The 45th Symposium on Ring Theory

and Representation Theory

ABSTRACT

Shinshu University, Matsumoto

September 7 – 9, 2012

Program

September 7 (Friday)

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Hidetoshi Marubayashi (Tokushima Bunri University)
Akira Ueda (Shimane University)

Skew Rees rings which are maximal orders

9:40–10:10 Noritsugu Kameyama (Shinshu University)
Mitsuo Hoshino (University of Tsukuba)
Hirotaka Koga (University of Tsukuba)

Constructions of Auslander-Gorenstein local rings

10:20–10:50 Kenta Ueyama (Shizuoka University)

Noncommutative graded Gorenstein isolated singularities

11:00–11:30 Kazunori Matsuda (Nagoya University)

Characterization of Gorenstein strongly Koszul Hibi rings by F-invariants

11:40–12:10 Akihiro Higashitani (Osaka University)

Toric rings arising from cyclic polytopes

13:30–14:00 Ryo Kanda (Nagoya University)

Classifying Serre subcategories via atom spectrum

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Derived Gabriel topology, localization and completion of dg-algebras

14:50–15:20 Yasuhiko Takehana (Hakodate National College of Technology)

A generalization of Goldie torsion theory

15:40–16:10 Kaoru Motose

Power residues

16:20–16:50 Satoshi Yamanaka (Okayama University)

On separable polynomials in skew polynomial rings

17:00–17:30 Yosuke Kuratomi (Kitakyushu National College of Technology)

On Goldie extending modules with finite internal exchange property

September 8 (Saturday)

9:00–9:30 Mayumi Kimura (Shizuoka University)
Hideto Asashiba (Shizuoka University)

Derived equivalence classification of generalized multifold extensions of piecewise hereditary algebras of tree type

9:40–10:10 Liu Yu (Nagoya University)

Quotients of exact categories by cluster tilting subcategories as module categories

10:20–10:50 Tokuji Araya (Tokuyama College of Technology)

Dimensions of triangulated categories with respect to subcategories

11:00–11:30 Ken-ichi Yoshida (Nihon University)
Kei-ichi Watanabe (Nihon University)

Ulrich modules and Special modules over 2-dimensional rational singularities

11:40–12:10 Yoshiyuki Kimura (Osaka City University)

Quiver varieties and quantum cluster algebras

13:30–14:00 Masahide Konishi (Nagoya University)

Cyclotomic KLR algebras of cyclic quivers

14:10–14:40 Luo Xueyu (Nagoya University)

Realizing cluster categories of Dynkin type A as stable categories of lattices

14:50–15:20 Gustavo Jasso (Nagoya University)

Cluster-tilted algebras of canonical type and quivers with potentials

15:40–16:10 Osamu Iyama (Nagoya University)

τ-tilting theory

16:20–16:50 Takahide Adachi (Nagoya University)

τ-tilting modules for self-injective Nakayama algebras

17:00–17:30 Martin Herschend (Nagoya University)

2-hereditary algebras and quivers with potential

17:40–18:10 Yuya Mizuno (Nagoya University)

Selfinjective algebras and quivers with potentials

18:40– Conference dinner

September 9 (Sunday)

9:00–9:30 Morio Uematsu (Jobu University)

On the relation of the upper bound of global dimension and the length of serial algebra which has finite global dimension

9:40–10:10 Laurent Demonet (Nagoya University)

Mutation of quiver with potential at several vertices

10:20–10:50 Erik Darpö (Nagoya University)

On the representation rings of the dihedral 2-groups

11:00–11:30 Obara Daiki (Tokyo University of Science)
Furuya Takahiko (Tokyo University of Science)

On Hochschild cohomology of a class of weakly symmetric algebras with radical cube zero

11:40–12:10 Furuya Takahiko (Tokyo University of Science)

On the existence of a bypass in the Auslaner-Reiten quiver

Skew Rees rings which are maximal orders

M.R. Helmi*, H. Marubayashi** and A. Ueda***

Let R be a Noetherian prime ring with its quotient ring Q, σ be an automorphism of R and X be an invertible ideal of R with $\sigma(X) = X$. A skew Rees ring $S = R[Xt; \sigma]$ is a subring of the skew polynomial ring $R[t; \sigma]$ in an indeterminate t. An $R-$ideal $\mathfrak{a}$ in Q is called $(\sigma; X)-$invariant if $X\sigma(\mathfrak{a}) = \mathfrak{a}X$. R is said to be a $(\sigma; X)-$maximal order if $O_l(\mathfrak{a}) = R = O_r(\mathfrak{a})$ for any $(\sigma; X)-$invariant ideal $\mathfrak{a}$ of R. It is shown that a skew Rees ring $S = R[Xt; \sigma]$ is a maximal order if and only if R is a $(\sigma; X)-$maximal order, which is proved by complete description of divisorial ideals of S. As applications of the result, we will give a necessary and sufficient conditions for S to be a generalized Asano ring and a unique factorization ring respectively. Of course, if R is a maximal order, then S is a maximal order. However the converse does not hold. We provide the following examples :

(1) R is a $(\sigma; X)-$maximal order but it is not a maximal order (in fact, it is not a $\sigma-$maximal order; by $\sigma-$maximal orders we mean $O_r(\mathfrak{a}) = R = O_l(\mathfrak{a})$ for any ideal $\mathfrak{a}$ with $\sigma(\mathfrak{a}) = \mathfrak{a}$. Note R is a $\sigma-$maximal order if and only if the skew polynomial ring $R[t; \sigma]$ is a maximal order.

(2) R is a $\sigma-$maximal order but it is not a $(\sigma; X)-$maximal order.

In non-commutative setting, Rees rings were studied under PI conditions ([2]) and recently Akalan proved in [1] that if R is a $PI-$Generalized Asano ring, then so is the Rees ring $R[Xt]$ which motivated us to study skew Rees rings.

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Constructions of Auslander-Gorenstein local rings

Mitsuo Hoshino, Noritsugu Kameyama and Hirotaka Koga

Auslander-Gorenstein rings appear in various areas of current research. However, little is known about constructions of Auslander-Gorenstein rings. It was shown in [2] that a left and right noetherian ring is an Auslander-Gorenstein ring if it admits an Auslander-Gorenstein resolution over another Auslander-Gorenstein ring. Generalizing the notion of crossed product, we will provide systematic constructions of Auslander-Gorenstein local rings starting from an arbitrary Auslander-Gorenstein local ring.

In order to provide the construction, we will define new multiplications on the ring of full matrices and the group ring of finite cyclic groups. Let $n \geq 2$ be an integer and set $I(n) = \{1, \dots, n\}$. We fix a cyclic permutation

$$\pi = \begin{pmatrix} 1 & 2 & \cdots & n \\ n & 1 & \cdots & n-1 \end{pmatrix}$$

of $I(n)$. Then the law of composition $I(n) \times I(n) \to I(n), (i, j) \mapsto \pi^{-i}(j)$ makes $I(n)$ a cyclic group.

We denote by $\Omega(n)$ the set of mappings $\omega : I(n) \times I(n) \to \mathbb{Z}$ satisfying the following conditions: (W1) $\omega(i, i) = 0$ for all $i \in I(n)$; (W2) $\omega(i, j) + \omega(j, k) \geq \omega(i, k)$ for all $i, j, k \in I(n)$; (W3) $\omega(i, j) + \omega(j, i) \geq 1$ unless $i = j$; and (W4) $\omega(i, j) + \omega(j, \pi(i)) = \omega(i, \pi(i))$ for all $i, j \in I(n)$. We fix $\omega \in \Omega(n)$ and a ring R together with a pair (σ, c) of $\sigma \in \mathrm{Aut}(R)$ and $c \in R$ such that $\sigma(c) = c$ and $xc = c\sigma(x)$ for all $x \in R$. Denote by $\Omega_+(n)$ the subset of $\Omega(n)$ consisting of $\omega \in \Omega(n)$ such that $\omega(1, i) = \omega(i, n) = 0$ for all $i \in I(n)$.

