HOM-ORTHOGONAL PARTIAL TILTING MODULES FOR DYNKIN QUIVERS

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Abstract. We count the number of the isomorphic classes of basic hom-orthogonal partial tilting modules for an arbitrary Dynkin quiver. This number is independent on the choice of an orientation of arrows, and the number for $\mathbb{A}_n$ or $\mathbb{D}_n$-type can be expressed as a special value of a hypergeometric function. As a consequence of our theorem, we obtain a minimum value of the number of basic relative invariants of corresponding regular prehomogeneous vector spaces.

Introduction

Let $Q = (Q_0, Q_1)$ be a Dynkin quiver having n vertices (i.e., its base graph is one of Dynkin diagrams of type $\mathbb{A}_n$ with $n \geq 1$, $\mathbb{D}_n$ with $n \geq 4$, or $\mathbb{E}_n$ with $n = 6, 7, 8$), where Q_0, Q_1 is the set of vertices, arrows of Q, respectively. We denote by $\Lambda = \mathbb{K}Q$ its path algebra over an algebraically closed field $\mathbb{K}$ of characteristic zero, and by $\operatorname{mod}\Lambda$ the category of finitely generated right Λ-modules.

Let $X \cong \bigoplus_{k=1}^{s} m_k X_k$ be the decomposition of $X \in \operatorname{mod}\Lambda$ into indecomposable direct summands, where $m_k X_k$ means the direct sum of m_k copies of X_k, and the X_k's are pairwise non-isomorphic. Then X is called *basic* if $m_k = 1$ for all indices k. We call X to be *hom-orthogonal* if $\operatorname{Hom}_\Lambda(X_i, X_j) = 0$ for all $i \neq j$. This notion is equivalent to that X is locally semi-simple in the sense of Shmelkin [8] when Q is a Dynkin quiver. In the case where X is indecomposable, we will say that X itself is hom-orthogonal. Since Λ is hereditary, we say that $X \in \operatorname{mod}\Lambda$ is a *partial tilting module* if it satisfies $\operatorname{Ext}^1_\Lambda(X, X) = 0$.

Each $X \in \operatorname{mod}\Lambda$ with dimension vector $\boldsymbol{d} = \operatorname{\mathbf{dim}} X$ can be regarded as a representation of Q; that is, a point of the variety $\operatorname{Rep}(Q, \boldsymbol{d})$ that consists of representations with dimension vector $\boldsymbol{d} = (d^{(i)})_{i \in Q_0} \in \mathbb{Z}^n_{\geq 0}$. Then the direct product $GL(\boldsymbol{d}) = \prod_{i \in Q_0} GL(d^{(i)})$ acts naturally on $\operatorname{Rep}(Q, \boldsymbol{d})$; see, for example, [3, §2]. Since Λ is representation-finite, $\operatorname{Rep}(Q, \boldsymbol{d})$ has a unique dense $GL(\boldsymbol{d})$-orbit; thus $(GL(\boldsymbol{d}), \operatorname{Rep}(Q, \boldsymbol{d}))$ is a prehomogeneous vector space (abbreviated PV). It follows from the Artin–Voigt theorem [3, Theorem 4.3] that the condition that X is a partial tilting module can be interpreted to that the $GL(\boldsymbol{d})$-orbit containing X is dense in $\operatorname{Rep}(Q, \boldsymbol{d})$; On the other hand, the condition that X is hom-orthogonal corresponds to that the isotropy subgroup (or, stabilizer) at $X \in \operatorname{Rep}(Q, \boldsymbol{d})$ is reductive. Therefore we are interested in hom-orthogonal partial tilting Λ-modules, because they correspond to generic points of *regular PVs* associated with Q; see [5, Theorem 2.28].

In this paper, we count up the number of the isomorphic classes of basic hom-orthogonal partial tilting Λ-modules for an arbitrary Dynkin quiver Q. In other words, this is nothing

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The detailed version of this paper has been submitted for publication elsewhere.

