ON SELFINJECTIVE ALGEBRAS OF STABLE DIMENSION ZERO

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ABSTRACT. This paper is based on our lecture giving at the '43rd Symposium on Ring Theory and Representation Theory' held at Naruto University of Education in September 2010. In this paper, we consider the stable dimension of selfinjective algebra, which is the dimension of its stable module category in the sense of Rouquier. We give a proof that a non-semisimple selfinjective algebra A is representation-finite if the stable dimension of A is zero. Moreover, we verify that selfinjective algebras obtained from some hereditary algebra have stable dimension at most one.

Key Words: Representation-finite algebra, Selfinjective algebra, Stable dimension.

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1. NOTATION

Throughout this article, k denotes an algebraically closed field, and all algebras are finite-dimensional associative k-algebras with an identity, unless otherwise stated.

For any k-algebra Λ, we denote by $\operatorname{mod}\Lambda$ the abelian category of finite-dimensional (over k) left Λ-modules and by $\Gamma(\Lambda)$ the Auslander-Reiten quiver of Λ. We may identify the vertices of $\Gamma(\Lambda)$ with the indecomposable Λ-modules. Then we have the Auslander-Reiten translation $\tau_\Lambda = D\operatorname{Tr}$ and $\tau_\Lambda^{-1} = \operatorname{Tr} D$, where $D: \operatorname{mod}\Lambda \to \operatorname{mod}\Lambda^{\mathrm{op}}$ is the standard duality $\operatorname{Hom}_k(\text{-}, k)$. Moreover, we denote by $\mathcal{D}^b(\operatorname{mod}\Lambda)$ the bounded derived category of Λ, by $\operatorname{gl.dim}\Lambda$ the global dimension of Λ, by $\mathrm{T}(\Lambda)$ the trivial extension of Λ and by $\hat{\Lambda}$ the repetitive category of Λ.

For any selfinjective k-algebra A, we denote by $\underline{\operatorname{mod}}A$ the stable module category of A. Let $\Omega = \Omega_A : \underline{\operatorname{mod}}A \to \underline{\operatorname{mod}}A$ be a syzygy functor. Note that if X is indecomposable, then $\Omega(X)$ remains indecomposable. And moreover, Ω is an equivalence and $\underline{\operatorname{mod}}A$ is a triangulated category with shift functor Ω^{-1} (see Happel [14]). Similarly, the stable module category $\underline{\operatorname{mod}}\hat{\Lambda}$ of a repetitive category $\hat{\Lambda}$ can be defined and then is a triangulated category.

Furthermore, we denote by ${}_s\Gamma(A)$ the stable Auslander-Reiten quiver of A, which is obtained from $\Gamma(A)$ by removing the projective-injective vertices and the arrows attached to them. Then the set ${}_s\Gamma(A)_0$ of vertices of ${}_s\Gamma(A)$ coincides with the set of isoclasses of non-projective indecomposable A-modules. It is well-known that we can recover $\Gamma(A)$ from ${}_s\Gamma(A)$. Note that the Auslander-Reiten translation τ_A is an automorphism of the quiver ${}_s\Gamma(A)$ with an inverse τ_A^{-1} and that $\tau_A \cong \Omega^2\nu \cong \nu\Omega^2$ since $\Omega\nu \cong \nu\Omega$, where $\nu = D\operatorname{Hom}_A(\text{-}, A)$ is the Nakayama functor.

The detailed version of this paper has been submitted for publication elsewhere.

2. Preliminaries

First, we define the dimension of triangulated category in the sense of Rouquier.

Definition 1 (Rouquier [21])**.** Let $\mathcal{T}$ be a triangulated category with shift functor [1]. Then the *dimension* of $\mathcal{T}$ is defined to be

$$\dim \mathcal{T} \;\; := \;\; \min\{n \geq 0 \mid \langle M \rangle_{n+1} = \mathcal{T} \text{ for some } M \in \mathcal{T}\}$$

or ∞ when there is no such an object M, where $\langle M \rangle_{n+1}$ is defined inductively:

$$\begin{aligned} &\text{for } n = 0, \quad \langle M \rangle_1 := \operatorname{add}\{M[i] \mid i \in \mathbb{Z}\}, \text{ and} \\ &\text{if } n > 0, \quad \langle M \rangle_{n+1} := \operatorname{add}\{M_{n+1} \mid \text{there is a triangle} : M_n \to M_{n+1} \to M_1 \to M_n[1], \\ &\qquad\qquad\qquad\qquad\qquad\qquad \text{where } M_n \in \langle M \rangle_n \text{ and } M_1 \in \langle M \rangle_1\}. \end{aligned}$$

Let $\mathcal{F}$ be another triangulated category, and let $F : \mathcal{T} \to \mathcal{F}$ be a triangle functor. Then we mention some fundamental remarks.

