The 46th Symposium on Ring Theory

and Representation Theory

ABSTRACT

Tokyo University of Science, Tokyo

October 12 – 14, 2013

プログラム

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A classification of cyclotomic KLR algebras of type $A_n^{(1)}$

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On some finiteness questions about Hochschild cohomology of finite-dimensional algebras

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10:45–11:15 亀山 統胤（信州大学）・星野光男・古賀 寛尚（筑波大学）
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16:30–17:20 Manuel Saorín Castaño (Universidad de Murcia)
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Support tau-tilting modules and preprojective algebras

10:15–10:45 足立 崇英（名古屋大学）
Classifying τ-tilting modules over Nakayama algebras

10:45–11:15 加瀬 遼一（大阪大学）
On the poset of pre-projective tilting modules over path algebras

11:30–12:00 板垣 智洋・眞田 克典（東京理科大学）
The dimension formula of the cyclic homology of truncated quiver algebras over a field of positive characteristic

One point extension of quiver algebras defined by two cycles and a quantum-like relation

Daiki Obara

Let k be a field, $\mathrm{HH}^*(A)$ the Hochschild cohomology ring of a finite dimensional k-algebra A and $\mathcal{N}$ the ideal of $\mathrm{HH}^*(A)$ generated by all homogeneous nilpotent elements. In [3], using the Hochschild cohomology ring modulo nilpotence $\mathrm{HH}^*(A)/\mathcal{N}$ Snashall and Solberg defined a support variety of A-module. And, in [2], Snashall gave the question to as whether we can give necessary and sufficient conditions on a finite dimensional algebra A for $\mathrm{HH}^*(A)/\mathcal{N}$ to be finitely generated as an algebra.

Let A_q be the quiver algebra defined by two cycles and a quantum-like relation depending on a nonzero element q in k. In [1], we determined the Hochchild cohomology ring of A_q modulo nilpotence $\mathrm{HH}^*(A_q)/\mathcal{N}$.

In this talk, we consider a one point extension algebra B of A_q. We determine the Hochschild cohomology ring modulo nilpotence $\mathrm{HH}^*(B)/\mathcal{N}$ and show that if q is a root of unity then $\mathrm{HH}^*(B)/\mathcal{N}$ is not finitely generated as an algebra.

For $s, t \geq 2$, let Γ be the quiver with $s+t$ vertices $1 = a(1) = b(1)$, $a(2), \ldots, a(s)$, $b(2), \ldots, b(t)$, 2 and $s+t+1$ arrows $\gamma\colon 1 \to 2$, $\alpha_i\colon a(i) \to a(i+1)$, $\beta_j\colon b(j) \to b(j+1)$ for $1 \leq i \leq s$, $1 \leq j \leq t$ where we regard the numbers i modulo s and j modulo t. Paths are written from right to left.

Let $B = k\Gamma/I_{q,v,u}$ where $I_{q,v,u}$ is the ideal of $k\Gamma$ generated by

$$X^{sa}, X^sY^t - qY^tX^s, Y^{tb}, \gamma X^{sv+u}$$

for $a, b \geq 2$, $0 \leq v \leq a-1$, $0 \leq u \leq s-1$ and $(v,u) \neq (0,0)$, $X := \alpha_1+\alpha_2+\cdots+\alpha_s$ and $Y := \beta_1+\beta_2+\cdots+\beta_t$. Then we have the following results.

Theorem 1. *If $s, t \geq 2$ and q is an r-th root of unity then*

$$\mathrm{HH}^*(B)/\mathcal{N} \cong \begin{cases} k \oplus k[W^{2r}_{0,0,0}, W^{2r}_{2r,0,0}]W^{2r}_{0,0,0} & \text{if } \bar{a} \neq 0, \bar{b} \neq 0, \\ k \oplus k[W^{2}_{0,0,0}, W^{2r}_{2r,0,0}]W^{2}_{0,0,0} & \text{if } \bar{a} \neq 0, \bar{b} = 0, \\ k \oplus k[W^{2r}_{0,0,0}, W^{2}_{2,0,0}]W^{2r}_{0,0,0} & \text{if } \bar{a} = 0, \bar{b} \neq 0, \\ k \oplus k[W^{2}_{0,0,0}, W^{2}_{2,0,0}]W^{2}_{0,0,0} & \text{if } \bar{a} = \bar{b} = 0. \end{cases}$$

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A Classification of cyclotomic KLR algebras of type $A_n^{(1)}$

Masahide Konishi

Khovanov-Lauda-Rouquier algebras (KLR algebras) were independently invented by Khovanov and Lauda [1] , and Rouquier [2] in 2008. This invention was motivated by categorification of quantum algebras.

