The 42nd Symposium on Ring Theory and Representation Theory

ABSTRACT

Osaka Kyoiku University, Osaka
October 10 - 12, 2009

The 42nd Symposium onRing Theory and Representation Theory

Program

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George Szeto (Bradley University)
Lianyong Xue (Bradley University)

On Galois Extensions with an Inner Galois Group and a Galois Commutator Subring

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Hisaya Tsutsui (Embry-Riddle University)
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Fully Weakly Prime Rings

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On Galois Extensions with an Inner Galois Group and a Galois Commutator Subring

Shûichi Ikehata, George Szeto and Lianyong Xue

It was shown ([1], Theorem 3) that B is a central Galois algebra over its center C with an inner Galois group G if and only if it is an Azumaya projective group algebra CG_f where $f : G \times G \longrightarrow$ units of C is a factor set. We shall generalize the above theorem to any Galois extension B with an inner Galois group G where $G = \{g \in G \mid g(x) = U_g x U_g^{-1}$ for some $U_g \in B$ and for all $x \in B\}$. It is shown that B contains a projective group algebra CG_f. An equivalent condition for a central Galois algebra CG_f with Galois group induced by G is given, and characterizations for a Galois extension B with an inner Galois group G generated by $\{U_g \mid g \in G\}$ over B^G are obtained. When B is also an Azumaya algebra, some properties are given for a Galois extension B with an inner Galois group G.

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Fully Weakly Prime Rings

Edward Poon, Yasuyuki Hirano, and Hisaya Tsutsui

We define a proper ideal I of a ring R to be weakly prime if $0 \neq JK \subseteq I$ implies either $J \subseteq I$ or $K \subseteq I$ for any ideals J, K of R. In this talk, we investigate the structure of rings, not necessarily commutative,in which all ideals are weakly prime. (A ring whose zero ideal is prime is called prime. In this sense, any ring R is weakly prime since the zero ideal is always weakly prime.)

Crystalline Graded Rings

Fred van Oystaeyen

In joint work with V. Bavula the generalized Weyl algebras have been studied, their homological algebra properties like dimensions and regularity have been studied. The main ingredient in this study is the existence of a gradation with very nice properties e.g. the part of degree zero being a Dedekind domain or a Noetherian integrally closed domain of low dimension. The class of crystalline graded rings generalizes the class of generalized Weyl algebras, even in the case of gradings by integers interesting new examples appear. Actions by a noncommutative group of automorphisms of the degree zero part as well as generalized Z-cocycles appear naturally in this theory and a suitable localization at an Ore set yields a crossed product. The general theory then deals with orders and maximal orders in crossed products. In case of crystalline graded rings over Dedekind domains the ideal theory of Dedekind domains may be used to obtain spectrally twisted cocycles where by taking valuations at primes in the Dedekind domains a set of 2-cocycle like conditions appears. We classify all graded maximal orders containing a given crystalline graded ring and provide classification in some special cases. The main ingredient in this theory is the manipulation of spectrally twisted cocycles and orders constructed starting from them.

On the structure of Sally modules of rank one

Shiro Goto, Koji Nishida and Kazuho Ozeki

Let A be a Cohen-Macaulay local ring with the maximal ideal $\mathfrak{m}$ and $d = \dim A > 0$. We assume the residue class field $k = A/\mathfrak{m}$ of A is infinite. Let I be an $\mathfrak{m}$-primary ideal in A and choose a minimal reduction $Q = (a_1, a_2, \cdots, a_d)$ of I. Let

$$R = \mathrm{R}(I) := \bigoplus_{n\geq 0} I^n, \ T = \mathrm{R}(Q) := \bigoplus_{n\geq 0} Q^n \quad \text{and} \quad G = \mathrm{G}(I) := \bigoplus_{n\geq 0} I^n/I^{n+1}$$

respectively denote the Rees algebras of I and Q and the associated graded ring of I. We then define

$$S = IR/IT$$

and call it the Sally module of I with respect to Q. Let $B = T/\mathfrak{m}T$ which is the polynomial ring with d indeterminates over the field $k = A/\mathfrak{m}$.