Remark 2.

(a) If a functor F is dense, then $\dim \mathcal{T} \geq \dim \mathcal{F}$.
(b) If a functor F is an equivalence, then $\dim \mathcal{T} = \dim \mathcal{F}$.

Second, we define the stable dimension of selfinjective algebra.

Let A be a non-semisimple selfinjective algebra (over a field). Recall that the stable module category $\underline{\mathrm{mod}}A$ of A is a triangulated category. Thus we can define the stable dimension of A.

Definition 3. The *stable dimension* of A is defined to be

$$\operatorname{stab}.\dim A \;\; := \;\; \dim(\underline{\mathrm{mod}}A) \;\; \text{(in the sense of Definition 1).}$$

Recall also that $\underline{\mathrm{mod}}A$ and $\mathcal{D}^b(\mathrm{mod}\, A)/\operatorname{per} A$ are equivalent as triangulated categories (see Rickard [19]), where $\operatorname{per} A$ is the épaisse subcategory of $\mathcal{D}^b(\mathrm{mod}\, A)$ consisting of perfect complexes. Then by Remark 2, we have the fundamental properties for the stable dimension.

Remark 4.

(a) If there exists a dense functor $F : \mathcal{T} \to \underline{\mathrm{mod}}A$, where $\mathcal{T}$ is a suitable triangulated category: *e.g.*, $\mathcal{T} = \mathcal{D}^b(\mathrm{mod}\, A)$ and $\underline{\mathrm{mod}}\hat{\Lambda}$, then $\dim \mathcal{T} \geq \operatorname{stab}.\dim A$ (see Subsection 4.1).
(b) Let B be another selfinjective algebra. If $\underline{\mathrm{mod}}A$ and $\underline{\mathrm{mod}}B$ are equivalent as triangulated categories, then $\operatorname{stab}.\dim A = \operatorname{stab}.\dim B$; For instance, A and B are derived equivalent, then $\operatorname{stab}.\dim A = \operatorname{stab}.\dim B$.

Rouquier introduced a notion of dimension of a triangulated category in [21]. One of his aims was to give a lower bound for Auslander's representation dimension of selfinjective algebras (see Proposition 6), and then he gave the first example of algebras having representation dimension at least four (see Theorem 23).

Definition 5 (Auslander [3])**.** The representation dimension of a non-semisimple artin algebra Λ is defined to be

$$\operatorname{rep}.\dim \Lambda := \min\{\operatorname{gl}.\dim \operatorname{End}_\Lambda(M) \mid M \text{ is a generator and a cogenerator in} \operatorname{mod} \Lambda\}.$$

For semisimple artin algebra, the dimension is defined to be one.

In [20], Rouquier showed the following result.

Proposition 6 (Rouquier [20] cf. Auslander [3]). *Let A be a non-semisimple selfinjective algebra* (*over a field*). *Then*

$$\mathrm{LL}(A) \geq \mathrm{rep}\,.\dim A \geq \mathrm{stab}\,.\dim A + 2,$$

where the Loewy length $\mathrm{LL}(A)$ *is the smallest integer* r *such that* $\mathrm{rad}(A)^r = 0$.

After Auslander proved in [3] (see Proposition p.55) that $\mathrm{LL}(A) + 1 \geq \mathrm{rep}\,.\dim A$, Rouquier has improved it by indicating that the equality does not occur, and hence the first inequality in Proposition 6.

Remark 7. The stable dimension is always finite by the first inequality in Proposition 6. Recall also that for any artin algebra, the representation dimension is always finite (see Iyama [15]).

Auslander introduced the representation dimension in [3], and hoped that the representation dimension should be a good measure of how far a representation-infinite algebra is from being representation-finite. Actually, he showed the following result.

