The 43rd Symposium on Ring Theory and Representation Theory

ABSTRACT

Naruto University of Education

September 10 – 12, 2010

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High order centers and left differential operators

Hiroaki Komatsu

Let A be an algebra over a commutative ring K. We shall denote by $\mathfrak{M}_K(A)$ the category of bimdules over a K-algebra A. If $M \in \mathfrak{M}_K(A)$, for $u \in M$ and $a \in A$, we use the notation $[u, a] = ua - au$. For $U \subseteq M$, we set

$$[U, A] = \{[u, a] \mid u \in U,\ a \in A\}.$$

Furthermore, we set $[U, A]_0 = U$ and we set $[U, A]_{n+1} = [[U, A]_n,\ A]$ for nonnegative integers n. If $U = \{u\}$ is a singleton, then we use the notations $[u, A]$ and $[u, A]_n$ instead of $[\{u\},\ A]$ and $[\{u\},\ A]_n$, respectively.

Let $\mathbf{A} = (A_1, \ldots, A_n)$ be an n-tuple of algebras over a commutative ring K and let $\hat{\mathbf{A}}$ denote the tensor product algebra $A_1 \otimes_K \cdots \otimes_K A_n$. Let $M \in \mathfrak{M}_K(\hat{\mathbf{A}})$ and let $\mathbf{p} = (p_1, \ldots, p_n)$ be an n-tuple of nonnegative integers. Since $M \in \mathfrak{M}_K(A_i)$ via the canonical algebra homomorphism $A_i \to \hat{\mathbf{A}}$, we can consider the set

$$\mathcal{C}^{\mathbf{p}}_{\mathbf{A}}(M) = \{u \in M \mid [\cdots[[u,\ A_1]_{p_1},\ A_2]_{p_2},\ \cdots,\ A_n]_{p_n} = 0\},$$

which is called the *center* of M of *type* $\mathbf{p}$.

Theorem 1. *There exists $\mathcal{J}^{\mathbf{p}}_{\mathbf{A}} \in \mathfrak{M}_K(\hat{\mathbf{A}})$ and $j^{\mathbf{p}}_{\mathbf{A}} \in \mathcal{J}^{\mathbf{p}}_{\mathbf{A}}$ such that, for each $M \in \mathfrak{M}_K(\hat{\mathbf{A}})$, the K-linear mapping*

$$\mathrm{Hom}_{\mathfrak{M}_K(\hat{\mathbf{A}})}(\mathcal{J}^{\mathbf{p}}_{\mathbf{A}},\ M) \ni \varphi \mapsto \varphi(j^{\mathbf{p}}_{\mathbf{A}}) \in \mathcal{C}^{\mathbf{p}}_{\mathbf{A}}(M)$$

is an isomorphism.

We shall introduce some results on $\mathcal{J}^{\mathbf{p}}_{\mathbf{A}}$. One of the results generalizes the Sweedler's theory on left differential operators. Let M and N be left $\hat{\mathbf{A}}$-modules. Then $\mathrm{Hom}_K(M, N)$ belongs to $\mathfrak{M}_K(\hat{\mathbf{A}})$ by the usual manner. We set

$$\mathcal{D}^{\mathbf{p}}_{\mathbf{A}}(M, N) = \mathcal{C}^{\mathbf{p}}_{\mathbf{A}}\big(\mathrm{Hom}_K(M, N)\big).$$

An element in $\mathcal{D}^{\mathbf{p}}_{\mathbf{A}}(M, N)$ is called a *left differential operator* of type $\mathbf{p}$.

In case of $n = 1$, $\mathcal{D}^{\mathbf{p}}_{\mathbf{A}}(M, N)$ coincides with the set of left differential operators of order $p_1 - 1$ in the sense of Sweedler [2].

In case of $\mathbf{A} = (A,\ A^\circ)$, where A° is the opposite algebra of A, the category of left $\hat{\mathbf{A}}$-modules coincides with $\mathfrak{M}_K(A)$. For $M \in \mathfrak{M}_K(A)$, a K-linear mapping $f : A \to M$ is a left differential operator of type $(1, 1)$ if and only if f satisfies $f(xy) = xf(y) + f(x)y - xf(1)y$ for all $x,\ y \in A$. In particular, all K-derivations of A to M are left differential operators of type $(1, 1)$.

The next theorem generalizes [2, Theorems 1.17 and 1.18] and is an immediate consequence of Theorem 1.

Theorem 2. *For each left $\hat{\mathbf{A}}$-modules M and N, the K-linear mapping*

$$\mathrm{Hom}_{\hat{\mathbf{A}}}(\mathcal{J}^{\mathbf{p}}_{\mathbf{A}} \otimes_{\hat{\mathbf{A}}} M,\ N) \ni \varphi \mapsto \varphi(j^{\mathbf{p}}_{\mathbf{A}} \otimes -) \in \mathcal{D}^{\mathbf{p}}_{\mathbf{A}}(M, N)$$

is an isomorphism.

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Faculty of Computer Science and System Engineering
Okayama Prefectural University
Soja, Okayama 719-1197 JAPAN
Email: komatsu@cse.oka-pu.ac.jp

Prime factor rings of Ore extensions over a commutative Dedekind domain

Yunxia Wang and Hidetoshi Marubayashi

Let $R = D[x; \sigma]$ be a skew polynomial ring over a commutative Dedekind domain D and let P be a minimal prime ideal of R, where σ is an automorphism of D.