The main result of my talk is the following, which is a complete structure theorem of the Sally modules of $\mathfrak{m}$-primary ideal I satisfying the equality

$$e_1 = e_0 - \ell_A(A/I) + 1,$$

where $e_i = \mathrm{e}_I^i(A)$ denotes the i-th Hilbert coefficients of I.

Theorem 1 ([?]). *The following three conditions are equivalent to each other.*

(1) $e_1 = e_0 - \ell_A(A/I) + 1$.
(2) $\mathfrak{m}S = (0)$ *and* $\mathrm{rank}_B\, S = 1$.
(3) $S \cong (X_1, X_2, \cdots, X_c)B$ *as graded T-modules for some* $0 < c \leq d$, *where* $\{X_i\}_{1\leq i\leq c}$ *are linearly independent linear forms of the polynomial ring B.*

When this is the case, $c = \ell_A(I^2/QI)$ *and* $I^3 = QI^2$, *and the following assertions hold.*

(a) $\mathrm{depth}\, G \geq d - c$ *and* $\mathrm{depth}_T\, S = d - c + 1$.
(b) $\mathrm{depth}\, G = d - c$, *if* $c \geq 2$.
(c) *Suppose $c < d$. Then*

$$\ell_A(A/I^{n+1}) = e_0\binom{n+d}{d} - e_1\binom{n+d-1}{d-1} + \binom{n+d-(c+1)}{d-(c+1)}$$

for all $n \geq 0$. Hence

$$e_i = \begin{cases} 0 & \text{if } i \neq c+1, \\ (-1)^{c+1} & \text{if } i = c+1 \end{cases}$$

for $2 \leq i \leq d$.

(d) *Suppose $c = d$. Then*

$$\ell_A(A/I^{n+1}) = e_0\binom{n+d}{d} - e_1\binom{n+d-1}{d-1}$$

for all $n \geq 1$. Hence $e_i = 0$ for $2 \leq i \leq d$.

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The Buchsbaum-Rim function of a parameter module

Futoshi Hayasaka

Let $(A, \mathfrak{m})$ be a commutative Noetherian local ring with the maximal ideal $\mathfrak{m}$ and $d = \dim A > 0$ the Krull dimension of A. Let $F = A^r$ be a free module of rank $r > 0$, and let $S = \mathcal{S}_A(F)$ be the symmetric algebra of F. For a submodule M of F, let $R = \mathcal{R}(M)$ be the image of the natural homomorphism $\mathcal{S}_A(M) \to \mathcal{S}_A(F)$. Let S_ν (resp. M^ν) be a homogeneous component of degree ν of S (resp. R). Assume that the quotient F/M has finite length and $M \subseteq \mathfrak{m}F$.

In 1964, in order to generalize the multiplicity to the situation of finitely generated modules F/M rather than just cyclic modules A/I, Buchsbaum and Rim introduced and studied the length function $\lambda(\nu) = \ell_A(S_\nu/M^\nu)$, and proved in [?] that the function λ is eventually a polynomial of degree $d+r-1$. The polynomial P corresponding to λ can then be written in the form $P(\nu) = \sum_{i=0}^{d+r-1}(-1)^i e_i \binom{\nu+d+r-2-i}{d+r-1-i}$ with integer coefficients e_i. *The Buchsbaum-Rim multiplicity of* F/M, denoted by $e(F/M)$, is now defined to be the leading coefficient e_0. The other coefficient e_i is called *the Buchsbaum-Rim coefficient of* F/M, and is denoted by $e_i(F/M)$ for $i = 1, 2, \ldots, d+r-1$.

Buchsbaum and Rim also introduced in [?] the notion of a parameter module (matrix), which generalizes the notion of a parameter ideal (system of parameters). The module N in F is said to be *a parameter module in* F, if the following three conditions are satisfied: (i) F/N has finite length, (ii) $N \subseteq \mathfrak{m}F$, and (iii) the minimal number of generators of N is just $d + r - 1$.