Let A be a free right R-module with a basis $\{v_i\}_{i \in I(n)}$ and define a multiplication on A subject to the following axioms: (G1) $v_i v_j = v_{\pi^{-j}(i)} c^{\omega(\pi^{-j}(i), j)}$ for all $i, j \in I(n)$; and (G2) $x v_i = v_i \sigma^{-\chi(i)}(x)$ for all $x \in R$ and $i \in I(n)$ and $\chi(i) = \sum_{k=1}^{i} \omega(k, \pi(k))$. Then A is an associative ring with $1 = v_n$ and R is considered as a subring of A via the injective ring homomorphism $R \to A, x \mapsto v_n x$; A/R is a split Frobenius extension; A is commutative if R is commutative and $\sigma^{\chi(i)} = \mathrm{id_R}$ for all $i \in I(n)$; and A is local if R is local and $c \in \mathrm{rad}(R)$.

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Noncommutative graded Gorenstein isolated singularities

Kenta Ueyama

Gorenstein isolated singularities play an essential role in representation theory of Cohen-Macaulay modules ([2], etc.). Furthermore, AS-Gorenstein algebras are the important class of algebras studied in noncommutative algebraic geometry ([3], etc.). In this talk, we define a notion of noncommutative graded isolated singularity by the smoothness of the noncommutative projective scheme defined in [1] (see also [4]), and study AS-Gorenstein isolated singularities and the categories of graded maximal Cohen-Macaulay modules over them. For an AS-Gorenstein algebra A of dimension $d \geq 2$, we show that A is a graded isolated singularity if and only if the stable category of graded maximal Cohen-Macaulay modules over A has the Serre functor. Using this result, we also show the existence of cluster tilting modules in the categories of graded maximal Cohen-Macaulay modules over Veronese subalgebras of certen AS-regular algebras. This gives examples of cluster tilting modules over non-orders.

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Characterization of Gorenstein strongly Koszul Hibi rings by F-invariants

Kazunori Matsuda

This is a partially joint work with T. Chiba.

F-thresholds and F-pure thresholds are invariants of commutative Noetherian rings of positive characteristic.

In this talk, we compute F-thresholds and F-pure thresholds of Hibi rings, and give a characterization of Hibi rings which satisfy the equality between these invariants in terms of its strongly Koszulness in the sense of Herzog-Hibi-Restuccia.

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Toric rings arising from cyclic polytopes

Akihiro Higashitani

First of all, this talk is based on a joint work with Takayuki Hibi, Lukas Katthän and Ryota Okazaki.

Let d and n be positive integers with $n \geq d+1$ and $\tau_1, \ldots, \tau_n$ real numbers with $\tau_1 < \cdots < \tau_n$. We write $C_d(\tau_1, \ldots, \tau_n) \subset \mathbb{R}^d$ for the convex hull of $\{(\tau_i, \tau_i^2, \ldots, \tau_i^d) \in \mathbb{R}^d : i = 1, \ldots, n\}$. The convex polytope $C_d(\tau_1, \ldots, \tau_n) \subset \mathbb{R}^d$ is called a *cyclic polytope* of dimension d with n vertices. In particular, we say that it is an *integral cyclic polytope* if $\tau_1, \ldots, \tau_n$ are all integers. A cyclic polytope is a simplicial polytope and its combinatorial type is independent of a choice of $\tau_1, \ldots, \tau_n$. Moreover, it is well known that a cyclic polytope is a convex polytope which attains the upper bound in the Upper Bound Theorem.

In this talk, we focus on *integral* cyclic polytopes and discuss some properties on toric rings arising from integral cyclic polytopes.

In general, for an integral convex polytope $\mathcal{P} \subset \mathbb{R}^N$, let $\mathcal{P}^* \subset \mathbb{R}^{N+1}$ be the convex hull of $\{(1, \alpha) \in \mathbb{R}^{N+1} : \alpha \in \mathcal{P}\}$ and $\mathcal{A}_\mathcal{P} = \mathcal{P}^* \cap \mathbb{Z}^{N+1}$. We say that $\mathcal{P}$ is *normal* if it satisfies

$$\mathbb{R}_{\geq 0}\mathcal{A}_\mathcal{P} \cap \mathbb{Z}\mathcal{A}_\mathcal{P} = \mathbb{Z}_{\geq 0}\mathcal{A}_\mathcal{P}.$$

We say that $\mathcal{P}$ is *very ample* if the set

$$\mathbb{R}_{\geq 0}\mathcal{A}_\mathcal{P} \cap \mathbb{Z}\mathcal{A}_\mathcal{P} \setminus \mathbb{Z}_{\geq 0}\mathcal{A}_\mathcal{P}$$

is finite. Thus, normal integral convex polytopes are always very ample.

The first main result of this talk is the following

Theorem 1. *Let d and n be positive integers with $n \geq d+1$ and $C_d(\tau_1, \ldots, \tau_n)$ an integral cyclic polytope, where $\tau_1 < \cdots < \tau_n$.*
(a) For each $1 \leq i \leq n-1$, if

$$\tau_{i+1} - \tau_i \geq d^2 - 1,$$

then $C_d(\tau_1, \ldots, \tau_n)$ is normal.
(b) Let $d \geq 4$. If

$$\textit{either} \quad \tau_3 - \tau_2 = 1 \quad \textit{or} \quad \tau_{n-1} - \tau_{n-2} = 1$$

is satisfied, then $C_d(\tau_1, \ldots, \tau_n)$ is not very ample.

For an integral convex polytope $\mathcal{P}$, $\mathbb{Z}_{\geq 0}\mathcal{A}_\mathcal{P}$ is an affine semigroup. Let K be a field. Then we set

$$K[\mathcal{P}] := K[\mathbb{Z}_{\geq 0}\mathcal{A}_\mathcal{P}]$$

and we call it a *toric ring* arising from $\mathcal{P}$.

The following is the second main theorem of this talk.

Theorem 2. *Let $\mathcal{P} = C_d(\tau_1, \ldots, \tau_n)$ be an integral cyclic polytope and $K[\mathcal{P}]$ its toric ring.*
(a) The following three conditions are equivalent:
(i) $K[\mathcal{P}]$ is Cohen–Macaulay;
(ii) $K[\mathcal{P}]$ is normal;
(iii) $K[\mathcal{P}]$ satisifes Serre's condition (S_2).
(b) $K[\mathcal{P}]$ is Gorenstein if and only if

$$d = 2,\ n = 3,\ (\tau_2 - \tau_1, \tau_3 - \tau_2) = (1, 2) \textit{ or } (2, 1)$$

is satisfied.

In addition, we also define another toric rings arising from integral cyclic polytopes. Let

$$Q := Q_d(\tau_1, \dots, \tau_n) := \mathbb{Z}_{\geq 0}\{(1, \tau_i, \tau_i^2, \dots, \tau_i^d) \in \mathbb{Z}^{d+1} : i = 1, \dots, n\}.$$

Then we write $K[Q]$ for the toric ring associated with the configuration Q. In other words, $K[Q]$ is a toric ring arising from the matrix

$$\begin{pmatrix} 1 & 1 & \cdots & 1 \\ \tau_1 & \tau_2 & \cdots & \tau_n \\ \vdots & \vdots & \vdots & \vdots \\ \tau_1^d & \tau_2^d & \cdots & \tau_n^d \end{pmatrix},$$

which is nothing but the Vandermonde matrix.