There are two different types of P, namely, either $P = \mathfrak{p}[x; \sigma]$ or $P = P' \cap R$, where $\mathfrak{p}$ is a σ-prime ideal of D, P' is a prime ideal of $K[x; \sigma]$ and K is the quotient field of D. In the first case R/P is a hereditary prime ring and in the second case, it is shown that R/P is a hereditary prime ring if and only if $M^2 \not\supseteq P$ for any maximal ideal M of R.

We give some examples of minimal prime ideals such that the factor rings are not hereditary or hereditary or Dedekind, respectively.

In the case $R = D[x; \sigma, \delta]$, an Ore extension, where δ is a left σ-derivation of D, we roughly speak of any prime ideal P of R which is not complete, by using Goodearl's classification.

Hohai University

Tokushima Bunri University

Graded Morita equivalences for AS-regular algebras

Kenta Ueyama

Let $T(V)$ be the tensor algebra on V over an algebraically closed field k where V is a finite dimensional vector space. We say that A is a quadratic algebra if A is a graded algebra of the form $T(V)/(R)$ where $R \subseteq V \otimes_k V$ is a subspace and (R) is the ideal of $T(V)$ generated by R. For a quadratic algebra $A = T(V)/(R)$, we define

$$\Gamma_2 := \{(p, q) \in \mathbb{P}(V^*) \times \mathbb{P}(V^*) \mid f(p, q) = 0 \text{ for all } f \in R\}.$$

Definition 1. [3] A quadratic algebra $A = T(V)/(R)$ is called geometric if there exists a geometric pair (E, σ) where $E \subseteq \mathbb{P}(V^*)$ is a closed subscheme and σ is an automorphism of E such that

(**G1**) $\Gamma_2 = \{(p, \sigma(p)) \in \mathbb{P}(V^*) \times \mathbb{P}(V^*) \mid p \in E\}$, and
(**G2**) $R = \{f \in V \otimes_k V \mid f(p, \sigma(p)) = 0 \text{ for all } p \in E\}$.

If A satisfies the condition (**G1**), then A determines a geometric pair (E, σ). If A satisfies the condition (**G2**), then A is determined by a geometric pair (E, σ), so we write $A = \mathcal{A}(E, \sigma)$.

Artin, Tate and Van den Bergh [1] classified 3-dimensional AS-regular algebras generated in degree 1 using geometric pairs. Classifying 4-dimensional AS-regular algebras is one of the most active projects in noncommutative algebraic geometry.

Let A be a d-dimensional AS-Gorenstein algebra, and $\mathfrak{m} := A_{\geq 1}$ the unique maximal homogeneous ideal of A. We define the graded A-A bimodule ω_A by

$$\omega_A := \mathrm{H}^d_{\mathfrak{m}}(A)^* = \underline{\mathrm{Hom}}_k(\lim_{n\to\infty} \underline{\mathrm{Ext}}^d_A(A/A_{\geq n}, A), k).$$

Then there exist a graded algebra automorphism ν called the generalized Nakayama automorphism, and an integer ℓ called the Gorenstein parameter, such that $\omega_A \cong {}_{\nu}A(-\ell)$ as graded A-A bimodules, where ${}_{\nu}A$ is the graded A-A bimodule defined by ${}_{\nu}A = A$ as a graded vector space with a new action $a * x * b := \nu(a)xb$ (see [4]).

The main result of this talk is the following theorem which gives a new criterion of graded Morita equivalences for geometric AS-regular algebras.

Theorem 2. *Let $A = \mathcal{A}(E, \sigma), A' = \mathcal{A}(E', \sigma')$ be geometric AS-regular algebras of Gorenstein parameters ℓ, ℓ' with the generalized Nakayama automorphisms ν, ν', and $\overline{A} = \mathcal{A}(E, \nu^*\sigma^\ell), \overline{A'} = \mathcal{A}(E, (\nu')^*(\sigma')^{\ell'})$. Then*

$$\mathrm{GrMod}\, A \cong \mathrm{GrMod}\, A' \Longrightarrow \overline{A} \cong \overline{A'}.$$

In particular, if A, A' are generic geometric 3-dimensional AS-regular algebras, then

$$\mathrm{GrMod}\, A \cong \mathrm{GrMod}\, A' \Longleftrightarrow \overline{A} \cong \overline{A'}.$$

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Department of Mathematics
Graduate School of Science
Shizuoka University
Shizuoka 422-8529 JAPAN
Email: r0930001@ipc.shizuoka.ac.jp

The first Hilbert coefficients of parameters

Kazuho Ozeki

This is based on a joint work with L. Ghezzi, S. Goto, J. Hong, T. T. Phuong, and W. V. Vasconcelos. To state the results, let A be a commutative Noetherian local ring with maximal ideal $\mathfrak{m}$ and $d = \dim A > 0$. Let $\ell_A(M)$ denote, for an A-module M, the length of M. Then for each $\mathfrak{m}$-primary ideal I in A we have integers $\{\mathrm{e}_I^i(A)\}_{0 \le i \le d}$ such that the equality

$$\ell_A(A/I^{n+1}) = \mathrm{e}_I^0(A)\binom{n+d}{d} - \mathrm{e}_I^1(A)\binom{n+d-1}{d-1} + \cdots + (-1)^d \mathrm{e}_I^d(A)$$

holds true for all $n \gg 0$, which we call the Hilbert coefficients of A with respect to I. We say that A is unmixed, if $\dim \widehat{A}/\mathfrak{p} = d$ for every $\mathfrak{p} \in \operatorname{Ass} \widehat{A}$, where $\widehat{A}$ denotes the $\mathfrak{m}$-adic completion of A. With this notation Wolmer V. Vasconcelos posed, exploring the vanishing of $\mathrm{e}_Q^1(A)$ for parameter ideals Q, in his lecture at the conference in Yokohama of March, 2008 the following conjecture.