With this notation, the purpose of this talk is to prove the following:

Theorem 1. *Let $(A, \mathfrak{m})$ be a Noetherian local ring of dimension $d > 0$. Then, for any rank $r > 0$ and any integer $\nu \geq 0$, the inequality*

$$\ell_A(S_{\nu+1}/N^{\nu+1}) \geq e(F/N)\binom{\nu+d+r-1}{d+r-1}$$

holds true for every parameter module N in $F = A^r$. Moreover, the ring A is Cohen-Macaulay, if the equality $\ell_A(S_{\nu+1}/N^{\nu+1}) = e(F/N)\binom{\nu+d+r-1}{d+r-1}$ holds for some integer $\nu \geq 0$ and some parameter module N in F.

This is a generalization of the result in [?]. Moreover, it seems that this result contains some new information even in the ideal case. Indeed, as a direct consequence of the inequality in Theorem 1, we have that $e_1(F/N) \leq 0$ hold true for any parameter module N in F. This is a generalization of the recent result of Mandal and Verma that $e_1(A/Q) \leq 0$ for any parameter ideal Q in A. Our proof based on the inequality in Theorem 1 is completely different from theirs and is considerably more simpler.

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Extension of the Matlis duality to a filtered Noetherian ring

Noritsugu Kameyama

We will give a duality over a certain filtered ring which includes, for example, Iwasawa algebras. We define a filtration as follows [?]. A family $\mathcal{F} = \{F_p : p \in \mathbb{N}\}$ of additive subgroups of a ring Λ is called a *filtration*, if (i) $1 \in \mathcal{F}_0\Lambda$, (ii) $\mathcal{F}_p\Lambda \subset \mathcal{F}_{p+1}\Lambda$, (iii) $(\mathcal{F}_p\Lambda)(\mathcal{F}_q\Lambda) \subset \mathcal{F}_{p+q}\Lambda$, (iv) $\Lambda = \bigcup_{p\in\mathbb{N}} \mathcal{F}_p\Lambda$. Let M be a (left) Λ-module. A family $\mathcal{F} = \{\mathcal{F}_pM : p \in \mathbb{Z}\}$ of additive subgroups of M is called a *filtration* of M, if (i) $\mathcal{F}_pM \subset \mathcal{F}_{p+1}M$, (ii) $\mathcal{F}_{-p}M = 0$ for $p >> 0$, (iii) $(\mathcal{F}_p\Lambda)(\mathcal{F}_qM) \subset \mathcal{F}_{p+q}M$, (iv) $M = \bigcup_{p\in\mathbb{Z}} \mathcal{F}_pM$.

Our setting is as follows. Let Λ be a left and right Noetherian filtered ring with a Zariskian filtration $F\Lambda = \{F_i\Lambda\}_{i\in\mathbb{Z}}$ such that

(a1) $H_i = F_i\Lambda$ is an ideal of Λ for every $i \in \mathbb{Z}$,
(a2) Λ is complete with respect to $F\Lambda$,
(a3) Λ/H_i is of finite length as a right and left Λ-module for every $i \in \mathbb{Z}$.

Let $(R, \mathfrak{m}, k)$ be a commutative local Noetherian ring and Λ an R-algebra which is finitely generated as an R-algebra. We consider that R is a subring of Λ via a structure map $R \to \Lambda$. Put $I_i := R \cap H_i (i \in \mathbb{Z})$ and $FR = \{I_i\} (i \in \mathbb{Z})$. Then FR is a filtration of R. We assume further that

(b1) R is complete with respect to FR,
(b2) R/I_i is a finite length R-module for every $i \in \mathbb{Z}$,
(b3) $\mathfrak{m}^n$ is open for all $n > 0$, i.e., $\mathfrak{m}^n \supset I_i$ for some $i \in \mathbb{Z}$,
(b4) Λ/H_i is a module-finite R/I_i-algebra for every $i \in \mathbb{Z}$, i.e., Λ/H_i is a finitely generated R/I_i-module.

