Tilting modules arising from two-term tilting complexes

Hiroki Abe

In the representation theory of Artin algebras, the connection between tilting modules and torsion theories has been well studied. Brenner and Butler introduced the notion of tilting modules and showed that tilting modules induce torsion theories for module categories ([1]). Conversely, several authors asked when torsion theories determine tilting modules. Hoshino gave a construction of tilting modules from torsion theories for under certain conditions ([2]). Smalø characterized torsion theories which determine tilting modules using the notion of covariantly finite subcategories and contravariantly finite subcategories ([5]). On the other hand, Rickard introduced the notion of tilting complexes as a generalization of tilting modules and showed that tilting complexes induce equivalences between derived categories of module categories, which are called derived equivalences ([4]). Then Hoshino, Kato, and Miyachi pointed out that two-term tilting complexes induce torsion theories for module categories and studied the connection between two-term tilting complexes and torsion theories ([3]). In this talk, we will see that the torsion theories introduced by Hoshino, Kato, and Miyachi determine tilting modules. Let A be an Artin algebra and $T^\bullet$ a two-term tilting complex of A. We prove that the 0-th homology group $\mathrm{H}^0(T^\bullet)$ is a tilting module of $A/\mathfrak{a}$, where $\mathfrak{a}$ is the annihilator of $\mathrm{H}^0(T^\bullet)$. Furthermore, we determine the endomorphism algebra of $\mathrm{H}^0(T^\bullet)$. Let B be the endomorphism algebra of $T^\bullet$. Then the endomorphism algebra of $\mathrm{H}^0(T^\bullet)$ is given as $B/\mathfrak{b}$, where $\mathfrak{b}$ is the annihilator of $\mathrm{H}^0(T^\bullet)$. Thus, we know that any derived equivalence given by arbitrary two-term tilting complex always induces a derived equivalence between the corresponding factor algebras.

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The representation rings of the dihedral 2-groups

Erik Darpö

The representation ring (or Green ring) of a group algebra encodes the behaviour of the tensor product on its module category. In the modular situation, only the representation rings of the cyclic p-groups and the Vierergruppe are reasonably understood. I shall give a short review of what is known about the representation ring of a dihedral 2-group over an algebraically closed field of characteristic 2.

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Categorification of cluster algebra structures of coordinate rings of simple Lie groups

Laurent Demonet

As an introduction, we will explain why so called cluster algebra structures are particularly important in the specific case of coordinate rings of simple Lie groups. We will quickly recall the problem of totally positive elements (those every non trivial generalized minors of which are positive) of such a group and how cluster algebra structures classify optimal criteria for detecting such an element. Thus, will we give the categorification of a specific instance of this problem using module categories of preprojective algebras. Then we will emphasize the necessary enhancement to pass from the simply laced case to the non simply laced case.

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Hochschild cohomology of cluster-tilted algebras of types $\mathbb{A}_n$ and $\mathbb{D}_n$

Takahiko Furuya and Takao Hayami

In [4], Buan, Marsh and Reiten defined cluster-tilted algebras as the endomorphism algebras of cluster tilting objects in a cluster category [2], and gave a relationship between cluster categories and the module theory of these algebras. The aim of this talk is to describe the Hochschild cohomology of cluster-tilted algebras of Dynkin types $\mathbb{A}_n$ and $\mathbb{D}_n$ [3].

In [5], Buan and Vatne gave the bound quivers for all cluster-tilted algebras of type $\mathbb{A}_n$, explicitly. This tells that cluster-tilted algebras of type $\mathbb{A}_n$ are precisely (D, A)-stacked monomial algebras with $D = 2$ and $A = 1$ of [7]. Then we apply the result of [7] to describe the Hochschild cohomology rings modulo nilpotence for all cluster-tilted algebras of type $\mathbb{A}_n$.

On the other hand, in [1], Bastian, Holm and Ladkani provided a derived equivalence classification of cluster-tilted algebras of type $\mathbb{D}_n$. We see from this classification that there are cluster-tilted algebras of type $\mathbb{D}_n$ which are derived equivalent to a (D, A)-stacked monomial algebra. Again we apply [7] to determine the structure of the Hochschild cohomology rings modulo nilpotence for these algebras.

The final part of the talk describes the Hochschild cohomology of algebras Λ in a class of some special biserial algebras which contains a cluster-tilted algebra of type $\mathbb{D}_4$. We give the dimension of the Hochschild cohomology group $\mathrm{HH}^i(\Lambda)$ $(i \geq 0)$ of Λ completely, and show that the Hochschild cohomology ring modulo nilpotence $\mathrm{HH}^*(\Lambda)/\mathcal{N}$ is isomorphic to the polynomial ring $K[x]$.

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Derived autoequivalences and braid relations

Joseph Grant

Much of the important homological information of an algebra is contained in its derived category, and we are interested in the symmetries of this category, known as derived autoequivalences. There are two related general procedures for constructing such autoequivalences: the *spherical twists* of Seidel and Thomas in algebraic geometry [ST] and the autoequivalences of Rouquier and Zimmermann in the representation theory of finite groups [RZ]. These constructions are very similar, and in certain cases can be seen as Koszul dual. Both satisfy braid relations under appropriate conditions on the defining objects.

Working in the setting of finite-dimensional algebras, we explain a more general construction called a *periodic twist* [G]. This recovers the autoequivalences of Rouquier and Zimmermann and algebraic examples of other known derived autoequivalences from algebraic geometry. It also allows us to construct braided autoequivalences in a single step. Finally, we hope to present some recent work on other relations between derived autoequivalences.

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n-representation infinite algebras

Martin Herschend

In my talk I will present parts of ongoing joint work with Osamu Iyama and Steffen Oppermann on higher-dimensional Auslander-Reiten (AR) theory.

A natural starting point of this theory is n-representation finite algebras. Let Λ be a finite-dimensional algebra over a field K of global dimension at most n. We say that Λ is n-representation finite if it admits an n-cluster tilting module M, i.e., a finitely generated Λ-module M satisfying

$$\begin{aligned} \operatorname{add} M &= \{X \in \operatorname{mod}\Lambda \mid \operatorname{Ext}^i_\Lambda(M,X) = 0 \ \text{ for all } \ 0 < i < n\} \\ &= \{X \in \operatorname{mod}\Lambda \mid \operatorname{Ext}^i_\Lambda(X,M) = 0 \ \text{ for all } \ 0 < i < n\}. \end{aligned} \tag{1}$$

If such a module M exists it is essentially unique. In higher AR theory $\operatorname{add} M$ plays the role of the module category in classical AR theory and only contains finitely many indecomposables. This motivates the name n-representation finite. To be more precise, for $n = 1$, the condition (1) is equivalent to that every indecomposable Λ-module is a direct summand of M. Thus 1-representation finite algebras are exactly hereditary representation finite algebras.

