategory of A-Mod such that $\mathfrak{C}$ contains A as an object. Since A is a k-algebra, every additive full subcategory $\mathfrak{C}$ is a k-category. We say that a covariant functor $\mathfrak{C} \to \mathfrak{C}$ is a k-linear automorphism of $\mathfrak{C}$ if it is a k-linear functor giving an auto-equivalence of the category $\mathfrak{C}$. In this talk, we are interested in the group of k-linear automorphisms of the category $\mathfrak{C}$. We denote the set of all the isomorphism classes of k-linear automorphisms of $\mathfrak{C}$ by $\mathrm{Aut}_k(\mathfrak{C})$. It forms a group by defining the multiplication to be the composition of functors.

We are motivated by the following computational result:

Theorem 1. *Let A be a complete local Cohen-Macaulay k-algebra which has only an isolated singularity, where k is a field. We denote by $\mathrm{CM}(A)$ the category of Cohen-Macaulay modules over A. Then we have the following equalities.*

$$\mathrm{Aut}_k(\mathrm{CM}(A)) = \begin{cases} \mathrm{Aut}_{k\text{-}alg}(A) & (\text{if } \dim A \neq 2), \\ \mathrm{Aut}_{k\text{-}alg}(A) \ltimes C\ell(A) & (\text{if } \dim A = 2). \end{cases}$$

In the theorem, $\mathrm{Aut}_{k\text{-alg}}(A)$ denotes the group of k-algebra automorphisms of A, and $C\ell(A)$ is the divisor class group of A. We can generalize this result to any additive full subcategories.

Theorem 2. *Let k be a commutative ring and let A be a commutative k-algebra. Assume that an additive full subcategory $\mathfrak{C}$ of A-Mod is stable under $\mathrm{Aut}_{k\text{-}alg}(A)$ and that $A \in \mathfrak{C}$. Then we have an isomorphism*

$$\mathrm{Aut}_k(\mathfrak{C}) \cong \mathrm{Aut}_{k\text{-}alg}(A) \ltimes \mathrm{Pic}(\mathfrak{C}).$$

In the theorem, $\mathrm{Pic}(\mathfrak{C})$ is the Picard group of $\mathfrak{C}$. The notion of Picard groups is originally defined for commutative rings, hence for schemes (cf. Bass [1]). Taking into consideration of this, we are able to define the Picard group for any additive full subcategories. To prove the theorem 2, we need several auxiliary results. In one of them, we can give explicit presentation for equivalence functors.

Theorem 3. *Let A and B be commutative k-algebras and let $\mathfrak{C}$ and $\mathfrak{D}$ be additive full subcategories of A-Mod and B-Mod respectively such that $A \in \mathfrak{C}$ and $B \in \mathfrak{D}$. Suppose that $\mathfrak{C}$ and $\mathfrak{D}$ are equivalent as k-categories and let $F : \mathfrak{C} \to \mathfrak{D}$ be a k-linear covariant functor which gives an equivalence between them. Then there is a k-algebra isomorphism $\sigma : B \to A$ such that F is isomorphic to the functor $\mathrm{Hom}_A(N_\sigma, -)|_{\mathfrak{C}}$, where $N \in \mathfrak{C}$ is chosen so that $F(N) \cong B$ in $\mathfrak{C}$.*

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Monogeny class, epigeny class, lower part, upper part

Alberto Facchini

Let R be an associative ring with identity. Two unital right modules A_R, B_R are said to *belong to the same monogeny class* ($[A_R]_m = [B_R]_m$) if there exist a monomorphism $A_R \to B_R$ and a monomorphism $B_R \to A_R$. Similarly, A_R and B_R *belong to the same epigeny class*, written $[A_R]_e = [B_R]_e$, if there exist an epimorphism $A_R \to B_R$ and an epimorphism $B_R \to A_R$.