Let $E := E_R(k)$ be an injective hull of k as an R-module. We put $(-)^\vee := \mathrm{HOM}_R(-, E)$, $(-)' := \mathrm{Hom}_R(-, E)$. $(-)'$ induces usual Matlis Duality. Let M be a filtered Λ-module with a filtration $FM = \{F_iM\}_{i\in\mathbb{Z}}$. We call M *pseudocompact*, if $M \cong \varprojlim M/F_iM$, that is, M is complete and $H_iM \subset F_iM$ for every $i \in \mathbb{Z}$. Dually, a filtered Λ-module N with a filtration $FN = \{F_iN\}_{i\in\mathbb{Z}}$ is called *copseudocompact*, if $N \cong \varinjlim F_iN$ and $H_{-i}F_iN = 0$ for every $i \in \mathbb{Z}$.

Let $\mathcal{F}_\Lambda$ be a category such that:

Objects: all filtered Λ-modules,
Morphisms: $\mathrm{HOM}_\Lambda(M, N)$ for $M, N \in \mathcal{F}_\Lambda$.

Here, we put $F_p\mathrm{HOM}_\Lambda(M, N) = \{f \in \mathrm{Hom}_R(M, N)\} \mid f(F_iM) \subset F_{i+p}N$ for all $i \in \mathbb{Z}\}$ and $\mathrm{HOM}_\Lambda(M, N) = \bigcup_{p\in\mathbb{Z}} F_p\mathrm{HOM}_\Lambda(M, N)$. Let $\mathcal{C}$ be a full subcategory of $\mathcal{F}_\Lambda$ consisting of all finitely generated pseudocompact Λ-modules, and $\mathcal{D}$ a full subcategory of $\mathcal{F}_\Lambda$ consisting of all finitely cogenerated copseudocompact Λ-modules. The main result is the following:

Theorem *The functor $(-)^\vee$ gives a duality between the categories $\mathcal{C}$ and $\mathcal{D}$.*

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Dualizing complex of the Stanley ring associated with a simplicial poset

Kohji Yanagawa

A poset (partially ordered set) P is called *simplicial*, if it has the smallest element $\hat{0}$, and the interval $[\hat{0}, x]$ is isomorphic to a boolean algebra (i.e., the power set of a finite set with order given by inclusion) for all $x \in P$. The face poset of a finite simplicial complex is clearly a simplicial poset. Similarly, any simplicial poset P is given by a regular cell complex $\Gamma(P)$. For example, if two d-simplices are glued along their boundaries, then it is not a simplicial complex, but gives a simplicial poset.

As is well-known, the Stanley-Reisner ring of a finite simplicial complex is a powerful tool for combinatorics. Generalizing this idea, Stanley ([**?**]) constructed a graded commutative ring A_P from a simplicial poset P.

M. Masuda and his coworkers studied A_P with a view from toric topology, since the equivariant cohomology ring of a torus manifold is of the form A_P (cf. [**?**]). In this talk, we give the following result by another approach.

(1) We describe a dualizing complex of A_P in a concise way.

(2) The theory of *squarefree modules* (see, for example, [**?**]) can be developed over A_P. To a squarefree module M over A_P, we can assign the constructible sheaf M^+ on $\Gamma(P)$. The dualizing complex of A_P is essentially a complex of squarefree modules, and its sheafification is Verdier's dualizing complex of (the underlying space of) $\Gamma(P)$, which give a Poincaré-Verdier duality.

(3) The Cohen-Macaulay (resp. Gorenstein*, Buchsbaum) property of A_P is a topological property of the underlying space of $\Gamma(P)$.

Some part of (3) is known result. But our argument is more systematic and direct (I believe).

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THICK SUBCATEGORIES OF THE STABLE CATEGORY OF COHEN-MACAULAY MODULES

Ryo Takahashi

Various classification theorems of thick subcategories (i.e., full triangulated subcategories closed under direct summands) of a triangulated category have been obtained in many areas of mathematics. In this talk, as a higher dimensional version of the classification theorem of thick subcategories of the stable category of finitely generated representations of a finite p-group due to Benson, Carlson and Rickard [**?**], we consider classifying thick subcategories of the stable category of Cohen-Macaulay modules over a (commutative) Gorenstein local ring.