The definition of n-representation infinite algebras is inspired by the above observation and properties of the Nakayama functor $\nu := D\mathbf{R}\mathrm{Hom}_\Lambda(-,\Lambda) : \mathcal{D}^b(\Lambda) \to \mathcal{D}^b(\Lambda)$. Recall that the n-AR translations

$$\tau_n := D\operatorname{Ext}^n_\Lambda(-,\Lambda) : \operatorname{mod}\Lambda \to \operatorname{mod}\Lambda \ \text{ and } \ \tau_n^- := \operatorname{Ext}^n_\Lambda(D\Lambda,-) : \operatorname{mod}\Lambda \to \operatorname{mod}\Lambda$$

are related to ν by $\tau_n = \mathrm{H}^0(\nu_n -)$ and $\tau_n^- = \mathrm{H}^0(\nu_n^{-1} -)$, where $\nu_n := \nu \circ [-n]$. For hereditary algebras, representation finiteness is characterized by the property that for each indecomposable projective module P there is $\ell \geq 0$ such that $\tau_1^{-\ell}(P)$ is indecomposable and injective (which implies that $\tau_1^{-(\ell+1)}(P) = 0$). This generalizes in the sense that Λ is n-representation finite if and only if each indecomposable projective Λ-module P satisfies that $\nu_n^{-\ell_P}(P)$ is an indecomposable injective Λ-module for some $\ell_P \geq 0$. In that case $\nu_n^{-i}(P) = \tau_n^{-i}(P)$ for all $0 \leq i \leq \ell_P$ and Λ has an n-cluster tilting module

$$M = \bigoplus_P \bigoplus_{i=0}^{\ell_P} \tau_n^{-i}(P).$$

It is worth noting that in this case $\nu_n^{-(\ell_P+1)}(P) = \nu^{-1}(\nu_n^{-\ell_P}(P))[n]$ is an indecomposable projective module shifted by n. In contrast, representation infinite hereditary algebras are characterized by the property that $\nu_1^{-i}(P)$ is a module for all projective modules P and $i \geq 0$. For this reason we say that Λ is n-representation infinite if $\nu_n^{-i}(\Lambda) \in \operatorname{mod}\Lambda$ for all $i \geq 0$.

This definition has the advantage that one can define higher analogues of the subcategories of preprojective and preinjective modules by

$$\mathcal{P} = \operatorname{add}\{\nu_n^{-i}(\Lambda) \mid i \geq 0\} \text{ and } \mathcal{I} = \operatorname{add}\{\nu_n^{i}(D\Lambda) \mid i \geq 0\}$$

respectively. I will present some properties of these subcategories and also explain what can be considered a higher analogue of regular modules.

Finally I will show how n-representation infinite algebras can be characterized using higher preprojective algebras and apply this result to present a large class of n-representation infinite algebras containing among others the Beilinson algebra.

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On a degeneration problem for Cohen-Macaulay modules

Naoya Hiramatsu

This is a joint work with Yuji Yoshino.

The degeneration problem of modules has been studied by many authors [3, 1, 4, 5]. In an Artinian case, it has been studied by Bongartz [1] in relation to the Auslander-Reiten quiver. In general, but in a commutative Noetherian case, Yoshino [4] generalized the theory of Bongartz for maximal Cohen-Macaulay modules and he has shown that any degenerations of maximal Cohen-Macaulay modules are fundamentally obtained by the degenerations of Auslander-Reiten sequences under some special conditions. For this, several order relations for modules, such as the hom order, the degeneration order, the extension order and the AR order, are introduced, and the connection among them has been studied.

The purpose of this talk is to give several examples of degenerations of maximal Cohen-Macaulay modules and to show how we can describe them. We will be able to give the complete description of degenerations over the hypersurface ring of type (A_n). More precisely, we have the following.

Theorem 1. *Let $R = k[[x_0, x_1, x_2, \cdots .x_d]]/(x_0^{n+1} + x_1^2 + x_2^2 + \cdots + x_d^2)$ where d is even and let M and N be maximal Cohen-Macaulay R-modules. Suppose that M degenerates to N. Then, M degenerates by extensions to N.*

Nowadays the stable analogue of degenerations for Cohen-Macaulay modules over a Gorenstein local ring is studied by Yoshino [6]. Our result relies heavily on his work.

We also investigate the relation among the extended versions of the degeneration order, the extension order and the AR order.

Theorem 2. *Let R be a Cohen-Macaulay complete local k-algebra which is of finite Cohen-Macaulay representation type. Then the following conditions are equivalent for maximal Cohen-Macaulay modules M, N:*

$$(1)\ M \leq_{DEG} N, \qquad (2)\ M \leq_{EXT} N, \qquad (3)\ M \leq_{AR} N.$$

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Quiver presentations of Grothendieck constructions

Mayumi Kimura

This is a joint work with H. Asashiba. Let I be a small category and $\Bbbk$ a commutative ring, and denote by $\Bbbk$-**Cat** the 2-category of all $\Bbbk$-categories.

In [2] Asashiba has shown that if (oplax) functors $X, X' \colon I \to \Bbbk\text{-}\mathbf{Cat}$ are derived equivalent, then so are their Grothendieck constructions $\mathrm{Gr}(X)$ and $\mathrm{Gr}(X')$. If categories $\mathcal{C}$ and $\mathcal{C}'$ are derived equivalent, then so are their diagonal functors $\Delta(\mathcal{C})$ and $\Delta(\mathcal{C}')$, and hence so are $\mathrm{Gr}(\Delta(\mathcal{C}))$ and $\mathrm{Gr}(\Delta(\mathcal{C}'))$. Therefore it will be important to investigate structures of Grothendieck constructions of functors.

Let A be a $\Bbbk$-algebra, which we regard as a $\Bbbk$-category with a single object, and let $\Delta(A) \colon I \to \Bbbk\text{-}\mathbf{Cat}$ be the diagonal functor (i.e., a functor sending all objects of I to A and all morphisms in I to the identity functor $\mathbb{1}_A$). In [2] it is noted that if I is a semigroup G, a poset S, or the free category $\mathbb{P}Q$ of a quiver Q, then the Grothendieck construction $\mathrm{Gr}(\Delta(A))$ of the diagonal functor $\Delta(A)$ is isomorphic to the semigroup algebra AG, the incidence algebra AS, or the path-algebra AQ, respectively. Further in [1] a quiver presentaion of the orbit category $\mathcal{C}/G$ for each $\Bbbk$-category $\mathcal{C}$ with a G-action is given in the case that $\Bbbk$ is a field (cf. [3] for the finite group case). This result can be seen as a computation of a quiver presentaion of the Grothendieck construction $\mathrm{Gr}(X)$ of each functor $X \colon G \to \Bbbk\text{-}\mathbf{Cat}$. In this talk, we generalize the two results above to obtain the following.

(1) We compute the Grothendieck construction $\mathrm{Gr}(\Delta(A))$ of the diagonal functor $\Delta(A)$ for each $\Bbbk$-algebra A and each small category I.

(2) We give a quiver presentation of the Grothendieck construction $\mathrm{Gr}(X)$ for each functor $X \colon I \to \Bbbk\text{-}\mathbf{Cat}$ and each small category I when $\Bbbk$ is a field.

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Quantum unipotent subgroup and dual canonical basis

Yoshiyuki Kimura

Let Q be an acyclic quiver and $\Lambda = \Lambda_Q$ be the associated preprojective algebra. For a Weyl group element w, C.Geiß, B.Leclerc and J.Schröer[1] gave an isomorphism between a cluster algebra structure and the coordinate ring of the unipotent subgroup using extension-closed Frobenius fullsubcategories $\mathcal{C}_w$ of the category of finite dimensional modules $\mathrm{f.d}\Lambda$ over Λ. Here an isomorphism is given by constructing a cluster tilting object of $\mathcal{C}_w$, which is associated with a reduced word $\overrightarrow{w}$ of w. They also proved that the *dual semicanonical basis* induces a basis of the cluster algebra and the set of cluster monomials are contained in the dual semicanonical basis. Moreover they propose a conjecture which the cluster monomials are contained in the set of the specialization of the *dual canonical basis*.