We say that a module A_R is:

- *uniserial* if, for any submodules B and C of A_R, we have $B \subseteq C$ or $C \subseteq B$;
- *uniform* if it has Goldie dimension 1, that is, it is non-zero and the intersection of any two non-zero submodules is a non-zero submodule;
- *couniform* if it has dual Goldie dimension 1, that is, it is non-zero and the sum of any two proper submodules is a proper submodule;
- *biuniform* if it uniform and couniform.

Let $\mathcal{C}$ be the class of all biuniform right R-modules.

Let A_R be a module in the class $\mathcal{C}$ and let $E = \mathrm{End}(A_R)$ be its endomorphism ring. Then E has two two-sided completely prime ideals I and K (given by all endomorphisms that are non-injective and non-surjective, respectively) such that every proper right ideal of E and every proper left ideal of E is contained either in I or in K, and one of the following two conditions hold: (a) Either E is a local ring, or (b) $E/J(E) \cong E/I \times E/K$, where E/I and E/K are division rings.

A weak form of the Krull-Schmidt Theorem holds for the class $\mathcal{C}$ [2]. Let U_1, $\dots$, U_n, V_1, $\dots$, V_t be modules in $\mathcal{C}$. Then the direct sums $U_1 \oplus \cdots \oplus U_n$ and $V_1 \oplus \cdots \oplus V_t$ are isomorphic if and only if $n = t$ and there are two permutations σ, τ of $\{1, 2, \dots, n\}$ such that $[U_i]_m = [V_{\sigma(i)}]_m$ and $[U_i]_e = [V_{\tau(i)}]_e$ for every $i = 1, 2, \dots, n$.

Similar results hold when $\mathcal{C}$ is the class of all cyclically presented modules and R is local [1], or when $\mathcal{C}$ is the class of all kernels of morphisms between indecomposable injective modules [4], or when $\mathcal{C}$ is the class of all couniformly presented modules [3]. We will discuss the existence of a general theory [5].

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Hochschild cohomology and support varieties of modules

Nicole Snashall

This talk begins with a brief introduction to Hochschild cohomology, an important invariant in the representation theory of algebras. This will include a discussion of a class of algebras which have a periodic projective resolution as a bimodule, thus enabling us to describe the structure of the Hochschild cohomology ring in these cases.

The main part of the talk discusses the notion of a support variety for a finitely generated module over finite-dimensional algebra. One of the motivations for this work was the theory of support (or cohomological) varieties of modules over group algebras, which has played a major role in the modular representation theory of a finite group. This latter concept was introduced by Carlson, and is defined in terms of the maximal ideal spectrum of the group cohomology ring. I will discuss the way in which we used the Hochschild cohomology ring to give a more general concept of a support variety for a finitely generated module over any finite-dimensional algebra (joint with Solberg, Proc LMS 2004) and the properties of these varieties. In particular, for selfinjective algebras under certain finite generation conditions, we have analogues of many of the properties of the group ring situation (joint with Erdmann, Holloway, Solberg and Taillefer, K-Theory 2004).

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Hochschild Cohomology of Brauer Algebras

Hiroshi Nagase

When we study Hochschild cohomology, it is natural to try relating cohomology of an algebra B to that of an 'easier' or 'smaller' algebra A. One such situation is that of B being a one-point extension of A, which was studied by Happel. Happel's long exact sequence has been generalized to triangular matrix algebras independently by C. Cibils and by S. Michelena and M. I. Platzeck and to algebras with heredity ideals by J. A. de la Peña and C. C. Xi.

In [3], these results are generalized to the case of algebras with stratifying ideals. An idempotent ideal BeB of a finite dimensional algebra B is called a *stratifying ideal* if the derived functor

$$D^+(\mathrm{mod}B/BeB) \to D^+(\mathrm{mod}B)$$

induced by the canonical algebra homomorphism $B \to B/BeB$ is fully faithful. For example, if BeB is projective as a left (or right) B-module, then BeB is a stratifying ideal.