Let R be a Gorenstein local ring. We denote by $\operatorname{mod} R$ the category of finitely generated R-modules, by $\mathrm{CM}(R)$ the full subcategory of $\operatorname{mod} R$ consisting of all Cohen-Macaulay R-modules, and by $\underline{\mathrm{CM}}(R)$ the stable category of $\mathrm{CM}(R)$. Then it is well-known that $\underline{\mathrm{CM}}(R)$ is a triangulated category. Let $\operatorname{Spec} R$ denote the prime ideal spectrum of R, that is, the set of prime ideals of R. Let $\operatorname{Sing} R$ denote the singular locus of R, that is, the set of prime ideals $\mathfrak{p}$ of R such that the local ring $R_{\mathfrak{p}}$ is not regular. A subset Φ of $\operatorname{Spec} R$ is called specialization-closed if $\mathfrak{p} \in \Phi$ and $\mathfrak{p} \subseteq \mathfrak{q} \in \operatorname{Spec} R$ imply $\mathfrak{q} \in \Phi$. A resolving subcategory is by definition a full subcategory closed under direct summands, extensions and syzygies.

Recall that a (local) hypersurface is defined to be a ring isomorphic to S/fS for a regular local ring S and an element f of S. A hypersurface is always a Gorenstein local ring. The main result of this talk is the following theorem.

Theorem 1. *Let R be a hypersurface. Then there are one-to-one correspondences among the following three sets:*

- *the set of nonempty thick subcategories of* $\underline{\mathrm{CM}}(R)$*,*
- *the set of specialization-closed subsets of* $\operatorname{Spec} R$ *contained in* $\operatorname{Sing} R$*,*
- *the set of resolving subcategories of* $\operatorname{mod} R$ *contained in* $\mathrm{CM}(R)$*.*

The bijective maps among the three sets are explicitly given in this talk.

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Ampleness of two-sided tilting complexes and Fano algebras

Hiroyuki Minamoto

From the view point of noncommutative algebraic geometry, two-sided tilting complexes are an analogue of line bundles. In algebraic geometry for line bundles, ampleness is an important notion. In this talk we introduce a notion of ampleness for two-sided tilting complexes over finite dimensional algebras of finite global dimension. A two-sided tilting complex σ is called very ample if $\mathrm{H}^i(\sigma) = 0$ for $i \geq 1$ and $\mathrm{H}^i(\sigma^n) = 0$ for $i \neq 0$ and $n \gg 0$. We justify this definition by using the theory of noncommutative projective schemes due to Artin-Zhang [**?**] and Polishchuk [**?**]. In the theory of noncommutative projective schemes, for a graded coherent ring R over k, we attach an imaginary geometric object $\mathrm{proj}R = \big(\mathrm{qcoh}R, \overline{R}, (1)\big)$. An abelian category $\mathrm{qcoh}R$ is considered as the category of coherent sheaves on $\mathrm{proj}R$. If σ is a very ample tilting complex over A, then the tensor algebra $T := T_A(\mathrm{H}^0(\sigma))$ of $\mathrm{H}^0(\sigma)$ over A is a graded connected coherent ring over A and there is a natural equivalence of triangulated categories $D^b(\text{mod-}A) \simeq D^b(\mathrm{qcoh}T)$.

From the view point of noncommutative algebraic geometry, Serre functors are considered as shifted canonical bundles. We define Fano algebras by the anti-ampleness of shifted Serre functor. Fano algebras have remarkable properties. Some classes of algebras studied before are Fano. For example a path algebra of a quiver of infinite representation type is Fano. Moreover we can characterize the representation type of a quiver from the noncommutative algebro-geometric point of view.

Theorem 1. *Let Q be a finite quiver without oriented cycles. Then Q has finite representation type if and only if its path algebra kQ is fractionally Calabi-Yau. Q has infinite representation type if and only if kQ is Fano.*

There is good supply of new Fano algebra. We can construct Fano algebras from AS-regular algebras (without finiteness of Gelfand-Kirillov dimension). This is a joint work with I.Mori.

Theorem 2 (M-Mori)**.** *Let A be an AS-regular algebra of gldim$A = d \geq 1$ and Gorenstein parameter e. Then the finite dimensional algebra*

$$F = \begin{pmatrix} A_0 & A_1 & \cdots & A_{e-1} \\ 0 & A_0 & \cdots & A_{e-2} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & A_0 \end{pmatrix}$$

is an extremely Fano algebra of dimension $d-1$.