Motivated by their works, we propose a conjecture which quantize the results in [1] and give a setting of it. (We note that our conjecture is related to some basic problems which concerns about the basis of cluster algebra.) As a first step, we give a definition of the quantum coordinate ring of unitpotent subgroup *(quantum unipotent subgroup)* using Poincaré-Birkhoff-Witt basis of quantum envelopping algebra. In particular, we prove the compatibility of the dual canonical basis and the quantum coordinate ring. Using the theory of Kashiwara's crystal basis, we prove the multiplicative properties of quantum minors which is a quantum analogue of the cluster-tilting object in [1]. Recently, weak form of a part of our conjecture is solved by Geiß-Leclerc-Schröer in symmetric case in [2].

This talk is based on the results in [3].

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Weak Gorenstein dimension for modules and Gorenstein algebras

Mitsuo Hoshino and Hirotaka Koga

Let A be a left and right coherent ring. Denote by $D(-)$ both $\mathbf{R}\mathrm{Hom}^{\bullet}_{A}(-, A)$ and $\mathbf{R}\mathrm{Hom}^{\bullet}_{A^{\mathrm{op}}}(-, A)$ and by $\eta_{X^\bullet} : X^\bullet \to D(DX^\bullet)$ the canonical homomorphism in $\mathcal{D}^{\mathrm{b}}(\text{mod-}A)$ for $X^\bullet \in \mathcal{D}^{\mathrm{b}}(\text{mod-}A)$. Denote by $\mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{dbh}}$ the full triangulated subcategory of $\mathcal{D}^{\mathrm{b}}(\text{mod-}A)$ consisting of $X^\bullet \in \mathcal{D}^{\mathrm{b}}(\text{mod-}A)$ with $DX^\bullet \in \mathcal{D}^{\mathrm{b}}(\text{mod-}A^{\mathrm{op}})$. A complex $X^\bullet \in \mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{dbh}}$ with $\sup\{\, i \mid \mathrm{H}^i(X^\bullet) \neq 0\} = d < \infty$ is said to have finite weak Gorenstein dimension if $\mathrm{H}^i(\eta_{X^\bullet})$ is an isomorphism for all $i < d$ and $\mathrm{H}^d(\eta_{X^\bullet})$ is a monomorphism.

Extending the fact announced by Avramov [2], we will characterize complexes of finite weak Gorenstein dimension. Denote by $\hat{\mathcal{G}}_A$ the full additive subcategory of mod-A consisting of modules $X \in$ mod-A with $\mathrm{Ext}^i_A(X, A) = 0$ for $i \neq 0$, by $\mathcal{P}_A$ the full additive subcategory of mod-A consisting of projective modules and by $\hat{\mathcal{G}}_A/\mathcal{P}_A$ the residue category of $\hat{\mathcal{G}}_A$ over $\mathcal{P}_A$. Also, denote by $\mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{bdh}}/\mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{fpd}}$ the quotient category of $\mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{bdh}}$ over the épaisse subcategory $\mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{fpd}}$. We will show that the embedding $\hat{\mathcal{G}}_A \to \mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{bdh}}$ gives rise to a full embedding

$$F : \hat{\mathcal{G}}_A/\mathcal{P}_A \to \mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{bdh}}/\mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{fpd}},$$

that a complex $X^\bullet \in \mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{bdh}}$ has finite weak Gorenstein dimension if and only if there exists a homomorphism $Z[m] \to X^\bullet$ in $\mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{bdh}}$ inducing an isomorphism in $\mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{bdh}}/\mathcal{D}^{\mathrm{b}}(\text{mod-}A)_{\mathrm{fpd}}$ for some $Z \in \hat{\mathcal{G}}_A$ and $m \in \mathbb{Z}$ and that F is an equivalence if and only if $\hat{\mathcal{G}}_A$ is equal to the full additive subcategory of mod-A consisting of modules of Gorenstein dimension zero (see [1]).

Using the notion of weak Gorenstein dimension, we will characterize Gorenstein algebras. Let R be a commutative noetherian local ring and A a module-finite R-algebra. Our main theorem states that the following are equivalent: (1) inj dim $A =$ inj dim $A^{\mathrm{op}} < \infty$; and (2) Every simple $X \in A_{\mathfrak{p}}$ has finite weak Gorenstein dimension for all $\mathfrak{p} \in \mathrm{Supp}_R(A)$. Furthermore, in case A is a local ring, we will show that for any $d \geq 0$ the following are equivalent: (1) inj dim $A =$ inj dim $A^{\mathrm{op}} = d$; (2) inj dim $A =$ depth $A = d$; and (3) $A/\mathrm{rad}(A)$ has weak Gorenstein dimension d. Note that if inj dim $A =$ depth $A < \infty$ then A is a Gorenstein R-algebra in the sense of Goto and Nishida [3].

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Weakly closed graph

Kazunori Matsuda

The purpose of this talk is to study the notion of F-purity of a commutative Noetherian ring of positive characteristic. The definition of F-purity is as follows.

Definition 1. ([HoR]) Let k be an F-finite field of characteristic $p > 0$, and let R be a commutative reduced Noetherian ring containing k.

R is said to be *F-pure* if the Frobenius map $F_R : R \to R$, $x \mapsto x^p$ is pure, where a ring homomorphism $R \to S$ is pure if $N = N \otimes_R R \to N \otimes_R S$, $n \mapsto n \otimes 1$ is injective for all R-module N.

In this talk, we introduce the *binomial edge ideal*. Binomial edge ideal was introduced by [HeHiHrKR] and [O] independently.

Definition 2. Let G be a connected simple graph, and let $V(G) = [n]$ be the vertex set of G and $E(G)$ the edge set of G. Let $S = k[X_1, \dots , X_n, Y_1, \dots , Y_n]$ be a polynomial ring in $2n$ variables.

We define the *binomial edge ideal* of G as

$$J_G := (X_iY_j - X_jY_i \mid \{i, j\} \in E(G)) \subset S.$$

In general, J_G is radical (see [HeHiHrKR], [O]), hence S/J_G is reduced. So the following question is natural.

Question. When is J_G F-pure ?

In order to answer this question, we introduce the notion of *weakly closed graph*. This notion is a generalization of closedness (see [HeHiHrKR], [EHeHi]). Due to limitations of space, we cannot state the definition of weakly closedness. Instead, we give a necessary and sufficient condition for a connected simple graph to be weakly closed.

Theorem 3. *Let G be a connected simple graph. Then the following conditions are equivalent:*

(1) *G is weakly closed.*
(2) *For all i, j such that $\{i, j\} \in E(G)$ and $j > i + 1$, the following assertion holds: for all $i < k < j$, $\{i, k\} \in E(G)$ or $\{k, j\} \in E(G)$.*

Corollary 4. *Closed graphs and complete r-partite graphs are weakly closed.*

Theorem 5. *If G is weakly closed, then S/J_G is F-pure.*

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POLYCYCLIC CODES AND SEQUENTIAL CODES

MANABU MATSUOKA

1. ABSTRACT

Let $\mathbf{F}$ be a finite field. A linear $[n,k]$-code over $\mathbf{F}$ is a k-dimensional subspace C of the vector space $\mathbf{F}^n = \{(a_0, \cdots, a_{n-1}) | a_i \in \mathbf{F}\}$. A linear code $C \subseteq \mathbf{F}^n$ is called cyclic if $(a_0, a_1, \cdots, a_{n-1}) \in C$ implies $(a_{n-1}, a_0, a_1, \cdots, a_{n-2}) \in C$. The notion of cyclicity has been generalized in several ways.