Theorem 8 (Auslander [3]). *For any artin algebra* Λ, Λ *is representation-finite if and only if* $\mathrm{rep}\,.\dim \Lambda \leq 2$.

Thus by Proposition 6 and Theorem 8, we observe that any (non-semisimple) representation-finite selfinjective algebra (over a field) has stable dimension zero, which also follows from definition immediately. Then we have a natural question whether the converse should also hold.

3. On selfinjective algebras of stable dimension zero

3.1. **Main results.** In this subsection, we assume that A is a non-semisimple selfinjective algebra over an algebraically closed field k, unless otherwise stated.

Our main result is to prove that if A has stable dimension zero, then A is representation-finite. Namely, we verify that the converse of the observation above indeed holds provided that the base field is algebraically closed. Although this was expected to hold by some experts, it had not been proved before.

Our main theorem is the following.

Theorem 9 (Yoshiwaki [24]). *Let A be a non-semisimple selfinjective finite-dimensional connected algebra over an algebraically closed field k. If the set ${}_s\Gamma(A)_0$ of isoclasses of non-projective indecomposable A-modules admits only finitely many Ω-orbits, then A is representation-finite.*

To prove this theorem, we need two critical results. The first result is a characterization of representation-finite algebras over an algebraically closed field due to Liu.

Proposition 10 (Liu [17] 3.11 Proposition p.52). *Let A be a finite-dimensional algebra over an algebraically closed field. Then A is representation-finite if and only if $\Gamma(A)$ admits only finitely many τ_A-orbits.*

This follows from the 2nd Brauer-Thrall conjecture. So, we require the assumption in Theorem 9 that the base field k is algebraically closed.

Second, we need the following well-known result due to Auslander.

Theorem 11 (Auslander [5]). *Let A be a finite-dimensional connected algebra (over a field), and let $\mathcal{C}$ be a connected component of $\Gamma(A)$. If the length of the modules in $\mathcal{C}$ is bounded, then A is representation-finite and $\mathcal{C} = \Gamma(A)$.*

So, we require the assumption in Theorem 9 that A is connected. As a consequence of Theorem 11, the 1st Brauer-Thrall conjecture follows. Namely, it may be to say that Theorem 9 follows from the two Brauer-Thrall conjectures.

Suppose that $\operatorname{stab.dim} A = 0$. Then by definition we have

$$\underline{\mathrm{mod}}A = \mathrm{add}\{\Omega^i M \mid i \in \mathbb{Z}\}$$

for some $M \in \underline{\mathrm{mod}}A$. Hence it is easy to see the following lemma.

Lemma 12. *The following are equivalent:*

(1) $\operatorname{stab.dim} A = 0$,
(2) ${}_s\Gamma(A)_0$ *admits only finitely many Ω-orbits.*

Thus we obtain the desired result by Theorem 9.

Corollary 13. *If $\operatorname{stab.dim} A = 0$, then A is representation-finite.*

Proof. Suppose that $\operatorname{stab.dim} A = 0$. Then by Lemma 12, ${}_s\Gamma(A)_0$ admits only finitely many Ω-orbits, and hence A is representation-finite by Theorem 9. $\square$

Moreover, we have the following result by Proposition 6, Theorem 8 and Corollary 13.

Corollary 14. *If $\operatorname{rep.dim} A = 3$, then $\operatorname{stab.dim} A = 1$.*

Proof. If $\operatorname{rep.dim} A = 3$, then $\operatorname{stab.dim} A \leq 1$ and A is not representation-finite by Proposition 6 and Theorem 8. Then by Corollary 13, $\operatorname{stab.dim} A = 1$. $\square$

In the last of this subsection, we give an example of selfinjective algebra having stable dimension zero.

Example 15. Let $A = kQ/I$, where Q is the quiver

$$\begin{tikzcd}
 & 2 \arrow[d, shift right, "\alpha_2"'] & \\
 & 0 \arrow[u, shift right, "\beta_2"'] \arrow[dl, shift right, "\beta_1"'] \arrow[dr, shift left, "\beta_3"] & \\
1 \arrow[ur, shift right, "\alpha_1"'] & & 3 \arrow[ul, shift left, "\alpha_3"]
\end{tikzcd}$$

and the ideal I is generated by

$$\alpha_1\beta_1 - \alpha_2\beta_2,\ \alpha_2\beta_2 - \alpha_3\beta_3,\ \beta_1\alpha_1,\ \beta_2\alpha_1,\ \beta_1\alpha_2,\ \beta_3\alpha_2,\ \beta_2\alpha_3,\ \beta_3\alpha_3.$$