As a byproduct, we prove the following corollary, which was conjectured by A.Bondal.

Corollary 3. *AS-regular algebras are graded coherent.*

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The Projective Scheme of the Blow-up Ring

Fred van Oystaeyen

Noncommutative Geometry has become a many-headed animal nowadays. In our version of noncommutative algebraic geometry we tried to actually keep the idea that there is a noncommutative space really present and not just virtual while one actually studies noncommutative algebras as the assumed ring of functions on the virtual space. This leads to noncommutative spaces (in fact endowed with noncommutative topologies) allowing a noncommutative version of the Serre global section theorem in scheme theory. The topological space is replaced by the set (lattice) of localizations on some nice Grothendieck category like modules or graded modules in the projective case, where intersection of opens corresponds to compositions of localization functors. The latter are exact pretorsion radicals but not localizations corresponding to torsion theories or Serre quotient categories. Viewing algebras as given by generators and relations they appear as quotients of the free algebra hence equipped with a standard filtration. The Rees ring of the filtration corresponds to a projective noncommutative scheme that acts as the projectivization of the affine noncommutative variety associated to the algebra. The condition for some noncommutative geometry to work for some noncommutative algebra is the so called "schematic" condition. We study the transfer of this condition from the associated graded algebra of nice (Zariskian) filtered ring to the blow-up or Rees ring of it. We establish the lifting of the projective schematic condition and obtain a statement about noncommutative geometries expressing how Proj of the Rees ring contains Spec of the filtered ring as a closed substructure, the latter being the part at "infinity" for the affine noncommutative variety

Then we also establish a noncommutative version of Serre's global section theorem relating noncommutative geometry to the quotient category of finitely generated graded modules modulo those of finite length. All kinds of dimensions and regularity conditions can then be studied algebraically and interpreted geometrically.

A list of relatively recent examples is provided.

Derived equivalences for endomorphism rings

Hiroki Abe and Mitsuo Hoshino

In [?], we have shown the following. Let $0 \to Y \xrightarrow{\mu} E \xrightarrow{\varepsilon} X \to 0$ be an exact sequence in an abelian category $\mathcal{A}$ and P an object of $\mathcal{A}$. Assume that $E \in \mathrm{add}(P)$ and that both $\mathrm{Hom}_{\mathcal{A}}(P, \varepsilon)$ and $\mathrm{Hom}_{\mathcal{A}}(\mu, P)$ are epic. Then $\mathrm{End}_{\mathcal{A}}(X \oplus P)$ and $\mathrm{End}_{\mathcal{A}}(Y \oplus P)$ are derived equivalent to each other. In this talk, we will provide several applications of this fact.

Let A be a representation-finite artin algerba with $M_1, \cdots, M_m \in$ mod-A a complete set of nonisomorphic indecomposable modules and $I = \{1, \cdots, m\}$. We assume that $m \geq 2$, i.e., A is not simple. Set

$$M = \bigoplus_{i \in I} M_i, \quad \Lambda = \mathrm{End}_A(M).$$

For each indecomposable module $X \in$ mod-A, since there exists a unique $i_X \in I$ such that $X \cong M_{i_X}$, we set $I(X) = I \setminus \{i_X\}$ and set

$$M_X = \bigoplus_{i \in I(X)} M_i, \quad \Lambda_X = \mathrm{End}_A(M_X).$$

We will show that if X is projective and injective then Λ_X is also an Auslander algebra, that if X is nonprojective then Λ_X and $\Lambda_{\tau X}$ are derived equivalent to each other, where $\tau = D\mathrm{Tr}$, and that if X is noninjective then Λ_X and $\Lambda_{\tau^{-1}X}$ are derived equivalent to each other, where $\tau^{-1} = \mathrm{Tr}D$.