KLR algebras have trivial idempotents called KLR idempotents. In general, there exists KLR idempotents which are not primitive. It is clear that when all KLR idempotents are primitive.

On cyclotomic KLR algebras, it is not easy to determine when all KLR idempotents are primitive. In this talk, we determine when that happen in type essentially $A_n^{(1)}$.

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On some finiteness questions about Hochschild cohomology of finite-dimensional algebras

Takahiko Furuya and Takao Hayami

Let K be an algebraically closed field, and let A be a finite-dimensional algebra over K. In this talk, we consider the following question **(1)** and conjecture **(2)** about the Hochschild cohomology of A:

(1) Happel's question [4]**.** If the Hochschild cohomology groups $\mathrm{HH}^n(A)$ of A vanish for all $n \gg 0$, then is the global dimension of A finite?

(2) Snashall-Solberg's conjecture [6]**.** The Hochschild cohomology ring modulo nilpotence, $\mathrm{HH}^*(A)/\mathcal{N}_A$, of A is finitely generated as an algebra.

Recently, in [1, 2, 5], a negative answer to **(1)** and, in [7, 8], some counterexamples to **(2)** have been obtained, where the authors computed the Hochschild cohomology groups or rings of several finite-dimensional Koszul algebras. In this talk we give other Koszul algebras which give a negative answer to **(1)** and counterexamples to **(2)**. We also pose new questions as to Hochschild cohomology rings modulo nilpotence associated with the conjecture **(2)**.

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Source algebras and cohomology of block ideals of fintie group algebras

Hiroki Sasaki

Let G be a finite group and k an algebraically closed field of characteristic p dividing the order of G. The group algebra kG is decomposed as a direct sum of indecomposable two sided ideals:

$$kG = B_0 \oplus B_1 \oplus \cdots \oplus B_s.$$

Direct summands above are called *block ideals*; only one block ideal survives under the augumentation map $\varepsilon : kG \to k$, which is called *the principal block* and is denoted by $B_0(G)$.

Let B be a block ideal of kG; let D be a defect group of B. An indecomosable direct summand of B as $k[G \times D^{\mathrm{op}}]$-bimodule having $\Delta(D) \leqslant G \times D^{\mathrm{op}}$ as a vertex is called *a source module* of B. A pair (Q, b_Q) of a p-subgroup $Q \leqslant G$ and a block ideal b_Q of $kC_G(Q)$ is called a B-subpair if the Brauer correspondent ${b_Q}^G$ of b_Q to G coincides with B. Let X be a source module of B; we can take a unique maximal B-subpair (D, b_D) associated with X and form the category $\mathcal{F}_{(D,b_D)}(B, X)$, which is called *a Brauer category*, with objects the subgroups $Q \leqslant D$. For an arbitrary subgroup Q of D there exits a uniqe B-subpair (Q, b_Q) contained in (D, b_D). Let Q, $P \leqslant D$; a morphism $\varphi : Q \to P$ is a conjugation map induced by an element $g \in G$ with ${}^g(Q, b_Q) \subseteq (P, b_P)$.

The cohomology ring $H^*(G, B; X)$ of B is defined in Linckelmann [2] to be the subring of $H^*(D, k)$ consisting of $\mathcal{F}_{(D,b_D)}(B, X)$-stable elements:

$$H^*(G, B; X) = \{\, \zeta \in H^*(D, k) \mid \operatorname{res}_Q \zeta \in H^*(Q, k) \text{ is } N_G(Q, b_Q)\text{-invariant for all } Q \leqslant D \,\}.$$

A defect group of the principal block $B_0(G)$ is a Sylow p-subgroup of G and the Brauer category is just the Frobenius category so that the cohomology of $B_0(G)$ is just the G-stable subring of the cohomology ring of a Sylow p-subgroup, which is isomorphic to the ordinary cohomology ring of G.