Then A is a selfinjective algebra of type $\mathbb{D}_4$. Indeed, let $\vec{\Delta}$ be the quiver

$$\begin{tikzcd}
1 \arrow[dr] & 2 \arrow[d] & 3 \arrow[dl] \\
& 0 &
\end{tikzcd},$$

and let $B = k\,\vec{\Delta}$ be the path algebra of $\vec{\Delta}$. Clearly, B is a tilted algebra of type $\mathbb{D}_4$. Then we have $\hat{B} = k\hat{\Delta}/\hat{I}$, where $\hat{\Delta}$ is the quiver

$$\begin{tikzcd}
(1,1) \arrow[dr] & (1,2) \arrow[d] & (1,3) \arrow[dl] \\
& (1,0) \arrow[dl, "\beta_{0,1}"'] \arrow[d, "\beta_{0,2}"'] \arrow[dr, "\beta_{0,3}"] & \\
(0,1) \arrow[dr, "\alpha_{0,1}"'] & (0,2) \arrow[d, "\alpha_{0,2}"'] & (0,3) \arrow[dl, "\alpha_{0,3}"] \\
& (0,0) \arrow[dl, "\beta_{-1,1}"'] \arrow[d, "\beta_{-1,2}"'] \arrow[dr, "\beta_{-1,3}"] & \\
(-1,1) \arrow[dr] \arrow[rr, leftrightarrow, bend right, "\rho"] & (-1,2) \arrow[d] & (-1,3) \arrow[dl] \arrow[uu, mapsto, bend right, "\nu_{\hat{B}}"'] \\
& (-1,0) \arrow[dl] \arrow[d] \arrow[dr] & \\
& &
\end{tikzcd}$$

and the ideal $\hat{I}$ is generated by

$$\alpha_{m,i}\beta_{m,i} - \alpha_{m,j}\beta_{m,j},\ \ \beta_{m-1,i}\alpha_{m,j},$$

with $m \in \mathbb{Z}$, $i, j \in \{1, 2, 3\}$, $i \neq j$. Let $\nu_{\hat{B}}$ be the Nakayama automorphism of $\hat{B}$, and let ρ be the automorphism of $\hat{B}$ given by the permutation $\{((m, 1),\ (m, 3))\}$. Then we obtain $A = \hat{B}/\langle \rho\nu_{\hat{B}} \rangle$, and thus the stable Auslander-Reiten quiver ${}_s\Gamma(A)$ of A is of the form:

$$\begin{tikzcd}
\bullet \arrow[dr] & & \bullet \arrow[dr] & & \bullet \arrow[dr] & & \bullet \arrow[dr] & & \bullet \arrow[dr] & & \underset{M}{\bullet} \\
\bullet \arrow[r] & \bullet \arrow[r] \arrow[ur] \arrow[dr] & \bullet \arrow[r] & \bullet \arrow[r] \arrow[ur] \arrow[dr] & \bullet \arrow[r] & \bullet \arrow[r] \arrow[ur] \arrow[dr] & \bullet \arrow[r] & \bullet \arrow[r] \arrow[ur] \arrow[dr] & \bullet \arrow[r] & \underset{L}{\bullet} \arrow[r] \arrow[ur] \arrow[dr] & \underset{N}{\bullet} \\
\underset{M}{\bullet} \arrow[ur] & & \bullet \arrow[ur] & & \bullet \arrow[ur] & & \bullet \arrow[ur] & & \underset{\Omega^2 M}{\bullet} \arrow[ur] & & \underset{\nu M = \Omega^5 M}{\bullet}
\end{tikzcd}.$$

Let $\{M,\ N,\ L\}$ be a complete set of representatives of τ_A-orbits in ${}_s\Gamma(A)$, and put $X = M \oplus N \oplus L$. Then $\underline{\mathrm{mod}}A = \mathrm{add}\{\Omega^i X \mid i \in \mathbb{Z}\}$ because the τ_A-orbits and the Ω-orbits in ${}_s\Gamma(A)_0$ coincide, and hence we have $\mathrm{stab\,.\,dim}\, A = 0$.