Let A be a ring, $P \in$ Mod-A and $0 \to Y \xrightarrow{\mu} E \xrightarrow{\varepsilon} X \to 0$ an exact sequence in Mod-A. Assume that $E \in \mathrm{add}(P)$ and that both $\mathrm{Hom}_A(P, \varepsilon)$ and $\mathrm{Hom}_A(\mu, P)$ are epic. We will show that $X \oplus P$ is a tilting module if and only if so is $Y \oplus P$, and that if $X \oplus P$ is a classical tilting module, then so is $Y \oplus P$.

Let A be a Noether algebra and $X \in$ mod-A. Assume that there exists $T \in$ mod-A such that $X \oplus T \in$ mod-A a tilting module. We will show the following.

(1) If there exists an epimorphism of the form $f : T^{(l)} \to X$, then there exists an epimorphism $\varepsilon : T^{(r)} \to X$ such that $\mathrm{Ker}\ \varepsilon \oplus T$ is a tilting module. In particular, if $X \oplus T$ is a classical tilting module, then so is $\mathrm{Ker}\ \varepsilon \oplus T$.

(2) If there exists a monomorphism of the form $g : X \to T^{(l)}$, then there exists a monomorphism $\mu : X \to T^{(r)}$ such that $\mathrm{Cok}\ \mu \oplus T$ is a tilting module.

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The classification of tilting modules over Harada algebras

Kota Yamaura

In the 1980s, Harada [?] introduced a new class of algebras now called Harada algebras. We recall left Harada algebras from a structural point of view as follows.

Definition. Let R be a basic finite dimensional algebra over a field K and $\mathrm{Pi}(R)$ be a complete set of orthogonal primitive idempotents of R. We call R a *left Harada algebra* if $\mathrm{Pi}(R)$ can be arranged such that $\mathrm{Pi}(R) = \{e_{ij}\}_{i=1}^{m}{}_{,j=1}^{n_i}$ where

(1) $e_{i1}R$ is an injective R-module for any $i = 1, \cdots, m$,
(2) $e_{ij}R \simeq e_{i,j-1}J(R)$ for any $i = 1, \cdots, m$, $j = 2, \cdots, n_i$.

Here $J(R)$ is the Jacobson radical of R.

In my talk, we give the classification of tilting modules over left Harada algebras. Tilting modules provide us a powerful tool in the representation theory of algebras (we refer to [?, ChapterVI]). We denote by mod$R$ the category of finitely generated module over an algebra R.

Definition. Let R be a finite dimensional algebra over a field K. We call $T \in \mathrm{mod}R$ a *tilting module* if T satisfies the following conditions.

(1) $\mathrm{proj.dim}T \leq 1$.
(2) $\mathrm{Ext}_R^1(T,T) = 0$.
(3) There exists an exact sequence $0 \longrightarrow R_R \longrightarrow T_0 \longrightarrow T_1 \longrightarrow 0$ where T_0 and T_1 are direct summands of a direct sum of some copies of T.

A tilting module T over an algebra R induces two category equivalences between certain full subcategories of mod$R$ and of $\mathrm{mod}(\mathrm{End}_R(T))$. As a consequence of these category equivalences, some problems about R can be shifted to those of $\mathrm{End}_R(T)$ (e.g. finiteness of global dimension). By this reason, finding a classification of tilting modules over a given algebra is an important problem in representation theory.

Now we present the main theorem of my talk. Let R be a left Harada algebra over an algebraically closed field K as in the above definition. We denote by $\mathrm{tilt}(R)$ the set of isomorphism classes of basic tilting R-modules and by $\mathrm{T}_n(K)$ the $n \times n$ upper triangular matrix algebra over K.

The following is our main theorem which asserts that tilting R-modules are described by tilting modules over a direct product of algebras of the form $\mathrm{T}_n(K)$.

Theorem [?]. There exists a bijection

$$\mathrm{tilt}(R) \longrightarrow \mathrm{tilt}(\mathrm{T}_{n_1}(K)) \times \mathrm{tilt}(\mathrm{T}_{n_2}(K)) \times \cdots \times \mathrm{tilt}(\mathrm{T}_{n_m}(K)).$$

It is known that tilting $\mathrm{T}_n(K)$-modules are described combinatorially by using non-crossing partitions of a regular $(n+2)$-polygon. This fact and our theorem allow us to classify tilting modules over a given Harada algebra.