Conjecture ([2, 5]) *Assume that A is unmixed. Then A is a Cohen-Macaulay local ring, once $\mathrm{e}_Q^1(A) = 0$ for some parameter ideal Q of A.*

In my talk I shall settle this conjecture affirmatively as follows. We note that $\mathrm{e}_Q^1(A) \le 0$ for every parameter ideal Q in arbitrary commutative Noetherian local rings A with $\dim A > 0$ (cf. [1, 4]).

Theorem 1 ([1])**.** *Suppose that A is unmixed. Then the following four conditions are equivalent to each other.*

(1) *A is a Cohen-Macaulay local ring.*
(2) *$\mathrm{e}_I^1(A) \ge 0$ for every $\mathfrak{m}$-primary ideal I in A.*
(3) *$\mathrm{e}_Q^1(A) \ge 0$ for some parameter ideal Q in A.*
(4) *$\mathrm{e}_Q^1(A) = 0$ for some parameter ideal Q in A.*

As a direct consequence of the result, one gets that, for a given commutative Noetherian local ring A with $d = \dim A > 0$, $\mathrm{e}_Q^1(A) = 0$ for every parameter ideal Q in A, once $\mathrm{e}_Q^1(A) = 0$ for some parameter ideal Q.

The second purpose of this paper is to study when the set

$$\Lambda = \{\mathrm{e}_Q^1(A) \mid Q \text{ is a parameter ideal in } A\}$$

is finite, or a singleton. I shall show that the local cohomology modules $\{\mathrm{H}_\mathfrak{m}^i(A)\}_{i \ne d}$ of A with respect to $\mathfrak{m}$ are all finitely generated, if the set Λ is finite and A is unmixed. It seems natural to conjecture that A is a Buchsbaum local ring, if A is unmixed and the set Λ is a singleton, which now the following theorem settles affirmatively.

Theorem 2 ([3])**.** *Suppose that $d = \dim A \ge 2$ and A is unmixed. Then the following two conditions are equivalent to each other.*

(1) *A is a Buchsbaum local ring.*
(2) *The first Hilbert coefficients $\mathrm{e}_Q^1(A)$ of A are constant and independent of the choice of parameter ideals Q in A.*

When this is the case, one has the equality

$$\mathrm{e}_Q^1(A) = -\sum_{i=1}^{d-1} \binom{d-2}{i-1} h^i(A)$$

for every parameter ideal Q in A, where $h^i(A)$ denotes for each $1 \le i \le d-1$ the length of the i-th local cohomology module $\mathrm{H}_\mathfrak{m}^i(A)$ of A with respect to $\mathfrak{m}$.

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Meiji Institute for Advanced Study of Mathematical Sciences
Meiji University
1-1-1 Higashi-mita, Tama-ku, Kawasaki 214-8571, Japan
Email: kozeki@math.meiji.ac.jp

Modules left orthogonal to modules of finite projective dimension

Tokuji Araya, Kei-ichiro Iima, Ryo Takahashi

Throughout this talk, let R be a commutative noetherian local ring with maximal ideal $\mathfrak{m}$ and residue field k. All modules considered in this talk are assumed to be finitely generated. An R-module C is said to be *semidualizing* if the natural homomorphism $R \to \mathrm{Hom}_R(C, C)$ is an isomorphism and $\mathrm{Ext}^i_R(C, C) = 0$ for all $i > 0$. Various homological dimensions with respect to a fixed semidualizing module C such as C-projective dimension are invented and investigated. Here the *C-projective dimension* of a nonzero R-module M, denoted by $C\text{-proj.dim}_R M$, is defined as the infimum of integers n such that there is an exact sequence of the form

$$0 \to C^{b_n} \to C^{b_{n-1}} \to \cdots \to C^{b_1} \to C^{b_0} \to M \to 0,$$

where each b_i is a positive integer. And an R-module X is said to be *totally C-reflexive* if the natural homomorphism $X \to \mathrm{Hom}_R(\mathrm{Hom}_R(X, C), C)$ is an isomorphism and $\mathrm{Ext}^i_R(X, C) = \mathrm{Ext}^i_R(\mathrm{Hom}_R(X, C), C) = 0$ for all $i > 0$. We denote by $\mathrm{mod}(R)$ the category of finitely generated R-modules, by $\mathcal{G}_C(R)$ the full subcategory of $\mathrm{mod}(R)$ consisting of all totally C-reflexive modules, by $\mathcal{I}(R)$ the full subcategory of $\mathrm{mod}(R)$ consisting of all modules of CI-dimension zero, by $\mathcal{X}_C(R)$ the full subcategory of $\mathrm{mod}(R)$ consisting of all modules X such that $\mathrm{Ext}^1_R(X, M) = 0$ for each module M of finite C-projective dimension. A free module of rank one is a typical example of a semidualizing module. We simply write $\mathcal{G}(R) = \mathcal{G}_R(R)$ and $\mathcal{X}(R) = \mathcal{X}_R(R)$.