In [4], Snashall and Solberg conjectured that, for any finite dimensional algebra A, the Hochschild cohomology ring modulo nilpotence $\overline{\mathrm{HH}}^*(A)$ is a finitely generated algebra. Green, Snashall and Solberg have shown that the conjecture holds for the case of self-injective algebras of finite representation type in [1] and the case of monomial algebras in [2].

In this talk, we verify the conjecture for some cases of Brauer algebra B with stratifying ideal BeB, by applying the following graded algebra homomorphism, which is obtained in [3],

$$\overline{\mathrm{HH}}^*(B) \hookrightarrow \overline{\mathrm{HH}}^*(B/BeB) \times \overline{\mathrm{HH}}^*(eBe).$$

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Derived equivalences for triangular matrix rings

Hiroki Abe and Mitsuo Hoshino

We generalize derived equivalences for triangular matrix rings induced by a certain type of classical tilting module introduced by Auslander, Platzeck and Reiten to generalize reflection functors in the representation theory of quivers due to Bernstein, Gelfand and Ponomarev.

Let R be a finite dimensional algebra over a field k and M a finitely generated projective right R-module. Set

$$A = \begin{pmatrix} k & M \\ 0 & R \end{pmatrix} \quad \text{and} \quad e = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \in A.$$

As pointed out by Brenner and Butler (see [3]), we know from [1] (cf. also [2]) that $\mathrm{Ext}^1_A(A/AeA, A) \oplus Ae \in \text{Mod-}A^{\mathrm{op}}$ is a classical tilting module, i.e., a tilting module of projective dimension at most one (see [4]) with

$$\mathrm{End}_{A^{\mathrm{op}}}(\mathrm{Ext}^1_A(A/AeA, A) \oplus Ae)^{\mathrm{op}} \cong \begin{pmatrix} R & \mathrm{Hom}_R(M, R) \\ 0 & k \end{pmatrix}.$$

Our aim is to extend this type of derived equivalence to the case where M_R has finite projective dimension. Let R, S be rings and M an S-R-bimodule such that M admits a projective resolution $P^\bullet \to M$ in Mod-R with $P^\bullet \in \mathcal{K}^{\mathrm{b}}(\mathcal{P}_R)$ and $\mathrm{Ext}^i_R(M, R) = 0$ for $i < d = \text{proj dim } M_R$. Set

$$A = \begin{pmatrix} S & M \\ 0 & R \end{pmatrix} \quad \text{and} \quad e = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \in A.$$

We will construct a tilting complex $T^\bullet \in \mathcal{K}^{\mathrm{b}}(\mathcal{P}_A)$ associated with e such that

$$\mathrm{End}_{\mathcal{K}(\text{Mod-}A)}(T^\bullet) \cong \begin{pmatrix} R & \mathrm{Ext}^d_A(M, R) \\ 0 & S \end{pmatrix}.$$

Assume further that ${}_SM$ is faithful and that if $d > 0$ then $S \xrightarrow{\sim} \mathrm{End}_R(M)$ canonically and $\mathrm{Ext}^i_R(M, M) = 0$ for $1 \le i < d$. Then we will see that

$$\mathrm{Hom}^\bullet_A(T^\bullet, A)[d+1] \cong \mathrm{Ext}^{d+1}_A(A/AeA, A) \oplus Ae$$

in $\mathcal{D}(\text{Mod-}A^{\mathrm{op}})$ and $\mathrm{Ext}^{d+1}_A(A/AeA, A) \oplus Ae \in \text{Mod-}A^{\mathrm{op}}$ is a tilting module of projective dimension $d+1$ (see [5]) with

$$\mathrm{End}_{A^{\mathrm{op}}}(\mathrm{Ext}^{d+1}_A(A/AeA, A) \oplus Ae)^{\mathrm{op}} \cong \begin{pmatrix} R & \mathrm{Ext}^d_A(M, R) \\ 0 & S \end{pmatrix}.$$

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QF-3' modules relative to torsion theories and others

Yasuhiko Takehana

Let R be a ring with identity, and let Mod-R be the category of right R-modules. Let M be a right R-module. We denote by $E(M)$ the injective hull of M. M is called *QF-3' module*, if $E(M)$ is M-torsionless, that is, $E(M)$ is isomorphic to a submodule of a direct product $\prod M$ of some copies of M.