The tensor product $A = X^* \otimes_B X$ of the k-dual of X and X endowed with ring structure is called *a source algebra* of the block B, which has so many common properties with the block. The cohomology ring of B is also determined by the source algebra A in the following sense.

Theorem 1. ([2, Theorem 5.6], [3, Theorem 1]) *An element $\zeta \in H^*(D, k)$ belongs to $H^*(G, B; X)$ if and only if the diagonal embedding $\delta_D(\zeta) \in HH^*(kD)$ is A-stable, where A is considered as a (kD, kD)-bimodule.*

Note that as a (kD, kD)-bimodule A is isomorphic to a direct sum of some $k[DgD]$s, since $A = X^* \otimes_B X$ is isomorphic to a direct summand kG as (kD, kD)-bimodule, which is decomposed into $kG = \bigoplus_{DgD \in D\backslash G/D} k[DgD]$.

We however have to confess that we do not have enough infomation on (kD, kD)-bimodule structures of source algebras. The following is an attempt to analyse source algebras by using cohomology theory.

For $g \in G$, the (kD, kD)-bimodule $k[DgD]$ defines a transfer map $t_{DgD} : HH^*(kD) \to HH^*(kD)$. The image of $\zeta \in H^*(D, k)$ under the diagonal embedding $\delta_D : H^*(D, k) \to HH^*(kD)$ followed by the transfer map t_{DgD} is again an image under δ_D; we can define a transfer map, which we also denote by the same symbol, $t_{DgD} : H^*(D, k) \to H^*(D, k)$;

$$\zeta \mapsto \operatorname{tr}^D \operatorname{res}_{D \cap {}^gD} {}^g\zeta.$$

Theorem 2. *Let $(P, b_P), (Q, b_Q) \subseteq (D, b_D)$; assume that $C_D(P)$ is a defect group of b_P or $C_D(Q)$ is a defect group of b_Q. For $g \in G$ with ${}^g(P, b_P) = (Q, b_Q)$ if the map*

$$t_g : H^*(D, k) \to H^*(D, k); \zeta \mapsto \operatorname{tr}^D \operatorname{res}_Q {}^g\zeta$$

is not the zero map, then there exists a double coset DxD with the property that $k[DxD]$ is isomorphic to a direct summand of the source algebra A, $Q = D \cap {}^xD$, ${}^x(P, b_P) = (Q, b_Q)$ and $t_g = t_{DxD}$.

Let B be a block ideal of tame representation type; it is known that $p = 2$ and a defect group D is isomorphic to a dihedral group, a semidihedral group, or a (generalized) quaternion group. In Kawai–Sasaki [1] we calculated the cohomology rings of these blocks and constructed transfer maps $t : H^*(D, k) \to H^*(D, k)$ whose images are just the cohomology rings of the blocks.

Applying Theorem 2 to these blocks, we obtain the following.

Theorem 3. *There exists a direct summand M of the source algebra A as (kD, kD)-bimodule such that*

(1) *the transfer maps obtained in Kawai–Sasaki* [1] *are induced from M,*
(2) *an element $\zeta \in H^*(D, k)$ belongs to $H^*(G, B; X)$ if and only if the diagonal embedding $\delta_D(\zeta) \in HH^*(kD)$ is M-stable.*

We can asign (D, D)-double cosets whose union defines (kD, kD)-bimodule isomorphic to M above. A detailed analysys on transfer maps defined by double cosets that could give rise to (kD, kD)-bimodules isomorphic to direct summands of the source algebra A implies that the transfer map defined by the source algebra A itself, of cource as (kD, kD)-bimodule, is described by using the transfer maps defined by double cosets that induce the module M.

We conjecture in general that for an arbitrary block ideal B its cohomology ring $H^*(G, B; X)$ is just the image under the transfer map defined by the source algebra $X^* \otimes_B X$.