In [3], S. R. López-Permouth, B. R. Parra-Avila and S. Szabo studied the duality between polycyclic codes and sequential codes. By the way, J. A. Wood establish the extension theorem and MacWilliams identities over finite frobenius rings in [7].

Definition 1. *Let C be a linear code of length n over $\mathbf{F}$. C is a polycyclic code induced by c if there exists a vector $c = (c_0, c_1, \cdots, c_{n-1}) \in \mathbf{F}^n$ such that for every $(a_0, a_1, \cdots, a_{n-1}) \in C$, $(0, a_0, a_1, \cdots, a_{n-2}) + a_{n-1}(c_0, c_1, \cdots, c_{n-1}) \in C$. In this case we call c an associated vector of C.*

Definition 2. *Let C be a linear code of length n over $\mathbf{F}$. C is a sequential code induced by c if there exists a vector $c = (c_0, c_1, \cdots, c_{n-1}) \in \mathbf{F}^n$ such that for every $(a_0, a_1, \cdots, a_{n-1}) \in C$, $(a_1, a_2, \cdots, a_{n-1}, a_0c_0 + a_1c_1 + \cdots + a_{n-1}c_{n-1}) \in C$. In this case we call c an associated vector of C.*

We generalize the notion of cyclicity of codes, that is, polycyclic codes and sequential codes. We study the relation between polycyclic codes and sequential codes. Furthermore, we characterized the family of some constacyclic codes.

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A note on dimension of triangulated categories.

Hiroyuki Minamoto

In [2] R. Rouquier introduced dimension of triangulated categories and showed that it relate by inequalities to other dimension in algebraic geometry or in representation theory(see also [3]). Dimension of triangulated categories is studied by many researchers.

We study the behavior of the dimension of the perfect derived category $\mathrm{Perf}(A)$ of a dg-algebra A over a field k under a base field extension K/k. For a field extension K/k, we denote $A \otimes_k K$ by A_K.

Theorem 1. (1) *For an algebraic extension K/k, we have*

$$\mathrm{tridimPerf}(A) \leq \mathrm{tridimPerf}(A_K).$$

(2) *If moreover K/k is separable, then equality holds.*

As an application we prove the following theorem, which shows evidence that dimension of triangulated categories captures representation theoretic property.

The stable category $\underline{\mathrm{mod}}A$ plays an important role in the study of self-injective algebra A (cf. [1, 3]). If a self-injective algebra A is of finite representation type then the dimension of the stable category $\underline{\mathrm{mod}}A$ is zero. Then a natural question arises as to whether the converse should also hold.

Theorem 2. *Let A be a self-injective finite dimensional algebra over a perfect field k. If $\mathrm{tridim}(\underline{\mathrm{mod}}A) = 0$, then A is of finite representation type.*

In the case when k is an algebraically closed field, this theorem is proved by M. Yoshiwaki in [4]. Our proof depends on his result.

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Hidden Hecke Algebra and Koszul Duality

H. Miyachi

The talk will be based on a joint work with **Joe Chuang** (City, London).

`http://www.math.nagoya-u.ac.jp/~miyachi/preprints/lrkszl20.pdf`

The q-Schur algebras $S(n,r)$ have been studied intensively for about 30 years. Many researchers got some motivation for studying $S(n,r)$. One reason for this is, for example, these algebras $S(n,r)$ completely describe the decomposition numbers for quantum general linear groups, infinite general linear groups in the defining characteristic and finite general linear groups in non-defining characteristic.

The Iwahori Hecke algebra $H_n(q)$ of symmetric group S_n appears in a q-Schur algebra by definition. Indeed, one definition of the q-Schur algebra is given as an endomorphism ring of a certain "permutation" module over $H_n(q)$.

In my talk, we shall introduce a hidden Hecke algebra which turns out to be a very degenerate Ariki-Koike algebra and which is a derived equivalence class invariant of (technically abelian defect) blocks in the q-Schur algebras. Our hidden Hecke algebra is hidden in q-Schur algebra and is defined as a Yoneda algebra of a certain semisimple module.

We expect that a series of highest weight covers of our hidden Hecke algebra naturally appear in GGOR category $\mathcal{O}$'s of rational Cherednik algebras and those O's are Koszul duals of q-Schur algebra blocks. We believe that their higher level analogue exists and this is a level-rank duality in a traditional terminology in Lie theory.

Our conjecture is almost proved by Shan-Varagnolo-Vasserot arXiv:1107.0146v1.

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APR tilting modules and quiver mutations

Yuya Mizuno

Recently tilting theory has been used as an essential tool in many areas of mathematics. In particular, tilting modules play important role in representation theory of algebras. Let T be a tilting module over an algebra Λ. Then two module categories over Λ and $\mathrm{End}_{\Lambda}(T)$ have similar structures and, in particular, they are derived equivalent. For these reasons, the relationship between Λ and $\mathrm{End}_{\Lambda}(T)$ has been widely studied by many researchers.

In this talk, we deal with APR tilting modules and investigate how the ordinary quiver with relations of Λ will change by taking the endomorphism algebras $\mathrm{End}_{\Lambda}(T)$.

Let us recall the definition of APR tilting modules. In 1979, Auslander-Platzeck-Reiten introduced the following notion, as a module-theoretic interpretation of reflection functors, and it is called APR tilting modules now.

Definition 1. Let Λ be a finite dimensional algebra and P be a simple projective noninjective Λ-module. Then Λ-module $T := \tau^{-}P \oplus \Lambda/P$ is called an *APR tilting module*, where τ^{-} denotes the inverse of the Auslander-Reiten translation.

One of the remarkable properties of APR tilting modules over path algebras is that the quiver of endomorphism algebra $\mathrm{End}_{\Lambda}(T)$ is obtained from the ordinary quiver of Λ by reversing all arrows associated with the source vertex.

On the other hand, in 2000, Fomin-Zelevinsky defined *mutations* and introduced the notion of cluster algebras. Roughly speaking, for a given quiver, the mutation is the operation of changing arrows corresponding to one vertex. In particular, if the vertex is a sink or a source, the operation is just reversing all the arrows at the vertex. Derksen-Weyman-Zelevinsky developed the mutations and applied to quivers with potentials. A Further Generalization was given by Amiot-Oppermann. They introduced graded right (left) mutations and extended the mutations to graded quivers with potentials.

In this talk we consider APR tilting module in a more general setting, and explain the connection between the quiver of the endomorphism algebra and graded mutations.

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The Example by Stephens

To the memory of Professor Goro Azumaya

Kaoru Motose

We set the next numbers $f,\ t$ for primes $p < q$,

$$f := \frac{q^p - 1}{q - 1} \text{ and } t := \frac{p^q - 1}{p - 1}.$$

Feit and Thompson [3] conjectured that f never divides t. If it would be proved, their odd order theorem [4] would be greatly simplified (see [1] and [5]).

Using computer, Stephens [10] found the example about a common prime divisor r of f and t as follows: for $p = 17$ and $q = 3313$, $r = 112643 = 2pq+1$ is the greatest common divisor of f and t. This example is so far of the only one with $(f, t) > 1$.