Here we remark that $K[Q]$ is equivalent to the polynomial ring in n variables if $n = d + 1$. Moreover, the case where $d = 1$ is well studied.

Theorem 3. *Let Q be as above.*
(a) If $d \geq 2$ and $n = d + 2$, then $K[Q]$ is not normal.
(b) When $n \geq d+3$, if $\prod_{k=1}^{d}(\tau_{d+1}-\tau_k) \nmid \prod_{k=1}^{d}(\tau_s-\tau_k)$ for some s with $d+2 \leq s \leq n$, then $K[Q]$ is not normal.

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Classifying Serre subcategories via atom spectrum

Ryo Kanda

A full subcategory of an abelian category $\mathcal{A}$ is called a *Serre subcategory* of $\mathcal{A}$ if it is closed under subobjects, quotient objects, and extensions. Classification of Serre subcategories of a module category has been studied by a number of authors (for example, [2], [3], [5], [6], and [1]). The prototype of such classification was given by Gabriel:

Theorem 1 (Gabriel [2]). *Let R be a commutative noetherian ring. Then there exists a one-to-one correspondence between Serre subcategories of the category* $\operatorname{mod} R$ *of finitely generated right R-modules and specialization-closed subsets of* $\operatorname{Spec} R$.

The aim of this talk is to give this type of classification for an arbitrary noetherian abelian category in terms of the atom spectrum. The atom spectrum of an abelian category is a topological space, and as a set (or class), it is defined as follows:

Definition 2. Let $\mathcal{A}$ be an abelian category.

(1) A nonzero object H in $\mathcal{A}$ is called *monoform* if for any nonzero subobject N of H, there exists no nonzero subobject of H/N isomorphic to a subobject of H.

(2) Two monoform objects H and H' in $\mathcal{A}$ are called *atom-equivalent* if there exists a nonzero subobject of H isomorphic to a subobject of H'.

(3) The *atom spectrum* of $\mathcal{A}$ is the quotient set (or class) of the class of all the monoform objects by the atom equivalence.

By using properties of monoform objects, we obtain the following theorem:

Theorem 3 (K [4]). *Let $\mathcal{A}$ be a noetherian abelian category. Then there exists a one-to-one correspondence between Serre subcategories of $\mathcal{A}$ and open subsets of the atom spectrum of $\mathcal{A}$.*

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Derived Gabriel topology, localization and completion of dg-algebras

Hiroyuki Minamoto

Gabriel topology is a special class of linear topology on rings, which plays an important role in the theory of localization of (not necessary commutative) rings[5]. Several evidences have suggested that there should be a corresponding notion for dg-algebras. In this talk I will introduce a notion of Gabriel topology on dg-algebras, derived Gabriel topology, and show its basic properties.

In the same way as the definition of derived Gabriel topology on a dg-algebra, we give the definition of topological dg-modules over a dg-algebra equipped with derived Gabriel topology. An important example of topology on dg-modules is the finite topology on the bi-dual module $S_J(M) := \mathbf{R}\mathrm{Hom}_{\mathcal{E}}(\mathbf{R}\mathrm{Hom}_A(M, J), J)$ of a dg-module M by another dg-module J.

We show that $S_J(M)$ equipped with the finite topology is the completion of M equipped with J-adic topology. This is inspired by the results of J. Lambek [3, 4]. However our formulation is new: the derived bi-duality module $S_J(M)$ is quasi-isomorphic to the homotopy limit of a certain tautological diagram. This is a simple observation, which seems to be true in wider context. However this provide us a fundamental understanding of derived bi-duality functor.

We give applications. 1. we give a generalization and an intuitive proof of Efimov-Dwyer-Greenlees-Iyenger Theorem which asserts that the completion of commutative ring satisfying some conditions is obtained as a derived bi-commutator [1, 2]. 2. We prove that every smashing localization of dg-category is obtained as a derived bi-commutator of some pure injective module. This is a derived version of the classical results in localization theory of ordinary rings.

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A generalization of Goldie torsion theory

Yasuhiko Takehana

Throughout this paper R is a ring with a unit element, every right R-module is unital and Mod-R is the category of right R-modules. Let $\mathcal{C}$ be a subclass of Mod-R. A torsion theory for $\mathcal{C}$ is a pair of ($\mathcal{T}$,$\mathcal{F}$) of classes of objects of $\mathcal{C}$ such that

(i) $\mathrm{Hom}_R(T,F)=0$ for all $T\in\mathcal{T}$, $F\in\mathcal{F}$.

(ii) If $\mathrm{Hom}_R(M,F)=0$ for all $F\in\mathcal{F}$, then $M\in\mathcal{T}$.

(iii) If $\mathrm{Hom}_R(T,N)=0$ for all $T\in\mathcal{T}$, then $N\in\mathcal{F}$.

Let $\mathcal{B}$ be a subclass of Mod-R, $\mathcal{P}=\{M|\mathrm{Hom}_R(b,M)=0$ for any $b\in\mathcal{B}\}$ and $\mathcal{S}=\{M|\mathrm{Hom}_R(M,p)=0$ for any $p\in\mathcal{P}\}$. Then $(\mathcal{S},\mathcal{P})$ is called to be a torsion theory generated by $\mathcal{B}$. A torsion theory cogenerated by a subclass of Mod-R is defined similary.

A torsion theory ($\mathcal{T}$,$\mathcal{F}$) is called to be hereditary if $\mathcal{T}$ is closed under taking submodules. It is well known that ($\mathcal{T}$,$\mathcal{F}$) is hereditary if and only if $\mathcal{F}$ is closed under taking injective hulls. A subfunctor of the identity functor of Mod-R is called a preradical. For preradical σ, $\mathcal{T}_\sigma:=\{M\in$ Mod-$R|\ \sigma(M)=M\}$ is the class of σ-torsion right R-modules, and $\mathcal{F}_\sigma:=\{M\in$ Mod-$R|\sigma(M)=0\}$ is the class of σ-torsion free right R-modules. A preradical t is called to be idempotent(a radical) if $t(t(M))=t(M)(t(M/t(M))=0)$. It is well known that $(\mathcal{T}_\sigma,\mathcal{F}_\sigma)$ is a torsion theory for an idempotent radical σ. A preradical t is called to be left exact if $t(N)=N\cap t(M)$ holds for any module M and its submodule N. For a preradical σ and a module M and its submodule N, N is called to be σ-dense submodule of M if $M/N\in\mathcal{T}_\sigma$. If N is an essential and σ-dense submodule of M, then N is called to be a σ-essential submodule of $M(M$ is a σ-essential extension of $N)$. For a left exact radical σ a module M is called to be σ-injective if the functor $\mathrm{Hom}_R(\_,M)$ preserves the exactness for any exact sequence $0\to A\to B\to C\to 0$ with $C\in\mathcal{T}_\sigma$. We denote $E(M)$ the injective hull of a module M. For an idempotent radical σ, $E_\sigma(M)$ is called the σ-injective hull of a module M, where $E_\sigma(M)$ is defined by $E_\sigma(M)/M:=\sigma(E(M)/M)$. Then $E_\sigma(M)$ is σ-injective and a σ-essential extension of M.

Let σ be a left exact radical. We call a torsion theory $(\mathcal{T},\mathcal{F})$ σ-hereditary if $\mathcal{T}$ is closed under taking σ-dense submodules. A torsion theory generated by σ-injective module is σ-hereditary. σ-hereditary torsion theory is studied in [1]. A torsion theory $(\mathcal{T},\mathcal{F})$ is called to be stable if $\mathcal{T}$ is closed under taking injective hulls. We call a torsion theory $(\mathcal{T},\mathcal{F})$ σ-stable if $\mathcal{T}$ is closed under taking σ-injective hulls. σ-stable torsion theory is studied in [3].