$e(n,s)$	$n=6$	7	8
$s=1$	36	63	120
2	108	315	945
3	72	336	1575
4	0	63	675

$e^0(n,s)$	$n=6$	7	8
$s=1$	7	16	44
2	35	120	462
3	35	170	924
4	0	40	462

TABLE 0.1. The values of $e(n,s)$ and $e^0(n,s)$

but essentially counting the number of regular PVs associated with. Our main theorem is the following:

Theorem 0.1. *Let Q be a quiver of type $\mathbb{A}_n$ with $n \geq 1$ (resp. $\mathbb{D}_n$ with $n \geq 4$, $\mathbb{E}_n$ with $n = 6, 7, 8$). Then the number $a(n,s)$ (resp. $d(n,s)$, $e(n,s)$) of the isomorphic classes of basic hom-orthogonal tilting $\mathbb{K}Q$-modules having s pairwise non-isomorphic indecomposable direct summands is given explicitely by the following:*

$$a(n,s) = \frac{(n+1)!}{s!\,(s+1)!\,(n+1-2s)!} \tag{0.1}$$

$$= C_s \cdot \binom{n+1}{2s} \tag{0.2}$$

if $1 \leq s \leq (n+1)/2$, and $a(n,s) = 0$ if otherwise. Here $C_s = \binom{2s}{s}/(s+1)$ denotes the s-th Catalan number.

$$d(n,s) = \frac{(n-1)!}{(s!)^2\,(n+2-2s)!} \cdot \left\{ s^2(s-1) + n(n+1-2s)(n+2-2s) \right\}$$

if $1 \leq s \leq (n+2)/2$, and $d(n,s) = 0$ if otherwise. The values of $e(n,s)$ for $1 \leq s \leq (n+1)/2$ are given in Table 0.1, and we have $e(n,s) = 0$ if otherwise.

Our approach to this theorem, which was inspired by Seidel's paper [7], is based on an observation of perpendicular categories introduced by Schofield [6]. Here we point out that the totality of $a(n,s)$ or $d(n,s)$ for fixed n can be expressed as a special value of a hypergeometric function. As mentioned in Remark 2.4, the formula (0.2) has a combinatorial interpretation.

According to Happel [4], if a Λ-module corresponding to a point contained in the dense orbit of a PV $(GL(\boldsymbol{d}), \mathrm{Rep}(Q, \boldsymbol{d}))$ has s pairwise non-isomorphic indecomposable direct summands, then the PV has exactly $n-s$ basic relative invariants. Thus we obtain a consequence of Theorem 0.1.

Corollary 0.2. *Each regular PV associated with a quiver of type $\mathbb{A}_n$ (resp. $\mathbb{D}_n$, $\mathbb{E}_6$, $\mathbb{E}_7$, and $\mathbb{E}_8$) has at least $(n-1)/2$ (resp. $(n-2)/2$, 3, 3, and 4) basic relative invariants.*

We say that $X \in \mathrm{mod}\,\Lambda$ is *sincere* if its dimension vector $\mathbf{dim}\,X$ does not have zero entry. Sincere modules are fairly interesting to the theory of PVs, because $(GL(\boldsymbol{d}), \mathrm{Rep}(Q, \boldsymbol{d}))$ with non-sincere dimension can be regarded as a direct sum of at least two PVs associated with proper subgraphs of Q. So we have counted them:

Theorem 0.3. *Let Q be a quiver of type $\mathbb{A}_n$ with $n \geq 1$ (resp. $\mathbb{D}_n$ with $n \geq 4$, $\mathbb{E}_n$ with $n = 6, 7, 8$). Then the number $a^0(n,s)$ (resp. $d^0(n,s)$, $e^0(n,s)$) of the isomorphic classes* *of basic sincere hom-orthogonal tilting $\mathbb{K}Q$-modules having s pairwise non-isomorphic indecomposables is given explicitely by the following:*

$$a^0(n,s) = \frac{(n-1)!}{s!\,(s-1)!\,(n+1-2s)!} = C_{s-1} \cdot \binom{n-1}{2s-2} \tag{0.3}$$