Let M be an R-module. Let

$$\cdots \to F_n \xrightarrow{\partial_n} F_{n-1} \to \cdots \to F_1 \xrightarrow{\partial_1} F_0 \to M \to 0$$

be a minimal free resolution of M. We define the *(Auslander) transpose* of M to be the cokernel of the map $\mathrm{Hom}(\partial_1, R) : \mathrm{Hom}(F_0, R) \to \mathrm{Hom}(F_1, R)$, and denote it by $\mathrm{Tr}\, M$. Then the following proposition holds.

Proposition 1. *Let C be a semidualizing R-module. Then*

$$C \otimes_R \mathrm{Tr}(\Omega^{\mathrm{depth}(R)+1}(k)) \in \mathcal{X}_C(R),$$

where $\mathrm{depth}(R) = \inf\{\, i \geq 0 \mid \mathrm{Ext}^i_R(k, R) \neq 0 \,\}$.

The main result in this talk is the theorem below.

Theorem 2. *The following statements hold.*

(1) *The following four conditions are equivalent.*
  (a) *R is regular.*
  (b) *$\mathcal{X}(R) = \mathrm{add}(R)$ holds.*
  (c) *There exist a semidualizing R-module C such that $\mathcal{X}_C(R) = \mathrm{add}(C)$.*
  (d) *There exist a semidualizing R-module C such that $\mathcal{X}_C(R) = \mathrm{add}(R)$.*

(2) *The following two conditions are equivalent.*
  (a) *R is complete intersection.*
  (b) *$\mathcal{X}(R) = \mathcal{I}(R)$ holds.*

(3) *The following two conditions are equivalent.*
  (a) *R is Gorenstein.*
  (b) *$\mathcal{X}(R) = \mathcal{G}(R)$ holds.*

(4) *The following two conditions are equivalent for a fixed semidualizing R-module C.*
  (a) *C is s dualizing module.*
  (b) *$\mathcal{X}_C(R) = \mathcal{G}_C(R)$ holds.*

An R-module M is called a *strong test module for projectivity* if every R-module N with $\mathrm{Ext}^1(N, M) = 0$ is projective. We give here an application of the above theorem.

Corollary 3. *The following seven conditions are equivalent.*

(a) *R is regular.*
(b) *Every R-module of depth at most one is a strong test module for projectivity.*
(c) *Every R-module of depth zero is a strong test module for projectivity.*
(d) *Every R-module of depth zero and of finite projective dimension is a strong test module for projectivity.*
(e) *There exists a strong test R-module for projectivity of depth zero and of finite projective dimension.*
(f) *There exists a strong test R-module for projectivity of finite projective dimension.*
(g) *There exist a semidualizing R-module C and a strong test R-module for projectivity of finite C-projective dimension.*

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Nara University of Education, Nara City, Nara 630-8528, JAPAN
Email: araya@math.okayama-u.ac.jp

Department of Liberal Studies, Nara National College of Technology, Yamatokoriyama, Nara 639-1080, JAPAN
Email: iima@libe.nara-k.ac.jp

Department of Mathematical Sciences, Faculty of Science, Shinshu University, Matsumoto, Nagano 390-8621, JAPAN
Email: takahasi@math.shinshu-u.ac.jp

t-structures and local cohomology functors

Takeshi Yoshizawa

This is a joint work with Yuji Yoshino.

Let R be a commutative noetherian ring. We denote the category of all R-modules by R-Mod, the derived category consisting of all left bounded complexes of R-modules by $\mathcal{D}^+(R\text{-Mod})$ and also denote the set of prime ideals of R by $\mathrm{Spec}(R)$.

A radical functor, or more generally a preradical functor, has its own long history in the theory of categories and functors. The section functor Γ_W with support in specialization closed subset W of $\mathrm{Spec}(R)$ is one of the most important radical functors and basic tools not only for the theory of commutative algebras but also for algebraic geometry. The aim of this talk is to characterize the section functor Γ_W (resp. the right derived functor $\mathbf{R}\Gamma_W$ of Γ_W) as elements of the set of all functors on R-Mod (resp. $\mathcal{D}^+(R\text{-Mod})$).

Let us recall some definitions for functors from the theory of categories.

Definition 1. Let γ be a functor on R-Mod.

(1) A functor γ is called a preradical functor if γ is a subfunctor of identity functor $\mathbf{1}$.
(2) A preradical functor γ is called a radical functor if $\gamma(M/\gamma(M)) = 0$ for every R-module M.
(3) A functor γ is said to preserve injectivity if $\gamma(I)$ is an injective R-module whenever I is an injective R-module.

Example 2. Let W be a subset of $\mathrm{Spec}(R)$. Recall that W is said to be specialization closed (or closed under specialization) if $\mathfrak{p} \in W$ and $\mathfrak{p} \subseteq \mathfrak{q} \in \mathrm{Spec}(R)$ imply $\mathfrak{q} \in W$.