A similar but more complicated calculations was also done in [1] for 2-blocks with defect groups isomorphic to wreathed 2-groups of rank 2:

$$W = \langle\, a,\, b,\, t \mid a^{2^n} = b^{2^n} = t^2 = 1,\, ab = ba,\, tat = b\,\rangle.$$

There exists a direct summand M of the source algebra A as (kD, kD)-bimodules satisfying the property (1) in the theorem above.

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Geigle-Lenzing spaces and canonical algebras in dimension d

Osamu Iyama

Weighted projective lines were introduced by Geigle and Lenzing. One key property is that they have tilting bundles, whose endomorphism rings are Ringel's canonical algebras. They have been important objects in representation theory and studied intensively. In this talk we will introduce the notion of Geigle-Lenzing d-spaces, generalizing the concept of weighted projective lines. In this case we obtain a nice tilting bundle, whose endomorphism ring we call a d-canonical algebra. We will then focus on some properties of Geigle-Lenzing d-spaces and their Cohen-Macaulay representation theory. In particular we show that the stable category of Cohen-Macaulay modules also has a tilting object. This is based on joint works with Martin Herschend, Boris Lerner, Hiroyuki Minamoto and Steffen Oppermann.

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Stable set polytopes of trivially perfect graphs

Kazunori Matsuda

Let G be a simple graph on the vertex set $V(G) = [n]$ with the edge set $E(G)$. $S \subset V(G)$ is said to be **stable** if $\{i, j\} \notin E(G)$ for all $i, j \in S$. Note that $\emptyset$ is stable. Put $\mathcal{Q}_G = \{S \subset V(G) \mid S \text{ is stable }\}$, and we define

$$k[\mathcal{Q}_G] := k[T \cdot \textstyle\prod_{i \in S} X_i \mid S \in \mathcal{Q}_G] \subset k[T, X_1, \ldots, X_n].$$

$k[\mathcal{Q}_G]$ is called the *toric ring associated with the stable set polytope of a graph.*

In this talk, we consider the problem when $k[\mathcal{Q}_G]$ is strongly Koszul. The notion of strongly Koszulness was defined by Herzog-Hibi-Restuccia.

Definition 1 ([1])**.** A graded k-algebra R is said to be **strongly Koszul** if $\mathfrak{m}$ admits a minimal system of generators $\{u_1, \ldots, u_n\}$ which satisfies the following condition:

For all subsequence $u_{i_1}, \ldots, u_{i_r}$ of $\{u_1, \ldots, u_t\}$ $(i_1 \leq \cdots \leq i_r)$ and for all $j = 1, \ldots, r-1$, $(u_{i_1}, \ldots, u_{i_{j-1}}) : u_{i_j}$ is generated by a subset of elements of $\{u_1, \ldots, u_t\}$.

The main theorem of this talk is as follows.

Theorem 2. *Let G be a graph. Then the following assertions are equivalent:*

(1) *$k[\mathcal{Q}_G]$ is strongly Koszul.*
(2) *G is trivially perfect graph.*

In particular, if $k[\mathcal{Q}_G]$ is strongly Koszul then $k[\mathcal{Q}_G]$ is trivial.

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Triangulated subcategories of extensions and triangles of recollements

Kiriko Kato

This is joint work with Peter Jørgensen. Let $\mathcal{T}$ be a triangulated category with triangulated subcategories $\mathcal{X}$ and $\mathcal{Y}$. We show that the subcategory of extensions $\mathcal{X} * \mathcal{Y}$ is triangulated if and only if every morphism from $\mathcal{X}$ to $\mathcal{Y}$ is factored through an object of $\mathcal{X} \cap \mathcal{Y}$. In this situation, we show that there is a stable t-structure $\left(\frac{\mathcal{X}}{\mathcal{X}\cap\mathcal{Y}}, \frac{\mathcal{Y}}{\mathcal{X}\cap\mathcal{Y}}\right)$ in $\frac{\mathcal{X}*\mathcal{Y}}{\mathcal{X}\cap\mathcal{Y}}$. We use this to give a recipe for constructing recollements and triangles of recollements.