A torsion theory$(\mathcal{T},\mathcal{F})$ generated by $\{M/N|N$ is essential in $M\}$ is called to be Goldie torsion theory. Let $Z(M)$ denote the singular submodule of a module M. For an idempotent preradical σ, by Amitsur transfinite construction there exists the smallest radical $\overline{\sigma}$ containing σ. For a module M, $Z_2(M)$ is defined by $Z_2(M)/Z(M):=Z(M/Z(M))$. It is well known that $\mathcal{T}=\mathcal{T}_{Z_2}$ and $\mathcal{F}=\mathcal{F}_{Z_2}$.

For a left exact radical σ we call a torsion theory generated by $\{M/N|N$ is σ-essential in $M\}$ σ-Goldie torsion theory.

Theorem 1. *For a left exact radical σ, σ-Goldie torsion*

For a module M, we denote $Z_\sigma(M):=\{m\in M|mI=0$ for some σ-dense right ideal of $R\}$ and we define $(Z_\sigma)_2(M)$ by $(Z_\sigma)_2(M)/Z_\sigma(M):=Z_\sigma(M/Z_\sigma(M))$.

Theorem 2. *For a left exact radical σ, it holds that $Z_2\sigma=(Z_\sigma)_2=\overline{Z_\sigma}$.*

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Power residues

Kaoru Motose

Let p be a prime, let $\mathbb{F}_p$ be a prime field of order p and let $G = \{\sigma_a \mid 1 \leqq a < m,\ (a, m) = 1\}$ be the automorphism group of $K = \mathbb{Q}(\zeta_m)$, where $\zeta_m = e^{\frac{2\pi i}{m}}$ and $\sigma_a : \zeta_m \to \zeta_m^a$. We set $H = \{\sigma_a \in G \mid \sigma_a^n = \sigma_1\}$ the subgroup of G and L the corresponding subfield of K to H. We put $\mathbb{O}$ the integer ring of subfield L and P a prime ideal in $\mathbb{O}$ containing p.

The next simplifies the proof of [6, Theorem] without Artin map (see [7, p.96]) using Galois correspondence and basic items in [3, p.196 and p.182]. We should note that the condition $\mathbb{O}/P = \mathbb{F}_p$ is equivalent to that p splits completely in $\mathbb{O}$.

(PR) $p^n \equiv 1 \bmod m$ *if and only if* $\mathbb{O}/P = \mathbb{F}_p$.

We shall give results on Feit-Thompson conjecture (see [2]) for primes 3 and 5. Using (PR) and refering some matters in [1], [3], [4] and [7], we obtain (FT3). Here we gave a very simple and easy proof about a relating fact in [1] as showed at Okayama.

(FT3) *If* $\frac{q^3-1}{q-1}$ *divides* $3^q - 1$ *for a prime* $q > 3$, *then* $q \equiv -1 \bmod 72$.

Using some results in [5] by Eisenstein reciprocity law on power residue symbol, we have

(FT5) *If* $\frac{q^5-1}{q-1}$ *divides* $5^q - 1$ *for a prime* $q > 5$, *then*
$q \equiv -1$ *or* $q \equiv 2$ *or* $q \equiv 1/2 \bmod 25$.

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On separable polynomials in skew polynomial rings

Satoshi Yamanaka

K. Kishimoto, T. Nagahara, Y. Miyashita, G. Szeto, L. Xue, and S. Ikehata studied extensively separable polynomials in skew polynomial rings. The purpose of this talk is to show some recent results about separable polynomials in skew polynomial rings.

A ring extension A/B is called a separable extension if the A-A-homomorphism of $A \otimes_B A$ onto A defined by $a \otimes b \to ab$ splits, and A/B is called a Hirata separable extension if $A \otimes_B A$ is A-A-isomorphic to a direct summand of a finite direct sum of copies of A. Let G be a finite subgroup of $\mathrm{Aut}(A)$, then A/B is called a G-Galois extension if $B = A^G$ and there exist $x_i,\ y_i \in A$ such that $\sum_i x_i \sigma(y_i) = \delta_{1,\sigma}$ $(\sigma \in G)$.

Let B be a ring with identity element 1, ρ an automorphism of B, D a ρ-derivation of B, and $B[X;\rho,D]$ a skew polynomial ring in which the multiplication is given by $\alpha X = X\rho(\alpha) + D(\alpha)$ $(\alpha \in B)$. Let $R = B[X;\rho,D]$ and $f \in R$ with $Rf = fR$. Then the residue ring R/fR is a free ring extension of B. If R/fR is a *separable* (resp. *Hirata separable, G- Glois*) extension of B, then f is called a *separable* (resp. *Hirata separable, G-Galois*) polynomial in R. These provide typical and essential examples of separable, Hirata separable, and G-Galois extensions.

In this talk, we study these polynomials in skew polynomial rings, and we show some new results about them.

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On Goldie extending modules
with finite internal exchange property

Yosuke Kuratomi

Let R be a ring. A right R-module M is said to be *Goldie extending* (*u-Goldie extending*) if, for any (uniform) submodule X of M, there exist an essential submodule Y of X and a direct summand N of M such that Y is essential in N. A Goldie extending module is introduced by Akalan-Birkenmeier-Tercan [1]. Note that Goldie extending modules are dual to H-supplemented modules (cf. [2]).

In this talk, we show some characterizations of Goldie extending and consider generalizations of relative injectivity. And we apply them to the study of the open problems "When is a direct sum of Goldie extending (uniform) modules Goldie extending ?" and "Is the property Goldie extending inherited by direct summands ?" in Akalan-Birkenmeier-Tercan [1]. Main results are the following:

Result 1. Let A and B be modules. Then A is B-injective if and only if A is B-ejective and essentially B-injective.

Result 2. Let M be a module with the finite internal exchange property. Then the following holds:

(1) M is extending if and only if M is Goldie extending and B is essentially A-injective for any decomposition $M = A \oplus B$.
(2) Then M is Goldie extending if and only if any direct summand of M is Goldie extending.

Result 3. Let M_1 and M_2 be Goldie extending modules with the finite internal exchange property and put $M = M_1 \oplus M_2$. Then M is Goldie extending for $M = M_1 \oplus M_2$ if and only if M_1' is weakly M_2'-ojective for any direct summand M_i' of M_i $(i = 1, 2)$.

Result 4. Let $\{U_i \mid i \in I\}$ be a family of uniform modules and put $P = \oplus_I U_i$. If U_i is weakly mono-U_j-ojective for any $i \neq j \in I$ and there is no infinite sequence $f_1, f_2, f_3, f_4, \cdots$ of proper monomorphisms $f_k : U_{i_k} \to U_{i_{k+1}}$ with all $i_k \in I$ distinct, then P is Goldie extending for $P = \oplus_I U_i$.

Result 5. Let $\{U_i \mid i \in I\}$ be a family of uniform modules and put $P = \oplus_I U_i$. Then the following conditions are equivalent:

(1) P is extending with the (finite) internal exchange property,
(2) (a) P is u-extending for the decomposition $P = \oplus_I U_i$,
    (b) There is no infinite sequence $f_1, f_2, f_3, f_4, \cdots$ of proper monomorphisms $f_k : U_{i_k} \to U_{i_{k+1}}$ with all $i_k \in I$ distinct (or, P satisfies (LSS) or (lsTn)).
(3) (a) P is Goldie extending for the decomposition $P = \oplus_I U_i$,
    (b) U_i is essentially U_j-injective,
    (c) (A_2') holds for all U_i and $\{U_j \mid i \neq j \in I\}$.