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The Stickelberger relation and Loewy series of group algebras $\mathrm{Map}(\mathbb{F}_q, \mathbb{F}_q)$

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In this talk, I present a proof of the Stickelberger relation (see [1]) using Loewy series of a group algebra $\mathrm{Map}(\mathbb{F}_q, \mathbb{F}_q)$ of the additive group of a finite field $\mathbb{F}_q$ (see [2], [3] and [4]). This relation is essential in a proof of the Eisenstein reciprocity law (see [1]).

I also present the next result by a special case in this law, namely, the law of cubic reciprocity.

If $q^2 + q + 1$ *divides* $3^q - 1$ *for a prime* $q > 3$, *then* 9 *divides* $q + 1$.

This is a slight contribution to the Feit-Thompson conjecture for a prime 3 (see [5]).

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Homological conjectures and radical-full extensions

Changchang Xi

This is a series of two lectures. In these lectures, we shall consider the finitistic dimension conjecture and the strong no loop conjecture (and related other homological conjectures). We approach these conjectures by the so-called radical-full extensions, and reduce the verification of the conjectures to the following question: Suppose that $B \subseteq A$ is a radical-full extension such that the radical of B is a left ideal in A, and that these conjectures are true for A, is it possible to prove the conjectures for B ? We shall provide basic definitions and examples, and report results in this direction. Among them are the following two theorems.

Definition 1. An extension of two Artin algebras is a pair of Artin algebras B and A such that B is a subalgebra of A and that A and B have the same identity. In this case, the extension is denoted by $B \subseteq A$. If, in addition, the Jacobson radical of A is generated as a right ideal by the Jacobson radical of B, we say that the extension $B \subseteq A$ is radical-full.

Theorem 2. *Let $B \subseteq A$ be a radical-full extension of Artin algebras. Suppose that the radical of B is a left ideal in A. If the global dimension of A is at most 4, then the finitistic dimension conjecture is true for B.*

Theorem 3. *Let $B \subseteq A$ be a radical-full extension of Artin algebras. Suppose that the radical of B is a left ideal in A. If the global dimension of A is at most 2, then the strong no loop conjecture is true for B.*

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Almost comparability and related comparabilities in von Neumann regular rings

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Abstract. There are many comparabilities in (von Neumann) regular rings: general comparability, the comparability axiom, s-comparability, weak comparability, almost comparability etc.. We mainly treat almost comparability in the talk. Here, we recall the definition of almost comparability for regular rings. A regular ring R satisfies *almost comparability* if for each $x, y \in R$ either $xR \lesssim_a yR$ or $yR \lesssim_a xR$, where for any principal right ideals A and B of R, $A \lesssim_a B$ means that A is subisomorphic to $B \oplus C$ for all nonzero principal right ideals C of R. The notion of almost comparability for regular rings was first introduced by Ara and Goodearl [1], for giving an alternative proof of the epoch-making O'Meara's Theorem [4] that "directly finite simple regular rings with weak comparability are unit-regular". After that the study of almost comparability for simple regular rings was continued by Ara et al. [2], and moreover Ara et al. [3] touched the relation between s-comparability and almost comparability.

In the talk, we investigate the cancellation property and the unperforation property for regular rings with almost comparability or related comparabilities.

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The Clebsch-Gordan problem for quiver representations

Martin Herschend

Given a quiver Q (i.e. an oriented graph) one can consider its path algebra A. The finite dimensional modules over A are identified with representations of Q. Such a representation consist of vector spaces, one for each vertex in Q and linear maps corresponding to the arrows of Q.

Given two representations of Q one can construct a new representation by, at each vertex taking the tensor product of the corresponding vector spaces and similarly taking the tensor product of the linear maps corresponding to the arrows.

This tensor product of quiver representations gives rise to the following Clebsch-Gordan problem: For each two indecomposable representations of Q find the decomposition of their tensor product into indecomposables.

In my talk I will survey the known solutions to this problem with focus on string algebras, which are a fairly large class of algebras given by quivers with relations.