if $1 \le s \le (n+1)/2$, *and* $a^0(n,s) = 0$ *if otherwise.*

$$d^0(n,s) = \frac{(n-2)!}{s!\,(s-1)!\,(n+2-2s)!} \times \left\{ n(n-1-2s)(n-2s) + 2n(n-2) + (s-1)(s^2-9s+4) \right\}$$

if $1 \le s \le (n+2)/2$, *and* $d^0(n,s) = 0$ *if otherwise. The values of* $e^0(n,s)$ *for* $1 \le s \le (n+1)/2$ *are given in Table* 0.1, *and we have* $e^0(n,s) = 0$ *if otherwise.*

Now we will exceptionally define some values of $a(m,t)$ for simplicity:

$$a(m,\,-1) = 0,\ a(m,\,0) = 1,\ \text{and } a(l,\,t) = 0 \text{ for } l \le 0 \text{ and } t \ne 0.$$

Then we can express $d(n,s)$, $a^0(n,s)$, and $d^0(n,s)$ as the following simpler forms:

$$d(n,s) = (n-1)\cdot a(n-3,\,s-2) + (s+1)\cdot a(n-1,\,s), \tag{0.4}$$

$$a^0(n,s) = a(n-2,\,s-1),$$

$$d^0(n,s) = (s-1)\cdot a(n-3,\,s-2) + (n-2)\cdot a(n-3,\,s-1). \tag{0.5}$$

As will be mentioned in §1, the numbers presented in Theorems 0.1 and 0.3 are independent on the choice of an orientation of arrows of Q. Thus we may assume that its arrows are conveniently oriented.

1. Preliminaries

Let Q be a Dynkin quiver having n vertices, $\Lambda = \mathbb{K}Q$ its path algebra. For an indecomposable Λ-module M, its right perpendicular category $M^{\perp}$ is defined by

$$M^{\perp} = \{X \in \operatorname{mod}\Lambda;\ \operatorname{Hom}_\Lambda(M,X) = 0 \text{ and } \operatorname{Ext}^1_\Lambda(M,X) = 0\}.$$

The left perpendicular category ${}^{\perp}M$ is also defined similarly. To investigate hom-orthogonal partial tilting modules (or, regular PVs), we are interested in their intersection $\operatorname{Per} M = {}^{\perp}M \cap M^{\perp}$; we will simply call it the *perpendicular category* of M. Now we recall the Ringel form, which is defied on the Grothendieck group $K_0(\Lambda) \cong \mathbb{Z}^n$:

$$\begin{aligned}\langle \mathbf{dim}\,X,\, \mathbf{dim}\,Y\rangle &= \dim \operatorname{Hom}_\Lambda(X,Y) - \dim \operatorname{Ext}^1_\Lambda(X,Y)\\ &= {}^t(\mathbf{dim}\,X)\cdot R_Q \cdot (\mathbf{dim}\,Y)\end{aligned}$$

for $X, Y \in \operatorname{mod}\Lambda$, where $R_Q = (r_{ij})_{i,j\in Q_0}$ is the representation matrix with respect to the basis $\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n$ of $K_0(\Lambda) \cong \mathbb{Z}^n$ (here we put $\boldsymbol{e}_k = \mathbf{dim}\,S(k)$, which is the dimension vector of a simple module corresponding to a vertex $k \in Q_0$). This is defined as $r_{ii} = 1$ for all $i \in Q_0$; $r_{ij} = -1$ if there exists an arrow $i \to j$ in Q; and $r_{ij} = 0$ if otherwise.

Lemma 1.1. *For indecomposable Λ-modules X and Y, we have* $\langle \mathbf{dim}\,X,\, \mathbf{dim}\,Y\rangle = 0$ *if and only if* $\operatorname{Hom}_\Lambda(X,Y) = 0$ *and* $\operatorname{Ext}^1_\Lambda(X,Y) = 0$.