A subfunctor of the identity functor of Mod-R is called a *preradical*. For a preradical σ, $\mathcal{T}_\sigma := \{M \in \text{Mod-}R \ ; \ \sigma(M) = M\}$ is the class of σ-*torsion* right R-modules, and $\mathcal{F}_\sigma := \{M \in \text{Mod-}R \ ; \ \sigma(M) = 0\}$ is the class of σ-*torsionfree* right R-modules. A right R-module M is called σ-*injective* if the functor $\text{Hom}_R(M, \quad)$ preserves the exactness for any exact sequence $0 \to A \to B \to C \to 0$ with $C \in \mathcal{T}_\sigma$. A right R-module M is called σ-*QF-3' module* if $E_\sigma(M)$ is M-torsionless, where $E_\sigma(M)$ is defined by $E_\sigma(M)/M := \sigma(E(M)/M)$.

In this talk, we characterize σ-QF-3' modules and give some related facts, which are announced in [1], [2], [3], [4] and [5].

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Left differential operators of modules over rings

Hiroaki Komatsu

Left differential operators of modules over an algebra was introduced by Sweedler [1]. In this talk, we give a more general definition which contains not only Sweedler's left differential operators but also derivations of an algebra to bimodules.

Let $\mathcal{R} = \{R_1, \ldots, R_n\}$ be a finite family of algebras over a commutative ring K with an identity element. If a unitary K-module M has a left R_i-module structure for each i and, for any $i \neq j$, the actions of R_i and R_j commute, then we call M a *left $\mathcal{R}$-module*. (If all R_i have identity elements and M is unitary over all R_i's, then M is nothing but a unitary left $R_1 \otimes_K \cdots \otimes_K R_n$-module.) Let M and N be left $\mathcal{R}$-modules. If $f \in \mathrm{Hom}_K(M, N)$ and $a \in R_i$, then, by making use of R_i-bimodule structure of $\mathrm{Hom}_K(M, N)$, we define $\{f, a\} = fa - af$. If $F \subseteq \mathrm{Hom}_K(M, N)$ and $A \subseteq R_i$, we define $\{F, A\}$ to be the K-subspace generated by all $\{f, a\}$ where $f \in F$ and $a \in A$. Moreover, we put $\{F, A\}_0 = F$ and $\{F, A\}_m = \{\{F, A\}_{m-1}, A\}$ ($m = 1$, $2, \ldots$).

Let $p = (p_1, \ldots, p_n)$ be an n-tuple of nonnegative integers. Under the above notations, if $f \in \mathrm{Hom}_K(M, N)$ satisfies $\{\cdots\{\{f, R_1\}_{p_1}, R_2\}_{p_2}, \cdots, R_n\}_{p_n} = 0$, then we call f a *left differential operator* of type p. The set of all left differential operators of type p from M to N is denoted by $\mathcal{D}^p_{\mathcal{R}}(M, N)$. In case of $n = 1$, this coincides with Sweedler's definition. In case of $\mathcal{R} = \{R, R^\circ\}$, where R° is the opposite algebra of R, $\mathcal{D}^{(1,1)}_{\mathcal{R}}(R, M)$ contains all K-derivations of R to M.

We introduce some results related to our left differential operators.

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The moduli spaces of non-thick irreducible representations for the free group of rank 2

Yasuhiro Omoda (Akashi National College of Technology) [1]
Kazunori Nakamoto (University of Yamanashi) [2]

There are several types among irreducible representations. Considering such several types gives us rich problems on representation theories, and the first step to describe the moduli spaces of representations. The moduli of irreducible representations for the free group F_2 is very big, and difficult to be investigated. But by the following definitions we can describe some parts of the moduli of representations.