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Derived equivalence classification of generalized multifold extensions of piecewise hereditary algebras of tree type

Hideto Asashiba and Mayumi Kimura

Throughout this note $\mathbb{k}$ is an algebraically closed field, and all algebras considered here are assumed to be basic finite-dimensional associative $\mathbb{k}$-algebras, which are regarded as $\mathbb{k}$-categories by fixing a basic set of primitive idempotents of A. We denote by D the usual $\mathbb{k}$-duality $\mathrm{Hom}_{\mathbb{k}}(\text{-}, \mathbb{k})$. Let A be an algebra. If $\psi \in \mathrm{Aut}(A)$, then ψ canonically induces an automorphism $\hat{\psi}$ of the repetitive category $\hat{A}$. A category of the form $T_{\psi}^{n}(A) := \hat{A}/\langle \hat{\psi}\nu_A^n \rangle$ with $n \in \mathbb{Z}$ is called a *twisted n-fold extension* of A, where ν_A is the Nakayama automorphism of $\hat{A}$. An automorphism ϕ of $\hat{A}$ is said to *have a jump* $n \in \mathbb{Z}$ if $\phi(A_0^{[0]}) = A_0^{[n]}$. Note that in this case ϕ induces an automorphism $\phi_0 := (\mathbb{1}^{[0]})^{-1}\nu_A^{-n}\phi\mathbb{1}^{[0]}$ of A. A category of the form $\hat{A}/\langle \phi \rangle$ with ϕ an automorphism of $\hat{A}$ having a jump n is called a *generalized n-fold extension* of A (or a *generalized multifold extension* of A if n is not specified). Twisted n-fold extensions of A are generalized n-fold extensions of A.

An algebra A is called *piecewise hereditary* of *tree type* T if it is derived equivalent to a hereditary algebra H and the underlying graph of the quiver of H is a tree T. In [1] we gave a derived equivalence classification (and a stable equivalence classification) of twisted multifold extensions of piecewise hereditary algebras of tree type by giving complete invariants under derived equivalences. We would like to extend this classification to the class of their generalized multifold extensions. The following theorem enables us to reduce this problem to the corresponding one for twisted multifold extensions.

Theorem 1. *Let A be a piecewise hereditary algebra of tree type and $n \in \mathbb{Z}$. Then any generalized n-fold extension $\hat{A}/\langle \phi \rangle$ of A (ϕ an automorphism of $\hat{A}$ having a jump n) is derived equivalent to a twisted n-fold extension $T_{\phi_0}^{n}(A)$ of A.*

Definition 2. Let $\Lambda := \hat{A}/\langle \phi \rangle$ be a generalized n-fold extension of piecewise hereditary algebra A of tree type T ($\phi \in \mathrm{Aut}(\hat{A})$ having a jump $n \in \mathbb{Z}$). Then the *derived equivalence type* $\mathrm{type}(\Lambda)$ of Λ is a triple $(T, n, \overline{\pi}(\phi_0))$, where the last entry was defined for the twisted multifold extension $T_{\phi_0}^{n}(A)$ in [1].

Theorem 3. *Let Λ, Λ' be generalized multifold extensions of piecewise hereditary algebras of tree type. Then the following are equivalent.*

(1) Λ *and* Λ' *are derived equivalent;*
(2) Λ *and* Λ' *are stably equivalent;*
(3) $\mathrm{type}(\Lambda) = \mathrm{type}(\Lambda')$.

Remark 4. When $n = 0$, the action of ϕ or $\hat{\psi}\nu_A^n$ above can be not free, and it was not possible to deal with this case in [1]. But now it became possible to deal also with this case by the result of [2].

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Quotients of exact categories by cluster tilting subcategories as module categories

Liu Yu

The quotient category of a Frobenius category by a cluster tilting subcategory is abelian by a result of Koenig-Zhu and a classical result by Happel.

We generalize this result to non necessarily Frobenius exact categories by an n-cluster tilting subcategory $\mathcal{M}$. More precisely,

Theorem 1. *Let $\mathcal{B}$ be an exact category with enough projectives and injectives.*

(1) *If $\mathcal{M}$ is a 2-cluster tilting subcategory of $\mathcal{B}$, then $\mathcal{B}/[\mathcal{M}] \simeq \operatorname{mod}\underline{\mathcal{M}}$.*

(2) *If $\mathcal{M}$ is an n-cluster tilting subcategory of $\mathcal{B}$, then ${}^{\perp_{n-2}}\mathcal{M}/[\mathcal{M}] \simeq \operatorname{mod}\underline{\mathcal{M}}$ where ${}^{\perp_{n-2}}\mathcal{M} = \{X \in \mathcal{B} \mid \forall i \in \{1, \dots, n-2\}, \operatorname{Ext}^i_{\mathcal{B}}(X, \mathcal{M}) = 0\}$.*

(3) *If $\mathcal{M}$ is a rigid subcategory of $\mathcal{B}$, then $\mathcal{M}_L \simeq$ mod $\underline{\mathcal{M}}$ where $\mathcal{M}_L = \{X \in \mathcal{B} \mid X$ admits a left 2-resolution by $\mathcal{M}\}$.*

Moreover, if $\mathcal{B}$ admits a $(n-1)$-AR translation τ_{n-1}, we can embed the second equivalence in a commutative diagram of equivalences of categories

$$\begin{tikzcd} {}^{\perp_{n-2}}\mathcal{M}/[\mathcal{M}] \arrow[r] \arrow[d, "\tau_{n-1}"] & \operatorname{mod}\underline{\mathcal{M}} \arrow[d] \\ \mathcal{M}^{\perp_{n-2}}/[\Omega'\mathcal{M}] \arrow[r] & \operatorname{mod}\overline{\mathcal{M}} \end{tikzcd}$$

where Ω' is the cosyzygy.

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Dimensions of triangulated categories with respect to subcategories

Tokuji Araya

This is a joint work with T. Aihara, O. Iyama, R. Takahashi and M. Yoshiwaki.

The notion of the dimension of a triangulated category has been introduced by Rouquier [5] based on work of Bondal and Van den Bergh [2] on Brown representability. It measures how many extensions are needed to build the triangulated category out of a single object, up to finite direct sum, direct summand and shift.

In this talk, we will introduce the concept of the dimension of a triangulated category with respect to a fixed full subcategory.

For the bounded derived category of an abelian category, upper bounds of the dimension with respect to a contravariantly finite subcategory and a resolving subcategory are given. Our methods not only recover some known results on the dimensions of derived categories in the sense of Rouquier, but also apply to various commutative and non-commutative noetherian rings.

Our main result is the following theorem.

Theorem 1. *Let $\mathcal{A}$ be an abelian category, and $\mathcal{X}$ a contravariantly finite subcategory which generates $\mathcal{A}$. Then*

$$\mathcal{X}\text{-tri.}\dim \mathsf{D}^{\mathrm{b}}(\mathcal{A}) \leq \mathrm{gl.}\dim(\mathrm{mod}\mathcal{X}).$$

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Ulrich modules and Special modules over 2-dimensional rational singularities

Ken-ichi Yoshida and Kei-ichi Watanabe

Throughout this talk, let $(A, \mathfrak{m}, k)$ be a commutative Noetherian local ring with the unique maximal ideal $\mathfrak{m}$ and the residue field k. Put $d = \dim A$, the Krull dimension of A. Let I be an $\mathfrak{m}$-primary ideal and Q its minimal reduction (i.e., Q is a parameter ideal which is contained in I and $I^{r+1} = QI^r$ for some $r \geq 0$). The length (resp. the minimal number of generators, the multiplicity with respect to I) of M is denoted by $\ell_A(M)$ (resp. $\mu_A(M)$, $e_I^0(M)$).

The notion of Ulrich modules (with respect to maximal ideals) was introduced by [2, 7]. An A-module M is called an Ulrich (MGMCM) A-module if $\mu_A(M) = e_{\mathfrak{m}}^0(M)$.