When W is specialization closed, we can define the section functor Γ_W with support in W as

$$\Gamma_W(M) = \{x \in M \mid \mathrm{Supp}(Rx) \subseteq W\},$$

for all $M \in R$-Mod. Then Γ_W is a left exact radical functor that preserves injectivity.

The notion of stable t-structure introduced by J. Miyachi.

Definition 3. A pair $(\mathcal{U}, \mathcal{V})$ of full subcategories of a triangulated category $\mathcal{T}$ is called a stable t-structure on $\mathcal{T}$ if it satisfies the following conditions:

(1) $\mathrm{Hom}_{\mathcal{T}}(\mathcal{U}, \mathcal{V}) = 0$.
(2) $\mathcal{U} = \mathcal{U}[1]$ and $\mathcal{V} = \mathcal{V}[1]$.
(3) For any $X \in \mathcal{T}$, there is a triangle $U \to X \to V \to U[1]$ with $U \in \mathcal{U}$ and $V \in \mathcal{V}$.

For a triangle functor δ on triangulated category $\mathcal{T}$, we define two full subcategories of $\mathcal{T}$

$$\begin{aligned} \mathrm{Im}(\delta) &= \{X \in \mathcal{T} \mid X \cong \delta(Y) \ \text{ for some } \ Y \in \mathcal{T}\}, \\ \mathrm{Ker}(\delta) &= \{X \in \mathcal{T} \mid \delta(X) \cong 0\}. \end{aligned}$$

The following theorem proved by J. Miyachi in [1] is a key to this talk.

Theorem 4. *Let $\mathcal{T}$ be a triangulated category and $\mathcal{U}$ a full triangulated subcategory of $\mathcal{T}$. Then the following conditions are equivalent for $\mathcal{U}$.*

(1) *There is a full subcategory $\mathcal{V}$ of $\mathcal{T}$ such that $(\mathcal{U}, \mathcal{V})$ is a stable t-structure on $\mathcal{T}$.*

(2) *The natural embedding functor $i : \mathcal{U} \to \mathcal{T}$ has a right adjoint $\rho : \mathcal{T} \to \mathcal{U}$.*

If it is the case, setting $\delta = i \circ \rho : \mathcal{T} \to \mathcal{T}$, we have the equalities

$$\mathcal{U} = \mathrm{Im}(\delta) \quad \textit{and} \quad \mathcal{V} = \mathcal{U}^{\perp} = \mathrm{Ker}(\delta).$$

We define an abstract local cohomology functor which is a main theme of this talk.

Definition 5. We denote $\mathcal{T} = \mathcal{D}^{+}(R\text{-Mod})$ in this definition. Let $\delta : \mathcal{T} \to \mathcal{T}$ be a triangle functor. We call that δ is an abstract local cohomology functor if the following conditions are satisfied:

(1) The natural embedding functor $i : \mathrm{Im}(\delta) \to \mathcal{T}$ has a right adjoint $\rho : \mathcal{T} \to \mathrm{Im}(\delta)$ and $\delta \cong i \circ \rho$. (Hence, by Miyachi's Theorem, $(\mathrm{Im}(\delta), \mathrm{Ker}(\delta))$ is a stable t-structure on $\mathcal{T}$.)

(2) The stable t-structure $(\mathrm{Im}(\delta), \mathrm{Ker}(\delta))$ divides indecomposable injective R-modules, by which we mean that each indecomposable injective R-module belongs to either $\mathrm{Im}(\delta)$ or $\mathrm{Ker}(\delta)$.

Example 6. Let W be a specialization closed subset of $\mathrm{Spec}(R)$. Then $\mathbf{R}\Gamma_W$ is an abstract local cohomology functor on $\mathcal{D}^{+}(R\text{-Mod})$.

The main result of this talk is the following.

Theorem 7. (1) *The following conditions are equivalent for a left exact preradical functor γ on R-Mod.*

(a) *γ is a radical functor.*

(b) *γ preserves injectivity.*

(c) *γ is a section functor with support in a specialization closed subset of $\mathrm{Spec}(R)$.*

(d) *$\mathbf{R}\gamma$ is an abstract local cohomology functor.*

(2) *Given an abstract local cohomology functor δ on $\mathcal{D}^{+}(R\text{-Mod})$, there exists a specialization closed subset W of $\mathrm{Spec}(R)$ such that δ is isomorphic to the right derived functor $\mathbf{R}\Gamma_W$ of the section functor Γ_W.*

Definition 8. (1) We denote by $\mathbb{S}(R)$ the set of all left exact radical functors on R-Mod.

(2) We denote by $\mathbb{A}(R)$ the set of the isomorphism classes $[\delta]$ where δ ranges over all abstract local cohomology functors on $\mathcal{D}^{+}(R\text{-Mod})$.

(3) We denote by $\mathrm{sp}(R)$ the set of all specialization closed subsets of $\mathrm{Spec}(R)$.