Definition 1. Let V be an n-dimensional vector space over k. For $0 < m < n$, we say that a representation $\rho : G \to \mathrm{GL}(V)$ of a group G is *m-thick* if for any subspaces V_1, V_2 of V with $\dim V_1 = m, \dim V_2 = n - m$, there exists $g \in G$ such that $(\rho(g)V_1) \oplus V_2 = V$. If ρ is m-thick for each $0 < m < n$, then we say that ρ is *thick*.

Definition 2. For $0 < m < n$, we say that a representation $\rho : G \to \mathrm{GL}(V)$ of a group G is *m-dense* if $\Lambda^m V$ is an irreducible representation of G. If ρ is m-dense for each $0 < m < n$, then we say that ρ is *dense*.

Irreducibility, thickness, and denseness are different properties. We introduce several results on them.

Proposition 3. *For n-dimensional representations, m-thickness is equivalent to $(n-m)$-thickness. And m-denseness is equivalent to $(n-m)$-denseness.*

Proposition 4.

$$m\text{-}dense \Longrightarrow m\text{-}thick \Longrightarrow 1\text{-}dense \Longleftrightarrow 1\text{-}thick \Longleftrightarrow irreducible.$$

In particular,

$$dense \Longrightarrow thick \Longrightarrow irreducible$$

For all representations of degree ≤ 3, irreducibility, denseness, and thickness are equivalent. For representations of degree ≥ 4, the differences appear.

Proposition 5. *For a 4-dimensional representation $\rho : G \to \mathrm{GL}_4(k)$, ρ is thick if and only if the exterior representation $\wedge^2 \rho$ has no invariant subspace W of k^4 with $2 \leq \dim W \leq 4$.*

Assume that k is an algebraically closed field. The moduli $\mathrm{Ch}_4(F_2)_{air}$ of equivalence classes of 4-dimensional irreducible representations for the free group F_2 of rank 2 is a 17-dimensional non-singular irreducible algebraic variety over k. Here we regard the moduli as the geometric object consisting of points which corresponds to equivalence classes of irreducible representations. The moduli $\mathrm{Ch}_4(F_2)_{air}$ has an open subset which consists of thick representations. Conversely, the non-thick irreducible representations form a close subset $\mathrm{Ch}_4(F_2)_{\mathrm{non-thick}}$ of the moduli. In this talk, we show how the closed subset $\mathrm{Ch}_4(F_2)_{\mathrm{non-thick}}$ is contained in $\mathrm{Ch}_4(F_2)_{air}$.

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Polynomial invariants of representation categories of semisimple and cosemisimple Hopf algebras

Michihisa Wakui

In this talk, we introduce an invariant defined as a polynomial for a finite-dimensional semisimple and cosemisimple Hopf algebra over a field $\boldsymbol{k}$. In fact, this polynomial is an invariant of the representation category of such a Hopf algebra under $\boldsymbol{k}$-linear tensor equivalences.

Let A be a Hopf algebra over the field $\boldsymbol{k}$. For a universal R-matrix R of A and a finite-dimensional left A-module M, we denote by $\underline{\dim}_R M$ the trace of the left action of the Drinfel'd element associated to R on M. By Etingof and Gelaki, if A is semisimple and cosemisimple of finite dimension, then the set of universal R-matrices of A, denoted by $\underline{\mathrm{Braiding}}(A)$, is finite, and if M is absolutely simple, then $(\dim M)1_{\boldsymbol{k}} \neq 0$. Thus, for such A and M, we have a polynomial $\tilde{\lambda}_{A,M}(x)$ defined by

$$\tilde{\lambda}_{A,M}(x) := \prod_{R\in\underline{\mathrm{Braiding}}(A)} \left(x - \frac{\underline{\dim}_R M}{\dim M}\right) \quad \in \quad \boldsymbol{k}[x].$$

Furthermore, for each positive integer d a polynomial $\tilde{\lambda}_A^{(d)}(x)$ is defined by

$$\tilde{\lambda}_A^{(d)}(x) := \prod_{i=1}^{t} \tilde{\lambda}_{A,M_i}(x) \quad \in \quad \boldsymbol{k}[x],$$

where $\{M_1, \cdots, M_t\}$ is a full set of non-isomorphic absolutely simple left A-modules with dimension d.