Hearts of twin cotorsion pairs on exact categories

Liu Yu

The cotorsion pairs were first introduced by Salce, and it has been deeply studied in the representation theory during these years, especially in tilting theory and Cohen-Macaulay modules. Recently, the cotorsion pairs are also studied in triangulated categories [2], in particular, Nakaoka introduced the notion of hearts of cotorsion pairs and showed that the hearts are abelian categories [4]. This is a generalization of the hearts of t-structure in triangulated categories [1] and the quotient of triangulated categories by cluster tilting subcategories [3]. Moreover, he generalized these results to a more general setting called twin cotorsion pair [5].

The aim of this talk is to give similar results for cotorsion pairs on Quillen's exact categories, which plays an important role in representation theory. We consider a *cotorsion pair* on an exact category, which is a pair $(\mathcal{U}, \mathcal{V})$ of subcategories of an exact category $\mathcal{B}$ satisfying $\mathrm{Ext}^1_{\mathcal{B}}(\mathcal{U}, \mathcal{V}) = 0$ (*i.e.* $\mathrm{Ext}^1_{\mathcal{B}}(U, V) = 0$, $\forall U \in \mathcal{U}$ and $\forall V \in \mathcal{V}$) and any $B \in \mathcal{B}$ admits two short exact sequences $V_B \rightarrowtail U_B \twoheadrightarrow B$ and $B \rightarrowtail V^B \twoheadrightarrow U^B$ where $V_B, V^B \in \mathcal{V}$ and $U_B, U^B \in \mathcal{U}$. Let

$$\mathcal{B}^+ := \{B \in \mathcal{B} \mid \mathcal{U}_B \in \mathcal{V}\}, \quad \mathcal{B}^- := \{B \in \mathcal{B} \mid \mathcal{V}^B \in \mathcal{U}\}.$$

We define the *heart* of $(\mathcal{U}, \mathcal{V})$ as the quotient category

$$\underline{\mathcal{H}} := (\mathcal{B}^+ \cap \mathcal{B}^-)/(\mathcal{U} \cap \mathcal{V}).$$

Now we state our first main result, which is an analogue of [4, Theorem 6.4].

Theorem 1. *Let $(\mathcal{U}, \mathcal{V})$ be a cotorsion pair on an exact category $\mathcal{B}$ with enough projectives and injectives. Then $\underline{\mathcal{H}}$ is abelian.*

Moreover, following Nakaoka, we consider pairs of cotorsion pairs $(\mathcal{S}, \mathcal{T})$ and $(\mathcal{U}, \mathcal{V})$ in $\mathcal{B}$ such that $\mathcal{S} \subseteq \mathcal{U}$, we also call such a pair a *twin cotorsion pair*. The notion of hearts is generalized to such pairs, and our second main result is the following, which is an analogue of [5, Theorem 5.4].

Theorem 2. *Let $(\mathcal{S}, \mathcal{T}), (\mathcal{U}, \mathcal{V})$ be a twin cotorsion pair on $\mathcal{B}$. Then $\underline{\mathcal{H}}$ is semi-abelian.*

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Serre subcategories of Artinian modules

Naoya Hiramatsu

Let R be a commutative noetherian ring and M be an R-module. We denote by $\mathrm{Mod}(R)$ the category of R-modules and R-homomorphisms and by $\mathrm{mod}(R)$ the full subcategory consisting of finitely generated R-modules. We also denote by Spec R the set of prime ideals of R and by $\mathrm{Ass}_R M$ the set of associated prime ideals of M. We say that a full subcategory is wide if it is closed under kernels, cokernels and extensions. A Serre subcategory is defined to be a wide subcategory which is closed under subobjects.

Classification theory of subcategories has been studied by many authors in many areas. Classically, Gabriel [1] gives a bijection between the set of Serre subcategories of $\mathrm{mod}(R)$ and the set of specialization closed subsets of Spec R. Recently, the following result was proved by Takahashi [5] and Krause [3].