In this talk, using Artin map (see [9]), we shall show that both 17 and 3313 are common index divisors (gemeinsamer ausserwesentlicher Discriminantenteiler) of some subfields of a cyclotomic field $\mathbb{Q}(\zeta_r)$ where $r = 112643$ and $\zeta_r = e^{\frac{2\pi i}{r}}$, and some results in [7, 8] shall be again proved using theorems in [2, 6, 9].

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Hom-orthogonal partial tilting modules for Dynkin quivers

Hiroshi NAGASE and Makoto NAGURA

Let $Q = (Q_0, Q_1)$ be a Dynkin quiver having n vertices, where Q_0, Q_1 is the set of vertices, arrows of Q, respectively. We denote by Λ its path algebra over an algebraically closed field of characteristic zero.

Each finitely generated Λ-module X with dimension vector $\boldsymbol{d} = \mathbf{dim}X$ can be regarded as a representation of Q; that is, a point of the vector space $\mathrm{Rep}(Q, \boldsymbol{d})$ that consists of representations with dimension vector $\boldsymbol{d} = (d^{(i)})_{i \in Q_0} \in \mathbb{Z}^n_{\geq 0}$. Then the direct product $GL(\boldsymbol{d}) = \prod_{i \in Q_0} GL(d^{(i)})$ acts naturally on $\mathrm{Rep}(Q, \boldsymbol{d})$. Since Λ is representation-finite, $\mathrm{Rep}(Q, \boldsymbol{d})$ has a unique dense $GL(\boldsymbol{d})$-orbit; thus $(GL(\boldsymbol{d}), \mathrm{Rep}(Q, \boldsymbol{d}))$ is a prehomogeneous vector space (abbreviated PV). It follows from the Artin–Voigt theorem that the condition that X is a *partial tilting* module can be interpreted to that the $GL(\boldsymbol{d})$-orbit containing X is dense in $\mathrm{Rep}(Q, \boldsymbol{d})$; On the other hand, the condition that X is *hom-orthogonal* corresponds to that the isotropy subgroup (or, stabilizer) at $X \in \mathrm{Rep}(Q, \boldsymbol{d})$ is reductive. Therefore we are interested in hom-orthogonal partial tilting Λ-modules, because they correspond to generic points of *regular PVs* associated with Q; see [2, Theorem 2.28].

According to Happel [1], if a Λ-module corresponding to a point of the dense orbit of a PV $(GL(\boldsymbol{d}), \mathrm{Rep}(Q, \boldsymbol{d}))$ has s pairwise non-isomorphic indecomposable direct summands, then the PV has exactly $n - s$ basic relative invariants. Therefore we are also interested in the number of the indecomposable direct summands of basic hom-orthogonal partial tilting Λ-modules.

In this talk, we will give the number of basic hom-orthogonal partial tilting Λ-modules as an explicite function in n and s for quivers of type $\mathbb{A}_n$ or $\mathbb{D}_n$. As a consequence, we obtain a minimum value of the number of basic relative invariants of corresponding regular PVs.

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The Noetherian properties of the rings of differential operators on central 2-arrangements

NAKASHIMA NORIHIRO

Let K be a field of characteristic zero. For a commutative K-algebra R, we define K-vector spaces of linear differential operators by

$$\mathcal{D}^0(R) := \{\theta \in \mathrm{End}_K(R) \mid a \in R, \theta a - a\theta = 0\},$$
$$\mathcal{D}^m(R) := \left\{\theta \in \mathrm{End}_K(R) \mid a \in R, \theta a - a\theta \in \mathcal{D}^{m-1}(R)\right\} \quad (m \geq 1).$$

We set $\mathcal{D}(R) := \bigcup_{m\geq 0} \mathcal{D}^m(R)$, and we call $\mathcal{D}(R)$ the ring of differential operators of R. Let $S := K[x_1, \ldots, x_n]$ denote the polynomial ring.

There has been a lot of research on finiteness properties of the rings of differential operators. For example, it is well known that $\mathcal{D}(R)$ is Noetherian, if R is a regular domain (see [1]).

Let $\mathcal{A} = \{H_i \mid i = 1, \ldots, r\}$ be a central (hyperplane) arrangement (i.e., all hyperplanes in $\mathcal{A}$ contain the origin) in K^n. Let I be the defining ideal of $\mathcal{A}$. Holm [2] proved that the ring of differential operators of the coordinate ring S/I is finitely generated when $\mathcal{A}$ is a generic hyperplane arrangement.

In this talk, I will talk about the Noetherian properties of $\mathcal{D}(S/I)$ when $\mathcal{A}$ is a central 2-arrangement. In addition, its graded ring associated to the order filtration is not Noetherian when the number of the consistuent hyperplanes is greater than 1

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Hochschild cohomology of quiver algebras defined by two cycles and a quantum-like relation

Daiki Obara

Let k be a field, $\mathrm{HH}^*(A)$ the Hochschild cohomology ring of a finite dimensional k-algebra A and $\mathcal{N}$ the ideal of $\mathrm{HH}^*(A)$ generated by all homogeneous nilpotent elements. In [3], using the Hochschild cohomology ring modulo nilpotence $\mathrm{HH}^*(A)/\mathcal{N}$ Snashall and Solberg defined a support variety of A-module. And, in [2], Snashall gave the question to as whether we can give necessary and sufficient conditions on a finite dimensional algebra A for $\mathrm{HH}^*(A)/\mathcal{N}$ to be finitely generated as an algebra. On the other hand, in [1], Erdmann, Holloway, Snashall, Solberg and Taillefer showed that some geometric properties of the support variety of A-modules and some representation theoretical properties of the algebra A under the following finiteness conditions:

- H is a commutative Noetherian graded subalgebra of $\mathrm{HH}^*(A)$ with $H^0 = \mathrm{HH}^0(A)$.
- $\mathrm{Ext}^*_A(A/J, A/J)$ is a finitely generated H-module.

where J denotes the Jacobson radical of A.

In this talk, we consider the k-algebra A_q defined by two cycles and a quantum-like relation depending on a nonzero element q in k. We determine the projective bimodule resolution of A_q and the Hochschild cohomology ring of A_q and give necessary and sufficient conditions for A_q to satisfy the finiteness conditions.

For $s, t \geq 1$, let Q be the two cycles with $s+t-1$ vertices $1 = a(1) = b(1)$, $a(2), \ldots, a(s)$, $b(2), \ldots, b(t)$ and $s+t$ arrows $\alpha_i{:}a(i) \to a(i+1)$, $\beta_j{:}b(j) \to b(j+1)$ for $1 \leq i \leq s$, $1 \leq j \leq t$ where we regard the numbers i modulo s and j modulo t. Paths are written from right to left.

Let $A_q = kQ/I_q$ where I_q is the ideal of kQ generated by

$$X^{sa}, X^sY^t - qY^tX^s, Y^{tb}$$

for $a, b \geq 2$, $X{:=}\, \alpha_1 + \alpha_2 + \cdots + \alpha_s$ and $Y{:=}\, \beta_1 + \beta_2 + \cdots + \beta_t$. Then we have the following results.