Theorem 1. *Let A, B be finite-dimensional semisimple and cosemisimple Hopf algebras over a field $\boldsymbol{k}$. If representation categories of A and B are equivalent as $\boldsymbol{k}$-linear tensor categories, then $\tilde{\lambda}_A^{(d)}(x) = \tilde{\lambda}_B^{(d)}(x)$ for any positive integer d. Furthermore, the polynomial invariant $\tilde{\lambda}_A^{(d)}(x)$ has the following properties.*

(1) *Let $K/\boldsymbol{k}$ be a finite Galois extension of fields, and let Z denote the integral closure of the prime ring of $\boldsymbol{k}$ in K. Then, $\tilde{\lambda}_{A^K}^{(d)}(x) \in (\boldsymbol{k} \cap Z)[x]$ $(d = 1, 2, \cdots)$ for the scalar extension A^K of A.*

(2) *There is a finite separable field extension $L/\boldsymbol{k}$ so that $\tilde{\lambda}_{A^E}^{(d)}(x) = \tilde{\lambda}_{A^L}^{(d)}(x)$ $(d = 1, 2, \cdots)$ for a field extension E/L.*

According to Masuoka's classification result, there are three types of the 8-dimensional non-commutative and semisimple Hopf algebras over an algebraically closed field $\boldsymbol{k}$ with characteristic 0 up to isomorphisms. Tambara and Yamagami, and also Masuoka showed that the representation rings of these Hopf algebras are same, whereas the representation categories are not mutually equivalent as $\boldsymbol{k}$-linear tensor categories. By comparing our polynomial invariants, we have an another proof of their result. Furthermore, there are other examples of Hopf algebras, whose representation rings are same, but polynomial invariants are different.

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Representation theory and the structure of the Hochschild cohomology ring modulo nilpotence

Nicole Snashall

This talk will continue with some of the representation-theoretic results obtained in the paper by EHSST (K-Theory 2004). One such result, for selfinjective algebras satisfying certain finiteness conditions, is that we have a generalised version of Webb's Theorem, and thus can describe the possible tree classes of a component of the stable Auslander-Reiten quiver of the algebra.

The second part of this talk considers the structure of the Hochschild cohomology ring modulo nilpotence. A recent example of Xu has shown that the Hochschild cohomology ring modulo nilpotence is not always finitely generated as an algebra, thus providing a counterexample to the conjecture made by myself and Solberg (Proc LMS 2004). Nevertheless, there are many cases when this quotient is finitely generated; these are being used to give new insight into the support variety of a module. I will finish by outlining some current research in this area.

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About decomposition numbers of J_4

Katsushi Waki

The decomposition numbers with some unknown parameters of non-principal blocks of the largest Janko group J_4 [2] for characteristic 3 are determined. We also concerned with the decomposition numbers of maximal 2-local subgroups [3] of J_4 in odd characteristics. We used the character table library in GAP[1]

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Notes on the Feit-Thompson conjecture

Kaoru Motose

In this talk, we shall present partial solutions to the Feit Thompson conjecture such that $(q^p-1)/(q-1)$ does not divide $(p^q-1)/(p-1)$ for distinct primes $p<q$.

I think also it is not so popular to mathematicians, even to finite group theorists and number theorists. As my results are very easy and almost trivial, the purpose of this talk is to make a propaganda and a motivation for persons to study this conjecture.

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