Corollary 9. *The mapping $\mathbb{S}(R) \to \mathbb{A}(R)$ which maps γ to $[\mathbf{R}\gamma]$ (resp. $\mathrm{sp}(R) \to \mathbb{A}(R)$ which sends W to $[\mathbf{R}\Gamma_W]$) gives an isomorphism of complete lattices.*

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Graduate School of
Natural Science and Technology
Okayama University
Okayama 700-8530, Japan
Email: tyoshiza@math.okayama-u.ac.jp

(θ, δ)-CODES WITH SKEW POLYNOMIAL RINGS

Manabu Matsuoka

Let $\mathbf{F}$ be a finite field. A linear $[n, k]$-code over $\mathbf{F}$ is a k-dimensional subspace C of the vector space $\mathbf{F}^n = \{(a_0, \cdots, a_{n-1}) | a_i \in \mathbf{F}\}$. We use polynomial representation of the code C, where we identify code words $(a_0, \cdots, a_{n-1}) \in C$ with coefficient tuples of polynomials $a_{n-1}X^{n-1} + \cdots a_1X + a_0 \in \mathbf{F}[X]$. Those polynomials can also be seen as elements of a quotient ring $\mathbf{F}[X]/(f)$ where f is a polynomial of degree n.

D. Boucher, W. Geiselmann and F. Ulmer [3] generalized the notion of codes to skew polynomial rings. In [4], D. Boucher and P. Solé studied skew constacyclic codes. They considered skew polynomial rings over Galois rings.

Definition 1. Let R be a finite ring, θ be an endomorphism of R, δ be a θ-derivation of R. Suppose $f \in R[X; \theta, \delta]$ is a nonzero polynomial with an invertible leading coefficient. Then $R[X; \theta, \delta]/(f)$ is a finite ring and a left ideal of $R[X; \theta, \delta]/(f)$ is called a skew (θ, δ)-code.

Theorem 2. *Let $f = hg \in R[X; \theta, \delta]$ and the leading coefficients of f and g be invertible. Suppose $C = (g)_l/(f)$ is a left ideal of $R[X; \theta, \delta]/(f)$. If f satisfies the condition $R[X; \theta, \delta]f = fR[X; \theta, \delta]$, then C is a free left R-module and $rankC = deg(f) - deg(g)$.*

We study skew codes of endomorphism type $R[X; \theta]$ and derivation type $R[X; \delta]$. And we give some examples.

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Yokkaichi Highschool

Email: e-white@hotmail.co.jp

Hochschild cohomology and Gorenstein Nakayama algebras

Hiroshi Nagase

Let k be an algebraically closed field and $\mathrm{HH}(A)$ the Hochschild cohomology algebra of a finite dimensional k-algebra A. In [1], Erdmann, Holloway, Snashall, Solberg and Taillefer showed that some geometric properties of the support variety of A-modules and some representation theoretic properties of those are related providing A satisfies the following finiteness conditions :

- $\mathrm{HH}(A)$ is noetherian,
- $\mathrm{Ext}_A^*(A/J, A/J)$ is a finitely generated $\mathrm{HH}(A)$-module,

where J denotes the Jacobson radical of A.

In this talk, we consider the finiteness conditions on Nakayama algebras and show that Gorenstein Nakayama algebras satisfy the finiteness conditions. It is proved by using the long exact sequense in [2].

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Tokyo Gakugei University
4-1-1 Nukui-Kitamachi, Koganei
Tokyo 184-8501 JAPAN
Email: nagase@u-gakugei.ac.jp

On selfinjective algebras of stable dimension zero

Michio Yoshiwaki

R. Rouquier has introduced a notion of dimension of a triangulated category in [6]. One of Rouquier's aims is to give a lower bound for Auslander's representation dimension of selfinjective algebras (see Proposition 1).

Let A be a non-semisimple selfinjective finite-dimensional algebra over a field k, and let $\Omega = \Omega_A$ be the syzygy functor. The *stable dimension* $\operatorname{stab.dim} A$ of A is the dimension of the stable module category of A (in the sense of Rouquier). Then Rouquier has shown the following result.

Proposition 1 (Rouquier [5] **cf.**Auslander [1])**.**

$$\operatorname{rep.dim} A \geq \operatorname{stab.dim} A + 2$$

where $\operatorname{rep.dim} A$ *is the representation dimension of* A*.*

Note that A is representation-finite if and only if it has representation dimension at most 2 (due to Auslander [1]). Thus any representation-finite selfinjective algebra (over a field) has stable dimension 0. Our result is that the converse holds under the assumption that k is algebraically closed. Namely, we prove the following theorem.

Theorem 2. *Let A be a non-semisimple selfinjective algebra over an algebraically closed field k, which is assumed to be connected. If the stable Auslander-Reiten quiver of A admits only finitely many Ω-orbits, then A is representation-finite.*

To prove Theorem 2, we need Liu's result on representation-finite algebras [4] and the first Brauer-Thrall conjecture (see [3]). As the consequence, we have the main result.

Corollary 3. *If* $\operatorname{stab.dim} A = 0$*, then A is representation-finite.*

Although this was expected to hold by some experts, it had not been proved before. Moreover, we have the following result as the consequence of Corollary 3.