Theorem 1. *If q is a root of unity, then $\mathrm{HH}^*(A_q)/\mathcal{N}$ is isomorphic to the polynomial ring of two variables.*

Theorem 2. *If q is not a root of unity, then $\mathrm{HH}^*(A_q)/\mathcal{N} \cong k$.*

Theorem 3. *A_q satisfies the finiteness conditions if and only if q is a root of unity.*

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Alternative polarizations of Borel fixed ideals and Eliahou-Kervaire type resolution

Ryota Okazaki (JST CREST) and Kohji Yanagawa

A *Borel fixed (monomial) ideal* I of a polynomial ring $K[x_1, \ldots, x_n]$ is very important in combinatorial/computational commutative algebra, since it appears as the *generic initial ideal* of a homogeneous ideal of S (if $\mathrm{char}(K) = 0$) and the "squarefree-zation" I^σ of I plays a role in combinatorics on simplicial complexes (c.f. [3]).

A minimal free resolution of a Borel fixed ideal was constructed by Eliahou and Kervaire [2]. While the minimal free resolution is unique up to isomorphism, its "description" depends on the choice of a free basis. Hence further analysis of the resolution is still an active topic in this area.

In this talk, we present a new description of the minimal free resolution of a Borel fixed ideal I which is also applicable to the squarefree-zation I^σ. This is a new feature the Eliahou-Kervaire resolution does not possesses. The main tool is the "alternative" polarization b-pol(I) of I ([5]). Our resolutions are supported by regular cell complexes given by *discrete Morse theory*, as in [1]. We also remark that our approach is a generalization of a recent work of Nagel and Reiner [4].

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Sharp bounds for Hilbert coefficients of parameters

Kazuho Ozeki

This is a joint work with Shiro Goto.

The purpose of my talk is to study the problem of when the Hilbert coefficients of parameter ideals in a Noetherian local ring have uniform bounds, and when this is the case, to ask for their sharp bounds.

To state the problem and the results also, let us fix some notation. In what follows, let A be a commutative Noetherian local ring with maximal ideal $\mathfrak{m}$ and $d = \dim A > 0$. We say that our local ring is a generalized Cohen-Macaulay ring, if the local cohomology modules $\mathrm{H}^i_{\mathfrak{m}}(A)$ are finitely generated for all $i \neq d$. Let $h^i(A) = \ell_A(\mathrm{H}^i_{\mathfrak{m}}(A))$ for each $i \in \mathbb{Z}$.

With this notation our first purpose is to study the problem of when the sets

$$\Lambda_i(A) = \{\mathrm{e}^i_Q(A) \mid Q \text{ is a parameter ideal in } A\}$$

are finite for all $1 \leq i \leq d$, where $\mathrm{e}^i_Q(A)$ denotes the Hilbert coefficients of A with respect to Q. Then the first main result of my talk is stated as follows.

Theorem 1. *Suppose that $d \geq 2$. Then the following conditions are equivalent.*

(1) *A is a generalized Cohen-Macaulay ring.*
(2) *The set $\Lambda_i(A)$ is finite for all $1 \leq i \leq d$.*

The heart of the proof of the implication (1) $\Rightarrow$ (2) is, in the case where A is a generalized Cohen-Macaulay ring, the existence of uniform bounds of the Castelnuovo-Mumford regularity $\operatorname{reg} \mathrm{G}(Q)$ of the associated graded rings $\mathrm{G}(Q) = \bigoplus_{n \geq 0} Q^n/Q^{n+1}$ of Q. Then we have the following, where $\mathbb{I}(A) = \sum_{j=0}^{d-1} \binom{d-1}{j} h^j(A)$.

Theorem 2. *Suppose that A is a generalized Cohen-Macaulay ring. Let Q be a parameter ideal in A and put $r = \operatorname{reg} \mathrm{G}(Q)$. Then*

(1) $|\mathrm{e}^1_Q(A)| \leq \mathbb{I}(A)$.
(2) $|\mathrm{e}^i_Q(A)| \leq 3 \cdot 2^{i-2}(r+1)^{i-1}\mathbb{I}(A)$ *for* $2 \leq i \leq d$.

Although the finiteness problem of $\Lambda_i(A)$ is settled affirmatively, the bounds in Theorem 2 is very huge. It seems to be a different problem to ask for the sharp bounds for the values of $\mathrm{e}^i_Q(A)$ of parameter ideals Q.

When A is a generalized Cohen-Macaulay ring with $d \geq 2$, the behavior of $\mathrm{e}^1_Q(A)$ for parameter ideals Q are rather satisfactorily understood by [1, 2, 3].

The second purpose is to study the question of how about $\mathrm{e}^2_Q(A)$. The sharp bound for $\mathrm{e}^2_Q(A)$ in the case where A is a generalized Cohen-Macaulay ring is given.

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Preprojective algebras and crystal bases of quantum groups
（前射影多元環と量子群の結晶基底）

Yoshihisa Saito

Abstract: Let Q be a finite quiver and $P(Q)$ the corresponding preprojective algebra. It is known that most preprojective algebras are of wild representation type. More precisely, $P(Q)$ is of finite (*resp.* tame) representation type if and only if Q is of Dynkin type A_n with $n \leq 4$ (*resp.* A_5 or D_4). Therefore, it is very difficult to study the category of finite dimensional modules of $P(Q)$ directly.

However, there is another way to study the structure of representations of $P(Q)$. It is given by the theory of "crystal bases" which was introduced by Kashiwara in the study of representation theory of quantum groups. In this talk, we try to explain how to use the theory of crystal bases in the study of representation theory of preprojective algebras.

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Matrix Factorizations, Orbifold Curves And Mirror Symmetry

Atsushi Takahashi

Mirror symmetry is now understood as a categorical duality between algebraic geometry and symplectic geometry. One of our motivations is to apply some ideas of mirror symmetry to singularity theory in order to understand various mysterious correspondences among isolated singularities, root systems, Weyl groups, Lie algebras, discrete groups, finite dimensional algebras and so on.

In my talk, I'll give an a summary of our results on categories of maximally-graded matrixfactorizations, in particular, on the existence of full strongly exceptional collections which gives triangulated equivalences to derived categories of finite dimensional modules over finite dimensional algebras. I'll also explain our motivations, the homological mirror symmetry conjecture, a relation between orbifold curves and cusp singularities via Orlov type semi-orthogonal decompositions.

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A generalization of costable torsion theory

Yasuhiko Takehana

In [R. L. Bernhardt, On Splitting in Hereditary Torsion Theories, P. J. M. Vol 39(1971),No1,31-38], a dualization of stable torsion theory is defined and characterized. In this talk we generalize this. Let R be a right perfect ring with identity and Mod-R be the categories of right R-modules. A subfunctor of the identity functor of Mod-R is called a preradical. For a preradical σ, we denote $\mathcal{T}_\sigma := \{M \in \text{Mod-}R\ ;\ \sigma(M) = M\}$ and $\mathcal{F}_\sigma := \{M \in \text{Mod-}R\ ;\ \sigma(M) = 0\}$. A right R-module M is called σ-projective if the functor $\text{Hom}_R(M, \quad)$ preserves the exactness for any exact sequence $0 \to A \to B \to C \to 0$ with $A \in \mathcal{F}_\sigma$. A preradical σ is idempotent[radical] if $\sigma(\sigma(M)) = \sigma(M)[\sigma\,(M/\sigma(M)) = 0]$ for a module M, respectively. $0 \to K(M) \to P(M) \xrightarrow{f} M \to 0$ is called a projective cover of a module M if $P(M)$ is projective and $K(M) := \ker f$ small in $P(M)$. We put $K_\sigma(M) = K(M)/\sigma(K(M))$ and $P_\sigma(M) = P(M)/\sigma(K(M))$ for a preradical σ. If σ is an idempotent radicisal, $0 \to K_\sigma(M) \to P_\sigma(M) \to M \to 0$ is called a σ-projective cover of a module M. We call a preradical t σ-costable if $\mathcal{F}_t$ is closed under taking σ-projective covers.