Now we will show that the numbers that are presented in our theorems do not depend on the choice of an orientation of arrows of Q. To do this, we need the following lemma:

Lemma 1.2. *For any sink $a \in Q_0$ and any Λ-module M, if $\mathrm{Hom}_\Lambda(S(a), M) = 0$ and $\mathrm{Ext}^1_\Lambda(M, S(a)) = 0$, then we have $\mathrm{Hom}_\Lambda(P(t\alpha), M) = 0$ for any arrow $\alpha : t\alpha \to a$ in Q.*

Let $\sigma = \sigma_a$ be the reflection functor (with the APR-tilting module T, see [2, VII Theorem 5.3]) at a sink $a \in Q_0$, and Q' the quiver obtained by reversing all arrows connecting with a in Q. For a basic hom-orthogonal partial tilting Λ-module $X \cong \bigoplus_{k=1}^{s} X_k$, we define a Λ'-module as follows (here we put $\Lambda' = \mathbb{K}Q'$):

$$\sigma X := S(a)_{\Lambda'} \oplus \sigma X_2 \oplus \cdots \oplus \sigma X_s$$

if X has a direct summand (say, X_1) isomorphic to the simple module $S(a)_\Lambda$; and

$$\sigma X := \sigma X_1 \oplus \sigma X_2 \oplus \cdots \oplus \sigma X_s$$

if X does not, where we put $\sigma X_k = \mathrm{Hom}_\Lambda(T, X_k)$ for each indecomposable X_k. Let $\mathcal{R}$, $\mathcal{R}'$ be the set of the isomorphic classes of basic hom-orthogonal partial tilting Λ-modules, Λ'-modules, having exactly s indecomposable direct summands, respectively. Then we have the following:

Proposition 1.3. *For a basic hom-orthogonal partial tilting Λ-module X having s indecomposable direct summands, so is Λ'-module σX. The correspondence $[X] \mapsto [\sigma X]$ gives a bijection from $\mathcal{R}$ to $\mathcal{R}'$. In particular, the numbers that are presented in Theorem 0.1 do not depend on the choice of an orientation of arrows.*

Proof. Let R_Q, $R_{Q'}$ be the representation matrix of the Ringel form of Λ, Λ', respectively. Let $r = r_a$ be the simple reflection on $\mathbb{Z}^n$ corresponding to the vertex a (we also denote by the same r its representation matrix). Then we have $R_{Q'} = {}^t r \cdot R_Q \cdot r$. On the other hand, we have $\mathbf{dim}\, \sigma X_k = r \cdot (\mathbf{dim}\, X_k)$ for X_k that is not isomorphic to $S(a)_\Lambda$, and $r(\boldsymbol{e}_a) = -\boldsymbol{e}_a$. Hence, by calculating with the Ringel form (recall Lemma 1.1), we see that σX is also a basic hom-orthogonal partial tilting Λ'-module. This correspondence $[X] \mapsto [\sigma X]$ is obviously a bijection. □

Next we define two subsets of $\mathcal{R}$ as follows:

$$\mathcal{R}_1 = \big\{[X] \in \mathcal{R};\ X \text{ is sincere, but } \sigma X \text{ is not sincere}\big\},$$
$$\mathcal{R}_2 = \big\{[X] \in \mathcal{R};\ X \text{ is not sincere, but } \sigma X \text{ is sincere}\big\}.$$

It follows from Lemma 1.2 that the condition "sincere" implies that any representative of each class of $\mathcal{R}_1$ or $\mathcal{R}_2$ does not have a direct summand isomorphic to the simple module $S(a)_\Lambda$.