Corollary 4. *If* $\operatorname{rep.dim} A = 3$*, then* $\operatorname{stab.dim} A = 1$*.*

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Graduate School of Science
Osaka City University
Osaka 558-8585 JAPAN
Email: yosiwaki@sci.osaka-cu.ac.jp

Reflection for selfinjective algebras

Hiroki Abe

Reflection functors introduced in [2] are induced by transformations of the quiver making a certain source vertex changed into a sink vertex. Let Λ be a finite dimensional algebra over a field K. In [1], it was shown that reflection functors are of the form $\mathrm{Hom}_\Lambda(T, -)$ with T a certain type of tilting modules. Let $P_1, \cdots, P_n$ be a complete set of nonisomorphic indecomposable projective modules in mod-Λ, the category of finitely generated right Λ-modules. Set $I = \{1, \cdots, n\}$. Assume that there exists a simple projective module $S \in$ mod-Λ which is not injective. Take $t \in I$ with $P_t \cong S$ and set

$$T = T_1 \oplus \tau^{-1} S \quad \text{with} \quad T_1 = \bigoplus_{i \in I \setminus \{t\}} P_i,$$

where τ denotes the Auslander-Reiten translation. Then T is a tilting module, called an APR-tilting module, and $\mathrm{Hom}_\Lambda(T, -)$ is a reflection functor.

In [3], APR-tilting modules were generalized as follows. Assume that there exists a simple module $S \in$ mod-Λ with $\mathrm{Ext}^1_\Lambda(S, S) = 0$ and $\mathrm{Hom}_\Lambda(D\Lambda, S) = 0$, where $D = \mathrm{Hom}_K(-, K)$. Let P_t be the projective cover of S and let T be the same as above. Then T is a tilting module, called a BB-tilting module. We are interested in a minimal projective presentation of T, which is a two-term tilting complex. Take a minimal injective presentation $0 \to S \to E^0 \xrightarrow{f} E^1$ and define a complex $E^\bullet$ as the mapping cone of $f : E^0 \to E^1$. Then $\mathrm{Hom}^\bullet_\Lambda(D\Lambda, E^\bullet)$ is a minimal projective presentation of $\tau^{-1}S$ and hence

$$T^\bullet = T_1 \oplus \mathrm{Hom}^\bullet_\Lambda(D\Lambda, E^\bullet)$$

is a minimal projective presentation of T. In this talk, we demonstrate that this type of tilting complexes play an important role in the theory of derived equivalences for selfinjective algebras. Consider the case where Λ is selfinjective and $S \in$ mod-Λ is a simple module with $\mathrm{Ext}^1_\Lambda(S, S) = 0$ and $\mathrm{Hom}_\Lambda(D\Lambda, S) \cong S$. Let $E^\bullet$ and $T^\bullet$ be the same as above. We will show that $T^\bullet$ is a tilting complex and $T^\bullet \cong T_1 \oplus E^\bullet$. In this talk, derived equivalences for selfinjective algebras induced by this type of tilting complexes are called reflections. Our main aim is to determine the transformations of Brauer trees associated with reflections.

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Institute of Mathematics, University of Tsukuba,
Ibaraki, 305-8571, Japan
Email: abeh@math.tsukuba.ac.jp

Auslander-Gorenstein resolution

Mitsuo Hoshino and Hirotaka Koga

In this talk, a noetherian ring A is a ring which is left and right noetherian, and a noetherian R-algebra A is a ring endowed with a ring homomorphism $R \to A$, with R a commutative noetherian ring, whose image is contained in the center of A and A is finitely generated as an R-module. Note that a noetherian algebra is a noetherian ring. We refer to [1] for the definition of Auslander-Gorenstein rings.

Let R be a commutative Gorenstein local ring and A a noetherian R-algebra with $\mathrm{Ext}^i_R(A, R) = 0$ for $i \neq 0$. Set $\Omega = \mathrm{Hom}_R(A, R)$. Then proj dim ${}_A\Omega < \infty$ and proj dim $\Omega_A < \infty$ if and only if Ω_A is a tilting module in the sense of [2]. Even if A is an Auslander-Gorenstein ring, it may happen that inj dim $A_A \neq$ dim R. For instance, if $A = \mathrm{T}_m(R)$, the ring of $m \times m$ upper triangular matrices over R, for $m \geq 2$, then A is an Auslander-Gorenstein ring with inj dim $A_A =$ dim $R + 1$. Also, consider the case where R is a complete Gorenstein local ring of dimension one and Λ is a noetherian R-algebra with $\mathrm{Ext}^i_R(\Lambda, R) = 0$ for $i \neq 0$. Denote by $\mathcal{L}_\Lambda$ the full subcategory of mod-Λ consisting of modules X with $\mathrm{Ext}^i_R(X, R) = 0$ for $i \neq 0$ and assume that $\mathcal{L}_\Lambda = \mathrm{add}(M)$ with $M \in$ mod-Λ non-projective. Then $A = \mathrm{End}_\Lambda(M)$ is an Auslander-Gorenstein ring of global dimension two.

Consider the case where Ω_A is a tilting module of arbitrary finite projective dimension. Take a projective resolution $P^\bullet \to \Omega$ in mod-A^{op}. Then, setting $Q^\bullet = \mathrm{Hom}^\bullet_R(P^\bullet, R)$, we have a right resolution $A \to Q^\bullet$ in mod-A such that every $Q^i \in$ mod-R is a reflexive module with $\mathrm{Ext}^j_R(\mathrm{Hom}_R(Q^i, R), R) = 0$ for $j \neq 0$, $\oplus_{i \geq 0} \mathrm{Hom}_R(Q^i, R) \in$ mod-A^{op} is a projective generator and proj dim $Q^i < \infty$ in mod-A for all $i \geq 0$. We will show that A is an Auslander-Gorenstein ring if proj dim $Q^i \leq i$ in mod-A for all $i \geq 0$ and that the converse holds true if R is complete and $P^\bullet \to \Omega$ is a minimal projective resolution.