The first aim of this talk is to introduce the notion of Ulrich modules with respect to a given ideal, and to classify all Ulrich modules over 2-dimensional rational double points. (If we have enough time, we want to discuss non-Gorenstein case).

Definition 1 (Ulrich module [4])**.** Let M be a finitely generated A-module. Then we say that M is an Ulrich A-module if M is an MCM A-module with $e_I^0(M) = \ell_A(M/IM)$ and M/IM is A/I-free.

Suppose that A is a Cohen-Macaulay local ring with minimal multiplicity, that is, $\mu(\mathfrak{m}) = e_{\mathfrak{m}}^0(A) + \dim A - 1$. Then syzygy modules $\mathrm{syz}_A^n(A/\mathfrak{m})$ are Ulrich A-modules for all $n \geq d$ (e.g. [2]). On the other hand, it remains open whether any Cohen-Macaulay local ring admits an Ulrich module.

Definition 2 (Ulrich ideal [4])**.** An $\mathfrak{m}$-primary ideal I is called an Ulrich ideal if $I^2 = QI$ and I/I^2 is A/I-free.

Now suppose that A is a complete 2-dimensional normal local ring and the residue field k is an algebraically closed field of characteristic 0. Then A is said to be a rational double point (i.e. a Gorenstein rational singularity) if it is isomorphic to the hypersurface $k[[x, y, z]]/(f)$, where f is one of the following polynomials:

$$(A_n)\ z^2 + x^2 + y^{n+1}\ (n \geq 1) \quad (D_n)\ z^2 + x^2y + y^{n-1}\ (n \geq 4)$$
$$(E_6)\ z^2 + x^3 + y^4 \quad (E_7)\ z^2 + x^3 + xy^3 \quad (E_8)\ z^2 + x^3 + y^5.$$

We can prove that $\mathrm{syz}_{\mathrm{A}}^{\mathrm{d}}(A/I)$ is an Ulrich A-module with respect to I for any Ulrich ideal I of A (see [4]). The converse is also true for any rational double point.

Theorem 3 ([4])**.** *Suppose that A is a 2-dimensional rational double point, and let I be an $\mathfrak{m}$-primary ideal with $\mu_A(I) > 2$. Then I is an Ulrich ideal if and only if there exists an Ulrich A-module with respect to I.*

By virtue of the theorem above, we consider only Ulrich ideals in order to classify all Ulrich modules with respect to some ideal. An Ulrich ideal I in a Gorenstein local ring is just a good ideal for which A/I is Gorenstein in the sense of [3]. In fact, our main theorem classifies all Ulrich modules with respect to a given Ulrich ideal I. The main tools are Riemann-Roch formula and McKay correspondence (due to Artin-Verdier).

The second aim of this talk is to generalize the notion of special Cohen-Macaulay modules in this context.

Definition 4. Suppose that A is a 2-dimensional rational singularity. An MCM A-module M is said to be a special CM module with respect to I if $\mathrm{syz}_A^1(M) \cong M^* := \mathrm{Hom}_A(M, A)$ and M/IM is A/I-free.

Wunram [9] defined the notion of (indecomposable) special Cohen-Macaulay modules, which corresponds to irreducible curves in the dual graph of resolution of singularities $X \to \mathrm{Spec}A$. Moreover, Iyama and Wemyss [6] classfied all special modules over any quotient surface singularity (which is a typical example of rational singularities).

Let $A = k[[x^4, x^3y, x^2y^2, xy^3, y^4]]$. Then $M_1 = Ax + Ay$ (resp. $M_3 = Ax^3 + Ax^2y + Axy^2 + Ay^3$) is the unique indecomposable special CM (resp. Ulrich) module. Also, $M_2 = Ax^2 + Axy + Ay^2$ is neither special nor Ulrich.

We can give a criterion for special modules for some (good) ideal in this case in terms of dual graphs, and prove the following theorem in the Gorenstein case.

Theorem 5. *Suppose that A is a 2-dimensional rational double point. Let M be an MCM A-module and I an $\mathfrak{m}$-primary ideal of A. Then M is an Ulrich A-module with respect to I if and only if it is a special CM module with respect to I.*

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Quiver varieties and quantum cluster algebras

Yoshiyuki Kimura

Let $A_v(\mathfrak{n}(w))$ be the quantum coordinate ring of the unipotent subgroup associated with a Weyl group element w of a symmetric Kac-Moody Lie algebra. It is shown that $A_v(\mathfrak{n}(w))$ is compatible with Kashiwara-Lusztig's dual canonical basis in [2]. Geiss-Leclerc-Schroer [1] has shown it has a quantum cluster algebra structure which is defined by the additive categorification using the preprojective algebras Λ. It is conjectured that the set of quantum cluster monomials are contained in the dual canonical basis.

Let Q be an acyclic quiver and c_Q be the corresponding acyclic Coxeter word and $w = c_Q^2$ with $\ell(w) = 2|Q_0|$. We consider the graded quiver varieties which is "adapted to Q" and a class of equivariant perverse sheaves on it. It can be shown that the quantum Grothendieck ring of the class of equivariant perverse sheaves has a structure of quantum cluster algebra and the (dual) basis of simple perverse sheaves contains the set of quantum cluster monomials using the Fourier-Sato-Deligne transform of it. As an application, it can be shown that the dual canonical basis of $A_v(\mathfrak{n}(w))$ contains the set of quantum cluster monomials.

This talk is based on a joint work [3] with Fan Qin (Université Paris Diderot, Paris 7, Institute de Mathématiques de Jussieu)

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Cyclotomic KLR algebras of cyclic quivers

小西正秀

ループと多重辺を持たない箙 Γ と，その頂点への非負整数値の重み付け α に対し，Khovanov-Lauda-Rouquier 多元環 (KLR 多元環) が定義される．また，頂点への非負整数値の重み付け Λ に対し，cyclotomic ideal が定義され，それによる商をとったものを cyclotomic KLR 多元環と呼ぶ．

KLR 多元環の生成系は図を用いて表される．ここで用いられる図は簡単に言えば各糸に色のついた組み紐であり，さらに各糸に有限個の黒玉を乗せることが許されているものである．

色は箙の頂点により定まり，図に各色の糸を何本ずつ使うかが α により定まる．また，それらの図は Γ から定まるいくつかの関係により同一視される．

積は図の結合として定義されるが，結合部の色が合致しない場合は 0 とする．特に糸を平行に並べた図は冪等となり，KLR 冪等元と呼ばれる．

cyclotomic ideal は各 KLR 冪等元の左端の糸に，Λ により定まる個数だけ黒玉をつけた図から生成される．

本講演では Γ を n 点巡回箙，即ち $\Gamma_0 = \{0, 1, \cdots, n-1\}$ であり，n を法として i から $i+1$ へ矢が伸びている箙とする．また，全ての頂点に 1 の重みをつけたものを α，頂点 0 にのみ 1 の重みをつけ，他の頂点の重みは 0 としたものを Γ とする．これらの Γ，α，Λ から定まる cyclotomic KLR 多元環を R_n と書くことにする．

このとき，原始冪等元の個数や，どの糸に黒玉が何個つけば 0 となるかが厳密に分かる．証明は KLR 多元環の言葉と，Brundan-Kleshchev による同型 [3] を用いた表現論の言葉の双方を用いて行われる．更にそれらから R_n の構造が規則的に変化することも示される．

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Realizing Cluster Categories of Dynkin type A as Stable Categories of Lattices

Luo Xueyu

We have already known that the cluster category $\mathcal{C}$ of Dynkin type A_n has some tight connections with triangulations of a regular polygon P with $n+3$ vertices by non-crossing diagonals. Given a triangulation of P, we associate a quiver with potential such that the associated Jacobian algebra has the structure of a $K[\![x]\!]$-order denoted as Λ_{n+3}, where $K[\![x]\!]$ is a formal power series ring over a field K. Then we show that $\mathcal{C}$ is equivalent to the stable category of the category of Λ_{n+3}-lattices.