Theorem 1. [5, Theorem 4.1][3, Corollary 2.6] *Let R be a noetherian ring. Then we have the following 1-1 correspondences;*

$$\{ \textit{ subcategories of } \mathrm{mod}(R) \textit{ closed under submodules and extensions } \} \cong \{ \textit{ subsets of } \ \mathrm{Spec}\ R\ \}.$$

Moreover this induces the bijection

$$\{ \textit{ Serre subcategories of } \mathrm{mod}(R)\ \} \cong \left\{ \textit{ specialization closed subsets of } \mathrm{Spec}\ R\ \right\}.$$

Krause [3] generalized the theorem to subcategories of $\mathrm{Mod}(R)$ which are closed under submodules, extensions and direct unions after Takahashi [5] proved it.

In addition, Takahashi [5] pointed out a property concerning wide subcategories of $\mathrm{mod}(R)$. Actually he proved the following theorem.

Theorem 2. [5, Theorem 3.1, Corollary 3.2] *Let R be a noetherian ring. Then every wide subcategory of $\mathrm{mod}(R)$ is a Serre subcategory of $\mathrm{mod}(R)$.*

In this talk we want to consider the artinian analogue of these results. We denote by $\mathrm{Att}_R M$ the set of the attached prime ideals of M. We shall show the following results.

Theorem 3. *Let R be a noetherian ring. Then every wide subcategory of $\mathrm{Art}(R)$ is a Serre subcategory of $\mathrm{Art}(R)$.*

Theorem 4. *Let R be a noetherian ring. Then one has an inclusion preserving bijection*

$$\begin{gathered}\{ \textit{ subcategories of } \mathrm{Art}(R) \textit{ closed under quotient modules and extensions } \} \\ \cong \{ \textit{ subsets of the set consisting of closed prime ideals of } \ \hat{R}\ \}.\end{gathered}$$

Moreover this induces the bijection

$$\{ \textit{ Serre subcategories of } \mathrm{Art}(R)\ \} \cong \left\{ \begin{array}{c} \textit{specialization closed subsets of} \\ \textit{the set consisting of closed prime ideals of } \ \hat{R} \end{array} \right\}.$$

We consider some completion of a ring, so that all of artinian modules can be regarded as modules over it.

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Specialization orders on atom spectra of Grothendieck categories

Ryo Kanda

The aim of this talk is to provide some methods to construct Grothendieck categories with certain structures and to establish a theory of partial orders on the spectra of Grothendieck categories.

In commutative ring theory, Hochster characterized topological spaces appearing as the prime spectra of commutative rings ([1, Theorem 6 and Proposition 10]). Speed [6] pointed out that Hochster's result induces the following characterization of partially ordered sets appearing as the prime spectra of commutative rings.

Theorem 1 (Hochster [1, Proposition 10] and Speed [6, Corollary 1])**.** *Let P be a partially ordered set. Then P is isomorphic to the prime spectrum of some commutative ring with the inclusion relation if and only if P is an inverse limit of finite partially ordered sets in the category of partially ordered sets.*

In [2] and [3], we investigated Grothendieck categories by using the associated topological spaces called the *atom spectra* of them. For a Grothendieck category $\mathcal{A}$, we have a partial order on the atom spectrum $\operatorname{ASpec}\mathcal{A}$. For a commutative ring R, the inclusion relation between prime ideals is a partial order on the prime spectrum $\operatorname{Spec}R$, and we have an isomorphism between $\operatorname{ASpec}(\operatorname{Mod}R)$ and $\operatorname{Spec}R$ as partially ordered sets. Hence we can consider that the atom spectrum of a Grothendieck category is a (noncommutative) generalization of the prime spectrum of a commutative ring, and it is natural to ask *what partially ordered sets appear as the atom spectra of Grothendieck categories.*

In order to answer this question, we introduce a construction of Grothendieck categories using colored quivers. A sextuple (Q_0, Q_1, C, s, t, u) is called a *colored quiver* if (Q_0, Q_1, s, t) is a quiver (not necessarily finite), C is a set (of colors), and $u\colon Q_1 \to C$ is a map. From a colored quiver which satisfies some condition of local finiteness, we construct a Grothendieck category associated to the colored quiver. By establishing a method to obtain appropriate colored quivers, we can show the following result, which is a complete answer to the above question.