3.2. **An application.** We now define the derived dimension of finite-dimensional algebra.

Definition 16. Let Λ be a finite-dimensional k-algebra. Then the *derived dimension* of Λ is defined to be

$$\operatorname{der.dim}\Lambda \;:=\; \dim(\mathcal{D}^b(\operatorname{mod}\Lambda)) \;\; \text{(in the sense of Definition 1).}$$

For the derived dimension, it is known that it has the following property.

Proposition 17 (Rouquier [20], Krause-Kussin [16], Oppermann [18])**.** *Let Λ be a finite-dimensional k-algebra. Then*

$$\operatorname{der.dim}\Lambda \leq \inf(\operatorname{gl.dim}\Lambda,\; \operatorname{rep.dim}\Lambda).$$

Remark 18. The derived dimension is always finite because the representation dimension is always finite (see Iyama [15]; also see Remark 7).

Furthermore, we introduce the iterated tilted algebra.

Definition 19. Let Q be a finite connected acyclic quiver, and let kQ be the path algebra of Q. Then Λ is an *iterated tilted algebra* of type Q if there exists a triangle equivalence between $\mathcal{D}^b(\operatorname{mod}\Lambda)$ and $\mathcal{D}^b(\operatorname{mod} kQ)$. If Q is a Dynkin quiver, Λ is called an iterated tilted algebra of Dynkin type.

There is the original definition of iterated tilted algebra (for instance, see Happel [14] Chapter IV 4.4 p.173). We, however, use Definition 19 for simplicity since Happel has shown that it is equivalent to the original definition (see Happel [14] Chapter IV 5.4 Theorem p.176).

As an application of our result above (see Corollary 13), we obtain the following result.

Theorem 20 (Chen-Ye-Zhang [8], Yoshiwaki [24])**.** *For any finite-dimensional k-algebra Λ, the following are equivalent*:

1. $\operatorname{der.dim}\Lambda = 0$*,*
2. $\operatorname{stab.dim}\mathrm{T}(\Lambda) = 0$*,*
3. $\mathrm{T}(\Lambda)$ *is representation-finite,*
4. Λ *is an iterated tilted algebra of Dynkin type.*

Sketch of Proof. Chen-Ye-Zhang have actually shown the implication from (1) to (2). It is easy to see that any iterated tilted algebra of Dynkin type has derived dimension zero. Therefore, since Corollary 13 means that the implication from (2) to (3) holds, the assertion follows from Assem-Happel-Roldán's result in [1] (also see Happel [14] Chapter V 2.1 Theorem p.199) that (3) is equivalent to (4). □

4. Some selfinjective algebras have stable dimension one

4.1. **A calculation for the stable dimension.** The proof of Theorem 20 gives us an idea for calculation of the stable dimension. In this subsection, we assume that Λ is an iterated tilted algebra of some finite connected acyclic quiver Q. Then we have the following facts due to Happel.

Facts (Happel [14]).

(a) There exists a triangle equivalence between $\underline{\mathrm{mod}}\hat{\Lambda}$ and $\mathcal{D}^b(\mathrm{mod}\,\Lambda)$ since Λ has finite global dimension (see [14] Chapter II 4.9 Theorem p.88).

(b) $\hat{\Lambda}$ is locally support-finite (see [14] Chapter V 3.1 Lemma p.201).
($i.e.$, for all $x \in \hat{\Lambda}$, $\sharp\{y \in \hat{\Lambda} \mid y \in \operatorname{supp} M \text{ with } M(x) \neq 0, M \in \operatorname{ind} \hat{\Lambda}\} < \infty$.)

Here, we need the critical result due to Dowbor-Skowroński (see [10] 2.5 Proposition p.319; also see [9] Lemma 2 p. 524).