Theorem For a radical t and an idempotent radical σ, the following conditions except (4) are equivalent. If moreover $\mathcal{F}_t$ is closed under taking $\mathcal{F}_\sigma$-factor modules(that is, M/N is in $\mathcal{F}_t$ for any $M \in \mathcal{F}_t$ and $N \in \mathcal{F}_{\sigma}$.), then all conditions are equivalent.

(1) t is σ-costable, that is, $\mathcal{F}_t$ is closed under taking σ-projective covers.

(2) The class of σ-projective modules is closed under taking the unique maximal t-torsionfree factor modules, that is, $P/t(P)$ is σ-projective for any σ-projective module P.

(3) Consiser the following commutative diagram.

$$\begin{array}{ccc} P_\sigma(M) & \xrightarrow{h} & M \to 0 \\ f\downarrow & & \downarrow j \\ P_\sigma(M/t(M)) & \underset{g}{\to} & M/t(M) \to 0 \end{array}$$

As $P_\sigma(M)$ is σ-projective, there exists an $f \in \text{Hom}_R(P_\sigma(M), P_\sigma(M/N))$ such that $h = gf$, where j is a canonical epimorphism and h and g is an epimorphism associated with its projective cover.

Then $t(P_\sigma(M))$ is contained in $\ker f$.

(4) $\mathcal{F}_t$ is closed under taking σ-coessential extensions, that is, for any small submodule $N \in \mathcal{F}_\sigma$ of a module M with $M/N \in \mathcal{F}_t$, it follows that M is in $\mathcal{F}_t$.

(5) For any σ-projective module P with $t(P) \in \mathcal{F}_\sigma$, $t(P)$ is a direct sumand of P.

Graded Frobenius algebras and quantum Beilinson algebras

Kenta Ueyama

Frobenius algebras are one of the important class of algebras studied in representation theory of finite dimensional algebras. The purpose of this talk is to study when given graded Frobenius Koszul algebras are graded Morita equivalent, that is, they have equivalent graded module categories.

In general, it is easier to determine if two graded algebras are isomorphic as graded algebras than to determine if they are graded Morita equivalent. In this talk, for every co-geometric Frobenius Koszul algebra A, we define another graded algebra $\overline{A}$, and see that if two co-geometric Frobenius Koszul algebras A, A' are graded Morita equivalent, then $\overline{A}, \overline{A'}$ are isomorphic as graded algebras.

Recently, Minamoto [2] introduced a nice class of finite dimensional algebras of finite global dimension, called (quasi-)Fano algebras, which are very interesting class of algebras to study and classify. It was shown that, for every graded Frobenius Koszul algebra A, we can define another algebra B , called the quantum Beilinson algebra (cf. [1], [4]), which turns out to be a quasi-Fano algebra ([3]). Moreover, we see that two graded Frobenius Koszul algebras A, A' are graded Morita equivalent if and only if B, B' are isomorphic as algebras. In general, it is not easy to check if two ungraded algebras are isomorphic as algebras by constructing an explicit algebra isomorphism. On the other hand, it is much easier to check if two graded algebras $T(V)/I$ and $T(V')/I'$ generated in degree 1 are isomorphic as graded algebras since any such isomorphism is induced by the vector space isomorphism $V \to V'$. In this sense, the results in this talk are useful for the classification of quantum Beilinson algebras.

This talk is based on joint work with Izuru Mori [6].

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Applications of Finite Frobenius Rings to Algebraic Coding Theory

Jay A. Wood

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These lectures will address some applications of finite Frobenius rings to prove theorems of interest in algebraic coding theory. The primary emphasis will be on generalizations to linear codes over rings of the MacWilliams identities and the MacWilliams extension theorem on linear isometries.

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Lecture 1. Two Theorems of MacWilliams over Finite Frobenius Rings

The basic language of algebraic coding theory will be introduced. The MacWilliams identities relate the weight enumerator of a linear code to that of its dual code. The MacWilliams extension theorem states that an isomorphism between linear codes that preserves Hamming weight extends to a monomial transformation. Both of these theorems have character-theoretic proofs that generalize from finite fields to finite Frobenius rings.

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Lecture 2. Using Coding Theory to Characterize Finite Frobenius Rings

The converse of the MacWilliams extension theorem characterizes finite Frobenius rings. That is, if a finite ring has the property that every linear isometry between linear codes extends to a monomial transformation, then the ring is Frobenius. The proof follows a strategy of Dinh and López-Permouth that depends upon producing counter-examples to the extension property over certain matrix modules.

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Tilting theory for stable graded module categories over graded self-injective algebras

Kota Yamaura

Our aim is to study the structure of algebraic triangulated categories. The notion of algebraic triangulated categories is defined to be a triangulated category constructed as the stable category of some Frobenius category (cf. [1]). This class of triangulated categories is important for representation theory of algebras because it contains the following examples of triangulated categories coming from algebras.

(1) The homotopy category $\mathcal{K}^{\mathrm{b}}(\mathrm{proj}\Lambda)$ of bounded chain complexes in the category $\mathrm{proj}\Lambda$ of finitely generated projective modules over an algebra Λ.
(2) The derived category $\mathcal{D}^{\mathrm{b}}(\mathrm{mod}\Lambda)$ of the category $\mathrm{mod}\Lambda$ of finitely generated modules over an algebra Λ.
(3) The stable category $\underline{\mathrm{mod}}A$ of the category $\mathrm{mod}A$ of finitely generated modules over a self-injective algebra A.

In the class of algebraic triangulated categories, one of the class of categories which can be easily treated is the above example (1). Therefore it is natural to ask when is a given algebraic triangulated category triangle-equivalent to example (1). B. Keller [2] gave an answer of this question by existence of tilting objects.

Definition. Let $\mathcal{T}$ be a algebraic triangulated category. An object T of $\mathcal{T}$ is called a *tilting object* if it satisfies the following conditions.

(1) $\mathrm{Hom}_{\mathcal{T}}(T, T[i]) = 0$ for $i \neq 0$.
(2) $\mathcal{T}$ coincides with the smallest full triangulated subcategory of $\mathcal{T}$ which is closed under direct summands and contains T.

Theorem. [2] *Let $\mathcal{T}$ be a algebraic triangulated category. If it has a tilting object T, then there exists a triangle-equivalence $\mathcal{T} \simeq \mathcal{K}^{\mathrm{b}}(\mathrm{projEnd}_{\mathcal{T}}(T))$.*

By this Keller's result, it is a fundamental problem for studying the structure of a given algebraic triangulated category that finding a tilting object in a given one.

In my talk, we consider this problem for the example (3). However it is easily seen that $\underline{\mathrm{mod}}A$ has no tilting objects. So we impose additional condition on A, and change triangulated categories which we consider as follows.

Let $A = \bigoplus_{i \geq 0} A_i$ be a positively graded self-injective algebra. We consider the category $\mathrm{mod}^{\mathbb{Z}}A$ of finitely generated $\mathbb{Z}$-graded A-modules. One can see that $\mathrm{mod}^{\mathbb{Z}}A$ is a Frobenius category. Therefore the stable category $\underline{\mathrm{mod}}^{\mathbb{Z}}A$ of $\mathrm{mod}^{\mathbb{Z}}A$ is regarded as an algebraic triangulated category.