This is my recent research under the guidance of my supervisor Prof. Iyama for my ongoing PhD program.

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Cluster-tilted Algebras of Canonical Type and Quivers with Potentials

Gustavo Jasso

We will describe the effect of mutation on the endomorphism algebras of cluster tilting objects in the cluster category associated with a canonical algebra in terms of quivers with potentials.

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τ-tilting theory

Osamu Iyama

This is a joint work with Takahide Adachi and Idun Reiten.

We introduce τ-rigid modules and (support) τ-tilting modules as generalizations of partial tilting modules and tilting modules.

Definition Let Λ be a finite dimensional algebra.

(a) We call M in $\mathrm{mod}\Lambda$ *τ-rigid* if $\mathrm{Hom}_\Lambda(M, \tau M) = 0$ for the AR translation τ of Λ.

(b) We call M in $\mathrm{mod}\Lambda$ *τ-tilting* if M is τ-rigid and $|M| = |\Lambda|$, where $|M|$ is the number of non-isomorphic indecomposable direct summands of M.

(c) We call M in $\mathrm{mod}\Lambda$ *support τ-tilting* if M is a τ-tilting $(\Lambda/\langle e \rangle)$-module for an idempotent e of Λ.

The notion of τ-rigid modules appear in work of Auslander-Smalo in 1981 without names. We show that they improve tilting theory from the viewpoint of mutation, i.e. any indecomposable summand of a support τ-tilting module can be replaced in a unique way to get a new support τ-tilting module:

Theorem *Let Λ be a finite dimensional algebra. Any basic τ-rigid Λ-module M with $|M| = |\Lambda| - 1$ is a direct summand of exactly two basic τ-tilting Λ-modules.*

Another main result is the following.

Theorem *Let Λ be a finite dimensional algebra. Then we have bijections between*

(a) *functorially finite torsion classes in* $\mathrm{mod}\Lambda$*,*

(b) *isomorphism classes of basic support τ-tilting Λ-modules,*

(c) *isomorphism classes of basic two-term silting complexes of Λ,*

(d) *isomorphism classes of basic cluster-tilting objects in $\mathbf{C}$ if Λ is a 2-CY tilted algebra associated with $\mathbf{C}$.*

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τ-tilting modules for self-injective Nakayama algebras

Takahide Adachi

The aim of this talk is to study support τ-tilting modules for self-injective Nakayama algebras. Recently the notion of a τ-tilting module was introduced by Iyama and Reiten. τ-tilting modules are a generalization of (classical) tilting modules.

Definition 1. Let Λ be a basic finite dimensional algebra over an algebraically closed field. Assume that Λ has n simple modules.

(1) We call $M \in \mathrm{mod}\Lambda$ *τ-tilting* if $\mathrm{Hom}_\Lambda(M, \tau M) = 0$ and $|M| = n$.

(2) We call $M \in \mathrm{mod}\Lambda$ *support τ-tilting* if there exists an idempotent $e \in \Lambda$ such that M is a τ-tilting $\Lambda/\langle e\rangle$-module.

The main result of this talk is the following.

Theorem 2. *Let Λ be a self-injective Nakayama algebra. There exists a one-to-one correspondence between*

(1) *τ-tilting Λ-modules.*

(2) *Proper support τ-tilting Λ-modules without projective Λ-modules as direct summand.*

Moreover, if Loewy length of algebras is more than $n-1$, we have the following result.

Corollary 3. *Let Λ be a self-injective Nakayama algebra with n simple modules and Loewy length $r \geq n$. There exists a one-to-one correspondence between*

(1) *τ-tilting Λ-modules.*

(2) *Proper support τ-tilting Λ-modules.*

As an application of these result, we shall give how to calculate all support τ-tilting modules by using tilting modules for path algebras of Dynkin type A.

In addition, we shall show that the number of basic support τ-tilting modules is equal to $\binom{2n}{n}$ for self-injective Nakayama algebras with n simple modules and Loewy length $r \geq n$.

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2-hereditary algebras and quivers with potential

Martin Herschend

This talk is based on joint work with Osamu Iyama and Steffen Oppermann [2]. From the viewpoint of higher dimensional Auslander-Reiten theory there is a natural analogue of hereditary algebras, called 2-hereditary, which are algebras of global dimension at most two satisfying a certain homological condition. The 2-hereditary algebras consist of two disjoint classes called 2-representation finite and 2-representation infinite. These algebras can be characterized by their higher preprojective algebras, which are selfinjective and 3-Calabi-Yau respectively.

I will present a description of higher preprojective algebras using quivers with potential and certain subsets of arrows called cuts. Using this description we obtain a structure theorem for 2-hereditary algebras. For the finite case several examples are constructed using this structure theorem in [1]. In my talk I will similarly treat the infinite case. I will present a source of examples coming from consistent dimer models, which give rise to 3-Calabi-Yau quivers with potential.

Finally I will discuss 2-APR tilting, which can be used to construct 2-hereditary algebras from given ones. In terms of quivers with potential and cuts it can be viewed as a change of cut called called cut mutation.

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Selfinjective algebras and quivers with potentials

Yuya Mizuno

Derived categories were introduced by Grothendieck and Verdier in the 1960s and they have been one of the important tools in the study of many areas of mathematics.

In the representation theory of algebras, tilting theory plays an essential role to control equivalences of derived categories. By the famous Rickard theorem [4], it is known that derived equivalences is obtained by tilting complexes. Tilting modules is considered as a special case of tilting complexes, which has been extensively investigated by many authors. Compared with tilting modules, it is hard to find tilting complexes which is not tilting modules. Moreover, it is quite difficult to give an explicit description of endomorphism algebras of tilting complexes because of the difficulty of the calculations for complexes. However it is known that there exists only trivial tilting module for selfinjective algebras, so that it is important to obtain explicitly derived equivalence classes of selfinjective algebras.

In this talk, we deal with selfinjective algebra given by quivers with potentials and explain that we can provide a derived equivalence classes of algebras by a combinatorial way.

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On the Relation of the Upper Bound of Global Dimension and the Length of Serial Algebra Which Has Finite Global Dimension

Morio UEMATSU

Let A be a finite dimensional basic connected serial algebra over an algebraically closed field, and n is the number of the non isomorphic simple left modules of A. If global dimension gl.dim.A of A is finite, then gl.dim.$A \leq 2n-2$ and length $l(A)$ of A is less than or equal to $2n-1$[3]. In this talk, we consider the relationship $l(A)$ and $gl.dim.A$.

Let $P_1, P_2, \cdots, P_n$ be the indecomposable left projective modules of A with P_{j+1} is a projective cover of $radP_j$ for $j = 1, 2, \cdots, n-1$, and P_1 is a projective cover of $radP_n$ if P_n is not simple. The sequence of posive integers $(a_1, \cdots, a_n)$ where $a_j = l(P_j)$ for $1 \leq j \leq n$ is called the admissible sequence of A and has the propety that $a_{j+1} \geq a_j - 1 \geq 1$ for all $j = 1, 2, \cdots, n-1$ and $a_1 \geq a_n - 1$. We call A is of cyclic type if P_n is not simple(or equivalently $a_n > 1$).

Now, let A be serial algebra of cyclic type with admissible sequence $(a_1, \cdots, a_n)$. f denote the function on $\{1, 2, \cdots, n\}$ defined by $f(i) = [i + a_i]$.Where

$$[i + a_i] = \begin{cases} (i + a_i) \mod n, & 1 \leq (i + a_i) \mod n \leq n-1 \\ n, & (i + a_i) \mod n = 0 \end{cases}.$$

We define admissible quiver Q of A by $\{1, 2, \cdots, n\}$ is set of vertices and an arrow i to j if $f(i) = j$. An admissible quiver is a disconnected union of left serial quiver which define below.