Proposition 1.4. *We have $\sharp\mathcal{R}_1 = \sharp\mathcal{R}_2$. In particular, the numbers for sincere modules that are presented in Theorem 0.3 do not depend on the choice of an orientation of arrows.*

Proof. Take the isomorphic class $[X] \in \mathcal{R}_1$ and let $X \cong \bigoplus_{k=1}^{s} X_k$ be its indecomposable decomposition. Then, since σX is not sincere, only the a-th entry of $\mathbf{dim}\, \sigma X = r \cdot (\mathbf{dim}\, X)$ is zero. Hence so is the a-th entry of each $r(\boldsymbol{\alpha}_k)$, where we put $\boldsymbol{\alpha}_k = \mathbf{dim}\, X_k$. On the other hand, since σX is a basic hom-orthogonal partial tilting Λ'-module, we have ${}^t r(\boldsymbol{\alpha}_i) \cdot R_{Q'} \cdot r(\boldsymbol{\alpha}_j) = 0$ for any pair of distinct indices. Then we see that ${}^t r(\boldsymbol{\alpha}_i) \cdot R_Q \cdot r(\boldsymbol{\alpha}_j) =$

0, because R_Q and $R_{Q'}$ are identical other than the a-th row and the a-th column. Let $\widetilde{X}$ be a Λ-module corresponding to the sum of positive roots $\sum_{k=1}^{s} r(\boldsymbol{\alpha}_k)$; this is not sincere, but $\sigma\widetilde{X}$ is sincere. Thus we see that the correspondence $[X] \mapsto [\widetilde{X}]$ gives a bijection from $\mathcal{R}_1$ to $\mathcal{R}_2$. □

2. $\mathbb{A}_n$-TYPE

Let Q be the equi-oriented quiver $\overset{1}{\circ} \longrightarrow \overset{2}{\circ} \longrightarrow \cdots \longrightarrow \overset{n}{\circ}$ of $\mathbb{A}_n$-type. In the following, we will sometimes consider the corresponding things of "$\mathbb{A}_0$-type" or "$\mathbb{A}_{-1}$-type" to be trivial for simplicity; for example, "$\mathbb{A}_{n-2} \times \mathbb{A}_{-1}$-type" means just "$\mathbb{A}_{n-2}$-type", and so on.

Proposition 2.1. *For each $k = 1, 2, \ldots, n$, the perpendicular category* $\operatorname{Per} I(k)$ *is equivalent to the module category of a path algebra of type $\mathbb{A}_{k-2} \times \mathbb{A}_{n-1-k}$.*

Proposition 2.2. *Let n and s be positive integers. The number $a(n, s)$ satisfies the following recurrence formula:*

$$a(n,\, s) = a(n-1,\, s) + \sum_{t=0}^{s-1} \sum_{m=-1}^{n-2} a(m,\, t) \cdot a(n-3-m,\, s-1-t). \tag{2.1}$$

Proof. Let $X = \bigoplus_{j=1}^{s} X_j$ be a basic hom-orthogonal partial tilting Λ-module having s distinct indecomposable summands. Note that X has at most one injective direct summand. If X does not have any injective, then the first entry of $\mathbf{dim}\, X$ is zero; that is, it is a sum of positive roots that come from $\mathbb{A}_{n-1}$-type. So the number for such modules is equal to $a(n-1,\, s)$. Assume that X has just one injective summand, say $I(k)$. Then, according to Proposition 2.1, X has t and $s-1-t$ direct summands that come from $\mathbb{A}_{k-2}$-type and $\mathbb{A}_{n-1-k}$-type, respectively. Thus we see that there exist $\sum_{t=0}^{s-1} a(k-2,\, t) \cdot a(n-1-k,\, s-1-t)$ such modules. Since k runs from 1 to n, we obtain our assertion. □

By using the recurrence formula above, we prove Theorem 0.1 for $\mathbb{A}_n$-type. Here we notice that the generating function of $a(n,s) = C_s \cdot \binom{n+1}{2s}$ can be immediately obtained from the generalized binomial expansion.