For the above algebraic triangulated category $\underline{\mathrm{mod}}^{\mathbb{Z}}A$, we will show that existence of tilting objects is characterized as follows.

Theorem. [Y] *Let A be a positively graded self-injective algebra. Then $\underline{\mathrm{mod}}^{\mathbb{Z}}A$ has a tilting object if and only if A_0 has finite global dimension.*

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On derived simple algebras

Dong Yang

Recollement is a way to deconstruct a triangulated category into smaller pieces. It helps to prove homological conjectures in representation theory of algebras (e.g. the finitistic dimension conjecture), to compute homological invariants (e.g. the Hochschild cohomology), to construct/classify t-structures (e.g. the perverse t-structure), and so on. In this talk I will focus on recollements of derived modules categories. A basic problem is to determine the derived simple algebras, that is, those algebras which do not admit nontrivial recollements by derived module categories. In works of Angeleri-Huegel, Koenig, Liu and myself, the following algebras are shown to be derived simple

1. local algebras (AKL)
2. simple artinian algebras (AKL)
3. indecomposable commutative algebras (AKLY)
4. blocks of finite group algebras (LY)

I will sketch the proof for blocks of finite group algebras.

Recollements generated by idempotents

Dong Yang

I will talk about the recollement of a derived module category generated by an idempotent of the algebra. Many classical examples of recollements are of this kind. For example, let A be a quasi-hereditary algebra and e an idempotent such that AeA is a heredity ideal; then there is a recollement of the derived category D(A) by D(eAe) and D(A/AeA). In general, the category D(A/AeA) should be replaced by D(B) for a dg algebra B, which is an 'enhancement' of A/AeA and satisfies many nice properties. Then I will move on to the application to the study of singularity categories.

Introduction to representations of Cohen-Macaulay modules and their degenerations

Yuji YOSHINO

The aim of my first talk is to give an introduction of Cohen-Macaulay representation theory. After giving several definitions concerning Cohen-Macaulay modules, we shall show some properties of Cohen-Macaulay modules. In particular, we shall concern ourselves about the relationship between the Cohen-Macaulay representation type and the singular locus of the ring. I am planning to give a proof of Huneke-Leuschke theorem which has solved the Schreyer's conjecture. Main references are [5, 3, 4].

In the second talk, I would like to discuss the degeneration problem of Cohen-Macaulay modules. Most of these results are published in the series of my papers [1, 2, 6, 7, 8, 9, 10].

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Subcategories of extension modules related to Serre subcategories

Takeshi Yoshizawa

In [1], P. Gabriel showed the existence of a lattice isomorphism between the set consisting of specialization closed subsets of the set of prime ideals and the set of Serre subcategories of the category consisting of finitely generated modules. In this talk, we consider subcategories consisting of the extensions of modules in two given Serre subcategories to find a method of constructing Serre subcategories of the category of modules. We shall give a criterion for this subcategory to be a Serre subcategory.

Throughout this talk, R is a commutative noetherian ring and we denote by R-Mod the category of R-modules.

Definition 1. Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be Serre subcategories of R-Mod. We denote by $(\mathcal{S}_1, \mathcal{S}_2)$ a subcategory consisting of extension modules of $\mathcal{S}_1$ by $\mathcal{S}_2$, that is

$$(\mathcal{S}_1, \mathcal{S}_2) = \left\{ M \in R\text{-Mod} \mid \begin{array}{c} \text{there are } S_1 \in \mathcal{S}_1 \text{ and } S_2 \in \mathcal{S}_2 \text{ such that} \\ 0 \to S_1 \to M \to S_2 \to 0 \text{ is exact.} \end{array} \right\}$$

Example 2. Let $\mathcal{S}_{f.g.}$ be the Serre subcategory consisting of finitely generated R-modules and $\mathcal{S}_{Artin}$ be the Serre subcategory consisting of Artinian R-modules. Modules in a subcategory $(\mathcal{S}_{f.g.}, \mathcal{S}_{Artin})$ (resp. $(\mathcal{S}_{Artin}, \mathcal{S}_{f.g.})$) are known as Minimax modules (resp. Maxmin modules). The subcategory $(\mathcal{S}_{f.g.}, \mathcal{S}_{Artin})$ is a Serre subcategory. However, the subcategory $(\mathcal{S}_{Artin}, \mathcal{S}_{f.g.})$ needs not be Serre.

Remark 3. Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be Serre subcategories of R-Mod.

(1) One has $\mathcal{S}_1, \mathcal{S}_2 \subseteq (\mathcal{S}_1, \mathcal{S}_2)$.

(2) It holds $\mathcal{S}_1 \supseteq \mathcal{S}_2$ if and only if $(\mathcal{S}_1, \mathcal{S}_2) = \mathcal{S}_1$.

(3) It holds $\mathcal{S}_1 \subseteq \mathcal{S}_2$ if and only if $(\mathcal{S}_1, \mathcal{S}_2) = \mathcal{S}_2$.

(4) A subcategory $(\mathcal{S}_1, \mathcal{S}_2)$ is closed under submodules, quotient modules and finite direct sums.

The main result of this talk is the following.

Theorem 4. *Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be Serre subcategories of R-Mod. Then the following conditions are equivalent:*

(1) *A subcategory $(\mathcal{S}_1, \mathcal{S}_2)$ is a Serre subcategory;*

(2) *One has $(\mathcal{S}_2, \mathcal{S}_1) \subseteq (\mathcal{S}_1, \mathcal{S}_2)$.*

Corollary 5. *A subcategory $(\mathcal{S}_{f.g.}, \mathcal{S})$ is a Serre subcategory for a Serre subcategory $\mathcal{S}$ of R-Mod.*

Corollary 6. *A subcategory $(\mathcal{S}, \mathcal{S}_{Artin})$ is a Serre subcategory for a Serre subcategory $\mathcal{S}$ of R-Mod.*

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On modules of finite complexity over selfinjective algebras

Dan Zacharia

Let R be a selfinjective finite dimensional algebra over an algebraically closed field. The *complexity* of a module measures the rate of growth of a minimal projective resolution. More precisely, if

$$P^{\bullet}: \cdots \to P^2 \xrightarrow{\delta_2} P^1 \xrightarrow{\delta_1} P^0 \xrightarrow{\delta_0} M \to 0$$

is a minimal projective resolution of a finitely generated R-module M, then we define the complexity of M as

$$\mathrm{cx} M = \inf\{n \in \mathbf{N} \mid \dim P_i \le c i^{n-1} \text{ for some positive number } c \in \mathbf{Q} \text{ and all } i \ge 0\}$$

If no such n exists, then we say that the complexity of M is infinite. For instance, a module has complexity 0 if and only if it is projective, and its complexity is 1 if and only if it has a "bounded" projective resolution.

A seemingly completely different notion, the Auslander-Reiten quiver of R contains a lot of information about the module category of R. It is a directed graph with vertices the isomorphism classes of the indecomposable finitely generated modules, and with arrows determined by some very special morphisms called irreducible maps. It has played an enormous role in representation theory in the last 20 years or more.

I will talk about the relationship between the notions of complexity over a selfinjective finite dimensional algebra, and that of its Auslander-Reiten quiver.

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