Formulating these facts, we will introduce the notion of Auslander-Gorenstein resolution. Let R, A be arbitrary noetherian rings. A right resolution $0 \to A \to Q^0 \to \cdots \to Q^m \to 0$ in Mod-A is said to be an Auslander-Gorenstein resolution of A over R if the following conditions are satisfied: (1) every Q^i is an R-A-bimodule; (2) every $Q^i \in$ Mod-R^{op} is a finitely generated reflexive module with $\mathrm{Ext}^j_R(\mathrm{Hom}_{R^{\mathrm{op}}}(Q^i, R), R) = 0$ for $j \neq 0$; (3) $\oplus_{i \geq 0} \mathrm{Hom}_{R^{\mathrm{op}}}(Q^i, R) \in$ Mod-A^{op} is faithfully flat; and (4) flat dim $Q^i \leq i$ in Mod-A for all $i \geq 0$. We will show that A is an Auslander-Gorenstein ring if it admits an Auslander-Gorenstein resolution over R and if R is an Auslander-Gorenstein ring. Also, we will provide several examples of Auslander-Gorenstein resolution.

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Institute of Mathematics
University of Tsukuba
Ibaraki, 305-8571, Japan
Email: hoshino@math.tsukuba.ac.jp

Institute of Mathematics
University of Tsukuba
Ibaraki, 305-8571, Japan
Email: koga@math.tsukuba.ac.jp

Weakly sectional paths and the shapes of Auslander-Reiten quivers

Takahiko Furuya

Recently the composite of irreducible maps in the module category of a finite dimensional algebra has been studied by Chaio, Coelho and Trepode [3] in terms of the radical of the module category (see also [2], [5]). In this talk we introduce weakly sectional paths in the Auslander-Reiten quiver of an artin algebra. These paths are generalizations of sectional paths as well as pre-sectional paths of Liu [7]. We study the composite of irreducible maps corresponding to arrows lying on a weakly sectional path. In particular it is shown that there are no weakly sectional oriented cycles in the Auslander-Reiten quiver of an artin algebra. The final part of the talk is devoted to considering the degrees of irreducible maps introduced in [7]. We use a weakly sectional path to give a necessary and sufficient condition for the degree of an irreducible map in a certain component to be finite.

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Department of Mathematics
Tokyo University of Science
1-3 Kagurazaka, Shinjuku-ku, Tokyo, JAPAN
Email: furuya@ma.kagu.tus.ac.jp

On a Generalization of Stable Torsion Theory

Yasuhiko Takehana

Let $(\mathcal{T}, \mathcal{F})$ be a torsion theory for Mod-R. $(\mathcal{T}, \mathcal{F})$ is called a stable torsion theory if $\mathcal{T}$ is closed under taking injective hulls. $(\mathcal{T}, \mathcal{F})$ is called a hereditary torsion theory if $\mathcal{T}$ is closed under taking submodules. P. Gabriel characterized a hereditary stable torsion theory in [Des categiries abeliennes 1962]. We generalize this by using torsion theoretic generalizations of hereditary torsion theories.

Hakodate National College of Technology

RINGS WITH INDECOMPOSABLE RIGHT MODULES UNIFORM

Surjeet Singh

A ring R is said to satisfy condition (**) if it is both sided artinian and every indecomposable right R-module of finite composition length is uniform. Such rings were first studied by Tachikawa (1959). The main purpose of this paper is to give a characterisation of such rings in terms of its right ideals. The approach here is quite different from that of Tachikawa. To start with a theorem on lifting of an isomorphism between simple homomorphic images of two uniform modules of over a ring satisfying (**) is proved. Let$M$ be a uniform modules of finite composition length with $S = soc(M)$, $D = End(S)$ and D' the division subring of D consisting of those $\sigma \in D$, which can be extended to some endomorphism of M. Then the pair (D, D') is called the *drpa* of M. The following is proved. A ring R with Jacobson radical J satisfies (**) if and only if it satisfies the following conditions: (1) R is a both sided artinian, right serial ring. (2) For any three indecomposable idempotents e, f, $g \in R$ with eJ, fJ, gJ non-zero the following hold: (i) If (D, D') is the *drpa* of $\frac{eR}{eJ^2}$ then the left and right dimensions of D over D' are less than or equal to 2; (ii) if e, f are non-isomorphic and $\frac{eJ}{eJ^2} \simeq \frac{fJ}{fJ^2}$, then $eJ^2 = 0$ or $fJ^2 = 0$; (iii) if e, f are non-isomorphic and $\frac{eJ}{eJ^2} \simeq \frac{fJ}{fJ^2} \simeq \frac{gJ}{gJ^2}$, then g is isomorphic to e or f; (iv) if $\frac{eR}{eJ^2}$ is not quasi-injective, then $eJ^2 = 0$ and $\frac{eJ}{eJ^2} \not\simeq \frac{fJ}{fJ^2}$, whenever e is not isomorphic to f. The structure of uniform modules over a ring R satisfying (**) is also determined.

King Saud University