Theorem 2 ([4])**.** *For any partially ordered set P, there exists a Grothendieck category $\mathcal{A}$ such that the atom spectrum $\operatorname{ASpec}\mathcal{A}$ is isomorphic to P as a partially ordered set.*

This construction of Grothendieck categories has several applications. As one of them, we can show the following result, which shows the existence of a counter-example to the question stated in [5, Note 4.20.3].

Theorem 3 ([4])**.** *There exists a nonzero Grothendieck category without a prime localizing subcategory.*

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Resolving subcategories of modules of finite projective dimension over a commutative ring

Manuel Saorín Castaño

Given a commutative Noetherian ring R, let us denote by $\mathrm{mod}-R$ its category of finitely generated modules and by $\mathcal{D}R$ the derived category of the category of all R-modules. I will show that the following three sets are in one-to-one correspondence, giving the precise definition of the bijection:

(1) The resolving subcategories of $\mathrm{mod}-R$ consisting of modules of finite projective dimension;
(2) The compactly generated t-structures of $\mathcal{D}R$ containing the stalk complex $R[1]$ in their heart;
(3) Decreasing sequences $Y_0 \supseteq Y_1 \supseteq ... \supseteq Y_n \supseteq ...$ of closed under specialization subsets of $\mathrm{Spec}(R)$ such that $Y_n \cap \mathrm{Ass}(E_n(R)) = \emptyset$, for all $n \geq 0$, where $E_n(R)$ is the n-th term of the minimal injective resolution of R.

The result relates to a recent claasification of the subcategories mentioned in the set 1 given by Dao and Takahashi, which uses grade-consistent functions. A byproduct of our result is a new interpretation of the (small) finitistic dimension of R in terms of the minimal injective resolution of R. The contents of the presentation appear in the joint paper with Lidia Angeleri-Hügel, "t-structures and cotilting modules over commutative noetherian rings" (http://profs.sci.univr.it/ angeleri/publ.html).

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On the ring of complex-valued functions on a finite ring

Yasuyuki Hirano

Greferath, Fadden and Zumbrägel [1] considered rings of complex-valued functions on some finite rings. Let R be a ring and let $\mathbf{C}$ denote the field of complex numbers. A function $f : R \to \mathbf{C}$ is said to be *finite* if $\{r \in R \mid f(r) \neq 0\}$ is a finite set. Consider the set $\mathbf{C}^R$ of all finite functions $\{f \mid f : R \to \mathbf{C} \text{ is finite}\}$. For $f,\ g \in \mathbf{C}^R$ and for $\lambda \in \mathbf{C}$, we define addition and scalar multiplication by

$$(f+g)(x) = f(x) + g(x)$$
$$(\lambda f)(x) = \lambda f(x).$$

Then $\mathbf{C}^R$ is a $\mathbf{C}$-vector space. We define multiplication by

$$(f * g)(x) = \sum_{\substack{a, b \in R \\ ab = x}} f(a)g(b).$$

Then $\mathbf{C}^R$ is a $\mathbf{C}$-algebra. For each element $r \in R$, we define the function δ_r by

$$\delta_r(x) = \begin{cases} 1 & \text{if } x = r \\ 0 & \text{otherwise} \end{cases}.$$

Then δ_1 is the identity of the $\mathbf{C}$-algebra $\mathbf{C}^R$. We can easily see that $\delta_r * \delta_s = \delta_{rs}$ for each $r,\ s \in R$. Also we can see that the set $\{\delta_r \mid r \in R\}$ forms a $\mathbf{C}$-basis of the vector space $\mathbf{C}^R$.

Let R be a finite ring. Recall that the semigroup ring $\mathbf{C}[R]$ is a $\mathbf{C}$-vector space with the $\mathbf{C}$-basis $\{\hat{r} \mid r \in R\}$ and multiplication defined by

$$\hat{r}\hat{s} = \hat{rs}$$

for each $r,\ s \in R$. We define a mapping $\phi : \mathbf{C}^R \to \mathbf{C}[R]$ by $\phi(\delta_r) = \hat{r}$ for each $r \in R$. Then ϕ is an isomorphism of $\mathbf{C}$-algebras.

