

NOTES ON THE FEIT-THOMPSON CONJECTURE

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ABSTRACT. In this paper, we present partial solutions about Feit Thompson Conjecture.

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Feit and Thompson [2] conjectured that $F = (q^p - 1)/(q - 1)$ does not divide $T = (p^q - 1)/(q - 1)$ for distinct odd primes $p < q$ (see also [6]).

In the paper [1, p.1], it was mentioned that if it could be proved, Odd paper [3] could be shortened by nearly 50 pages (see also [4, p.125]).

Stephan [6] conjectured that F and T are relatively prime. However, using computer, he found a common divisor $r = 112643 = 2pq + 1$ for a pair $p = 17, q = 3313$.

This is a rare example by the equation $q^{\frac{p-1}{2}} \equiv 1 \pmod{p^2}$ for this pair (see [5]).

He also confirmed that r is the greatest common divisor of F and T by computer, so this example leaves Feit-Thompson conjecture unresolved.

At the present, it is known by computer that no other such pairs exist for $p < q < 10^7$ and $p = 3 < q < 10^{14}$ (see [4]).

The next is easily proved as in my talk.

Proposition. *In either case of the next conditions, F does not divide T .*

- (1) $q \equiv 1 \pmod{p}$.
- (2) $p = 3 < q$ and F is composite.
- (3) $p \equiv 3$ and $q \equiv 1 \pmod{4}$.
- (4) $r = 2p + 1$ is prime, Legendre symbol $\left(\frac{p}{r}\right) = 1$, and $q \not\equiv 1 \pmod{r}$.

REFERENCES

- [1] Apostol, T. M., *The Resultant of the Cyclotomic Polynomials $F_m(ax)$ and $F_n(bx)$* , Math. Comput. **29** (1975), 1-6.
- [2] Feit, W. and Thompson, J. G., *A Solvability Criterion for Finite Groups and Some Consequences*, Proc. Nat. Acad. Sci. USA **48** (1962), 968-970.
- [3] Feit, W. and Thompson, J. G., *A Solvability of Groups of odd order*, Pacific J. Math. **13** (1963), 775-1029.
- [4] Guy, R. K., *Unsolved Problems in Number Theory*, 3rd ed., 2004, New York Springer.
- [5] Ribenboim, P., *1093*, Math. Intelligencer, **5** (1983), 28-34.
- [6] Stephens, N. M., *On the Feit-Thompson Conjecture*, Math. Comput. **25** (1971), 625.

The detailed version of this paper will be submitted for publication elsewhere.

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In Lemma 10, the assumption $|\Delta| = p + s$ with $s < p$ should be corrected to $|\Delta|$ is p or $p + 1$.