Theorem 21 (Dowbor-Skowroński)**.** *Let $F_\lambda : \mathrm{mod}\,\hat{\Lambda} \to \mathrm{mod}\,\hat{\Lambda}/G$ be the push-down functor, where G is an admissible torsion-free group of k-linear automorphisms of $\hat{\Lambda}$. If $\hat{\Lambda}$ is locally support-finite, then F_λ is dense.*

The push-down functor preserves the projective modules and the injective modules (see Bongartz-Gabriel [7] 3.2 Proposition p.344), so that the induced functor $\underline{F_\lambda} : \underline{\mathrm{mod}}\hat{\Lambda} \to \underline{\mathrm{mod}}\hat{\Lambda}/G$ is well-defined. Then we have the following commutative diagram

$$\begin{tikzcd} \underline{\mathrm{mod}}\hat{\Lambda} \arrow[d, "\text{push-down}"'] & \cong & \mathcal{D}^b(\mathrm{mod}\,\Lambda) \arrow[dll, dotted, two heads] \\ \underline{\mathrm{mod}}\hat{\Lambda}/G & & \end{tikzcd} .$$

Since we have the dense functor from $\mathcal{D}^b(\mathrm{mod}\,\Lambda)$ to $\underline{\mathrm{mod}}\hat{\Lambda}/G$, we obtain

$$1 \geq \mathrm{der}\,.\dim \Lambda \geq \mathrm{stab}\,.\dim \hat{\Lambda}/G$$

by Remark 4 and Proposition 17.

We call such an algebra $\hat{\Lambda}/G$ a selfinjective algebra of type Q. Thus we have the following result by the argument above.

Proposition 22. *Any selfinjective algebra of type Q has stable dimension at most one.*

4.2. **Conjectures.** It has been conjectured, or asked by many experts, whether the following holds.

Conjecture 1. *Any artin algebra of tame representation type has representation dimension at most three.*

This is true for some classes of tame algebras, such as special biserial algebras (see Erdmann-Holm-Iyama-Schröer [11]) and domestic selfinjective algebras socle equivalent to a weakly symmetric algebra of Euclidean type (see Bocian-Holm-Skowroński [6]). Note that the latter algebras have stable dimension at most one by Proposition 6.

Also, any hereditary algebra has representation dimension at most three (see Auslander [3] Proposition p.58). Namely, any wild hereditary algebra must have representation dimension at most three. Hence the converse does not hold in general.

By Corollary 14, we pose a new conjecture for the stable dimension.

Conjecture 2. *Any (non-semisimple) selfinjective k-algebra of tame representation type has stable dimension at most one.*

By Proposition 22, any selfinjective algebra of Euclidean type has stable dimension at most one. According to Skowroński [22], a selfinjective algebra is of Euclidean type if and only if it is standard domestic of infinite type. Therefore, any domestic selfinjective standard algebra of infinite type has stable dimension at most one. Namely, we obtain a partial result for Conjecture 2.

Even if Q is a wild quiver, then any selfinjective algebra of type Q has stable dimension at most one. Thus the converse does not hold in general, similar to Conjecture 1.

A basic connected algebra A is *standard* (see [23]) if there exists a Galois covering $R \to R/G = A$ (see [12]) such that R is a simply connected locally bounded category (see [2]) and G is an admissible torsion-free group of k-linear automorphism of R. Thus it will be possible to calculate the stable dimension of selfinjective standard algebra in the same way as subsection 4.1, so that we pose a new conjecture.

Conjecture 3. *Any (non-semisimple) selfinjective standard k-algebra of tame representation type has stable dimension at most one.*

This is a weak version of Conjecture 2.

5. A question

In [20], Rouquier gave the first example of algebras having representation dimension at least four. Namely, he showed the following.

Theorem 23 (Rouquier). *Let $A = \bigwedge(k^n)$ be an exterior algebra. Then* $\operatorname{rep.dim} = n+1$, $\operatorname{der.dim} A = n$ *and* $\operatorname{stab.dim} A = n-1$.

Moreover, Han showed the following result in [13].

Theorem 24 (Han). *Any representation-finite artin algebra has derived dimension at most one.*

Since any non-semisimple selfinjective algebra is not derived equivalent to a hereditary algebra, any (non-semisimple) representation-finite selfinjective algebra has derived dimension one by Theorem 20. So, by Theorem 8, we obtain the following result.

Theorem 25. *If a non-semisimple selfinjective k-algebra A is representation-finite, then* $\operatorname{rep.dim} A = 2$, $\operatorname{der.dim} A = 1$ *and* $\operatorname{stab.dim} A = 0$.

Thus we have the following natural question.

Question. What about $\operatorname{rep.dim} A - \operatorname{der.dim} A$ and $\operatorname{der.dim} A - \operatorname{stab.dim} A$?

Theorems 23 and 25 suggest that the difference in the question above may be at least one.

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