Lemma 2.3. *The generating function $F_s(x) = \sum_{n=0}^{\infty} a(n,s)x^n$ of $a(n,s)$ for fixed s is given by*

$$F_s(x) = \frac{C_s \cdot x^{2s-1}}{(1-x)^{2s+1}}.$$

Proof of Theorem 0.1 *for $\mathbb{A}_n$-type.* First we note that $a(n,1)$ is nothing but the number of positive roots of $\mathbb{A}_n$-type, which is equal to $n(n+1)/2 = C_1 \cdot \binom{n+1}{2}$. In the case of $n = 1$, our assertion is trivial. So we assume that the assertion (0.2) holds for all positive integers less than n (≥ 2). In the recurrence formula (2.1), we note that $a(m,t)$ (resp. $a(n-3-m,\, s-1-t)$) is the coefficient of degree m (resp. $n-3-m$) of $F_t(x)$ (resp. $F_{s-1-t}(x)$). The coefficient of degree $n-3$ of the Taylor expansion at the origin ($x = 0$) of

$$F_t(x) \times F_{s-1-t}(x) = C_t \cdot C_{s-1-t} \cdot \frac{x^{2s-4}}{(1-x)^{2s}}$$

is equal to $\binom{n}{2s-1}$; hence we have

$$\sum_{m=-1}^{n-2} a(m,\,t)\cdot a(n-3-m,\,s-1-t) = C_t\cdot C_{s-1-t}\cdot \binom{n}{2s-1}. \tag{2.2}$$

By the recurrence formula (2.1) and the assumption of induction, we have

$$\begin{aligned} a(n,s) &= a(n-1,\,s) + \binom{n}{2s-1}\sum_{t=0}^{s-1} C_t\cdot C_{s-1-t} \\ &= C_s\cdot\binom{n}{2s} + \binom{n}{2s-1}\cdot C_s = C_s\cdot\binom{n+1}{2s}. \end{aligned}$$

Next we prove that $a(n,s)=0$ if $s>(n+1)/2$. Let s be such an integer. Then we have $a(n-1,\,s)=0$ by the assumption of induction because $s>n/2$. Suppose that $t\le(m+1)/2$ and $s-1-t\le(n-3-m+1)/2$ for fixed t. Then we have $s-1\le(n-1)/2$; a contradiction. Hence $t>(m+1)/2$ or $s-1-t>(n-3-m+1)/2$, and so that $a(m,t)=0$ or $a(n-3-m,\,s-1-t)=0$. Thus we conclude $a(n,s)=0$ by the recurrence formula (2.1). Therefore we obtain our assertion for $\mathbb{A}_n$-type. □

Remark 2.4. The formula (0.2) has a combinatorial interpretation. According to Araya [1, Lemma 3.2], for distinct indecomposables $X,Y\in \operatorname{mod}\Lambda$, their direct sum $X\oplus Y$ is a hom-orthogonal partial tilting module (or, both (X,Y) and (Y,X) are exceptional pairs) if and only if the corresponding codes of a circle with $n+1$ points do not meet each other. It follows from a well-known combinatorics on codes that the number of such codes is equal to $C_2\cdot\binom{n+1}{4}=a(n,2)$. The formula for general $s\ge 2$ can be similarly obtained.

Proposition 2.5. *Let X be a basic sincere hom-orthogonal partial tilting Λ-module. Then X has exactly one direct summand isomorphic to $I(n)$.*

Proof of Theorem 0.3 for $\mathbb{A}_n$-type. Let X be a basic sincere hom-orthogonal partial tilting Λ-module. In the case of $s=1$ (that is, X itself is indecomposable), it must be isomorphic to $I(n)$. Hence we have $a^0(n,1)=1$ for any n. If $n=1$ or $n=2$, our assertion can be proved directly. So let $n\ge 3$ and $s\ge 2$. By Propositions 2.2 and 2.5, the other summands of X should be taken from a module category of $\mathbb{A}_{n-2}$-type. The number of such candidates is equal to $a(n-2,\,s-1)$. We can prove $a^0(n,s)=0$ for $s>(n+1)/2$ by a similar manner to the proof of Theorem 0.1. □

Theorems for $\mathbb{D}_n$-type and $\mathbb{E}_n$-type are shown in a similar way. The detailed proof is given in our paper which has been submitted for publication elsewhere

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