Definition 1. A quiver called a left serial if it has unique oriented cycle and when removing all arrows of this cycle, the remaing is a disconnected union of trees with a single root which is a vertex of the cycle.

Proposition 2. *Let A be a serial algebra of cyclic type. If $gl.dim.A < \infty$, then its admissible quiver is a left serial.*

Theorem 3. *Let A be a serial algebra of cyclic type with finite global dimension and $l = l(A)$.*

(1) *If $n < l < 2n$, then $gl.dim.A \leq 4n - 2l$.*
(2) *If $l(A) = n$, then $gl.dim.A \leq 2n - 3$.*
(3) *For any positive integer k with $n \geq 2k + 3$ and $\lceil \frac{n}{k+1} \rceil < \lceil \frac{n}{k} \rceil$, if $\lceil \frac{n}{k+1} \rceil \leq l < \lceil \frac{n}{k} \rceil$, then $gl.dim.A \leq 2n - 2k - 3$.*

Example 4. Let A be serial algebra with admissible sequence $(3, 3, 3, 3, 2, 2)$. In this case, $n = 6, l = \lceil \frac{6}{2} \rceil = 3 < \lceil \frac{6}{1} \rceil, k = 1$, and $gl.dim.A = 7 = 2 \cdot 6 - 2 \cdot 1 - 3$.

Example 5. Let A be serial algebra with admissible sequence $(4, 4, 4, 4, 4, 3, 3, 3, 3, 3, 3, 3, 3, 3)$. In this case, $n = 14, l = \lceil \frac{14}{4} \rceil = 4 < \lceil \frac{14}{3} \rceil, k = 3$, and $gl.dim.A = 19 = 2 \cdot 14 - 2 \cdot 3 - 3$.

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Mutation of quiver with potential at several vertices

Laurent Demonet

Derksen, Weyman and Zelevinsky introduced mutations of quivers with potentials in [DWZ]. These mutations have been proved to categorify cluster algebras mutations, which permitted to reach a better understanding of cluster algebras by solving several open conjectures. On the other hand, the Jacobian algebras of mutated quivers with potentials are canonically derived equivalent [KY].

In both of these cases, the mutation of Jacobian algebras behaves as the mutation of endomorphism rings of cluster tilting objects in well behaved triangulated categories. Moreover, it these triangulated categories, it makes sense to mutate several indecomposable summands of cluster tilting objects at the same time.

The aim of this talk is to introduce a conjectural formula of mutation of quiver with potential at several vertices. Thus, we will give some lemmas and propositions supporting the conjecture. For example, the fact that it numerically behaves has expected (*c.f.* green sequences) when the subquiver we mutate at belongs to some specified families of quivers (including the family of acyclic quivers).

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On the representation rings of the dihedral 2-groups

Erik Darpö

While the finite-dimensional modules of the dihedral 2-groups over fields of characteristic 2 were classified over 30 years ago, very little is known about the tensor products of such modules. In this talk, I shall present a formula for the Loewy length of the tensor product of any two modules of a dihedral 2-group.

This is joint work with C. C. Gill.

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On Hochschild cohomology of a class of weakly symmetric algebras with radical cube zero

Daiki Obara, Takahiko Furuya

In this talk, we consider the Hochschild cohomology of the following algebra A. Let k be a field and $m \geq 1$. Let Q be a quiver with m vertices and $2m$ arrows as follows:

$$\overset{a_0}{\circlearrowright} 0 \underset{\overline{a}_1}{\overset{a_1}{\rightleftarrows}} 1 \underset{\overline{a}_2}{\overset{a_2}{\rightleftarrows}} \cdots \underset{\overline{a}_{m-1}}{\overset{a_{m-1}}{\rightleftarrows}} m-1 \overset{a_m}{\circlearrowleft}$$

Let $A = kQ/I$ where I is the ideal of kQ generated by

$$a_1\overline{a}_1 - a_0^2, \quad a_m^2 - \overline{a}_{m-1}a_{m-1}, \quad \overline{a}_1 a_0, \quad a_m\overline{a}_{m-1},$$
$$a_i\overline{a}_i - \overline{a}_{i-1}a_{i-1}, \quad a_j a_{j+1}, \quad \overline{a}_{l+1}\overline{a}_l,$$
$$\text{for } 2 \leq i \leq m-1, 0 \leq j \leq m-1 \text{ and } 1 \leq l \leq m-2.$$

Let $\mathrm{HH}^*(A)$ be the Hochschild cohomology ring of A. In [1], Erdmann and Solberg showed that A satisfies the following finiteness conditions:

- H is a commutative Noetherian graded subalgebra of $\mathrm{HH}^*(A)$ with $H^0 = \mathrm{HH}^0(A)$.
- $\mathrm{Ext}^*_A(A/J, A/J)$ is a finitely generated H-module where J denotes the Jacobson radical of A.

So $\mathrm{HH}^*(A)$ is finite generated as a k-algebra. Moreover, in [2], Schroll and Snashall determined the Hochschild cohomology groups of A in the case $m = 2$.

In this talk, we introduce the Hochschild cohomology groups of A in the case $m = 1$ or $m \geq 3$.

Theorem 1. *In the case $m \geq 3$,*

$$\dim_k \mathrm{HH}^n(A) =$$
$$\begin{cases} m+3 & \textit{if } n = 0, \\ p + \begin{cases} 3 & \textit{if } p \textit{ is even and } \operatorname{char} k \neq 2, \\ 2 & \textit{if } p \textit{ is odd, } t \neq m-1 \textit{ and } \operatorname{char} k \neq 2, \\ 3 & \textit{if } p \textit{ is odd, } t = m-1 \textit{ and } \operatorname{char} k \neq 2, \\ 4 & \textit{if } p \textit{ is even and } \operatorname{char} k = 2, \\ 3 & \textit{if } p \textit{ is odd, } t \neq m-1 \textit{ and } \operatorname{char} k = 2, \\ 4 & \textit{if } p \textit{ is odd, } t = m-1 \textit{ and } \operatorname{char} k = 2, \end{cases} & \textit{if } n = pm+t \geq 1. \end{cases}$$

where p is the quotient and t is the remainder when we divide n by m.

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On the existence of a bypass in the Auslander-Reiten quiver

Takahiko FURUYA

Let A be an artin algebra over a commutative Artinian ring K, and let $\mathit{\Gamma}_A$ be the Auslander-Reiten quiver of A. The aim in this talk is to describe a bypass in $\mathit{\Gamma}_A$ defined in [1] (see also [2]).

Let $Y \to Z$ be an arrow in $\mathit{\Gamma}_A$, and let $n \geq 2$ be an integer. Then a path $Y = X_0 \to X_1 \to \cdots \to X_{n-1} \to X_n = Z$ in $\mathit{\Gamma}_A$ with $Y \neq X_{n-1}$ and $Z \neq X_1$ is called a bypass of the arrow $Y \to Z$. If a bypass is a sectional path, then it is said to be a sectional bypass, otherwise it is said to be a nonsectional bypass.

Bypasses occasionally appear in the Auslander-Reiten quiver, for example, the postprojective components and the preinjective components of the Auslander-Reiten quivers of certain hereditary algebras of Euclidean type $\widetilde{\mathbb{A}}_n$ have sectional bypasses. In [1, 2] the authors gave some properties of bypasses and studied the shapes of components in $\mathit{\Gamma}_A$ having a sectional bypass. In this talk we show that if a bypass in $\mathit{\Gamma}_A$ is a weakly sectional path ([3]), then it is actually a sectional bypass.

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