Using this fact, we study the structure of $\mathbf{C}$-algebras $\mathbf{C}^R$ for some finite rings R.

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QF rings and direct summand conditions

Manabu Matsuoka

In [7], T. Sumiyama studied maximal Galois subrings of finite local rings. Y. Hirano characterized finite frobenius rings in [3]. We shall generalized the notion of QF rings. For a ring R, we consider the condition that every finitely generated free submodule N of a finitely generated free R-module M is a direct summand of M. QF rings satisfy this condition. In [4], Y. Hirano studied the relation of artinian rings and this condition.

By the way, since several years, codes over finite Frobenius rings draw considerable attension in coding theory. In [2], M. Greferath investigated splitting codes over finite rings. In [1], A. A. Andrade and Palazzo Jr. studied linear codes over finite rings. J. A. Wood established the extension theorem and MacWilliams identities over finite Frobenius rings in [8]. K. Shiromoto and L. Storme gave a Griesner type bound for linear codes over finite QF rings in [6].

In this talk, we study the rings with the direct summand condition and give the applications to coding theory.

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Clifford extension

Mitsuo Hoshino, Noritsugu Kameyama and Hirotaka Koga

Auslander-Gorenstein rings appear in various fields of current research in mathematics (see [1], [2], [3] and [5]). However, little is known about constructions of Auslander-Gorenstein rings. We have already known that Frobenius extensions of Auslander-Gorenstein rings are Auslander-Gorenstein rings. In this note, formulating the construction of Clifford algebras (see e.g. [4]) we introduce the notion of Clifford extensions and show that Clifford extensions are Frobenius extensions. Consequently Clifford extensions of Auslander-Gorenstein rings are Auslander-Gorenstein.

Let $n \geq 2$ be an integer. We fix a set of integers $I = \{0, 1, \dots, n-1\}$ and a ring R. We use the notation A/R to denote that a ring A contains R as a subring. First, we will construct a Frobenius extension Λ/R using a certain pair (σ, c) of $\sigma \in \mathrm{Aut}(R)$ and $c \in R$. Namely, we will define an appropriate multiplication on a free right R-module Λ with a basis $\{v_i\}_{i \in I}$. Then we restrict ourselves to the case where $n = 2$ in order to recover the construction of Clifford algebras. For $m \geq 1$ we construct ring extensions Λ_m/R which we call Clifford extensions using the following data: a sequence of elements $c_1, c_2, \cdots$ in $\mathrm{Z}(R)$ and signs $\varepsilon(i, j)$ for $1 \leq i, j \leq m$. Namely, we will define an appropriate multiplication on a free right R-module Λ_m with a basis $\{v_x\}_{x \in I^m}$. We show that Λ_m is obtained by iterating the construction above m times, that Λ_m/R is a Frobenius extension, and that if $c_i \in \mathrm{rad}(R)$ for $1 \leq i \leq m$ then $R/\mathrm{rad}(R) \xrightarrow{\sim} \Lambda_m/\mathrm{rad}(\Lambda_m)$.

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Group-graded and group-bigraded rings

Mitsuo Hoshino, Noritsugu Kameyama and Hirotaka Koga

Let I be a non-trivial finite multiplicative group with the unit element e. Throughout this talk, we fix a ring A together with a family $\{\delta_x\}_{x\in I}$ in $\mathrm{End}_{\mathbb{Z}}(A)$ satisfying the following conditions:

(D1) $\delta_x\delta_y = 0$ unless $x = y$ and $\sum_{x\in I}\delta_x = \mathrm{id}_A$;

(D2) $\delta_x(a)\delta_y(b) = \delta_{xy}(\delta_x(a)b)$ for all $a, b \in A$ and $x, y \in I$.

Namely, setting $A_x = \mathrm{Im}\ \delta_x$ for $x \in I$, $A = \oplus_{x\in I} A_x$ is an I-graded ring. In this talk, we determine when the ring extension A of A_e is a Frobenius extension (see [2, 3]) using a constructed ring extension Λ of A.

Let Λ be a free right A-module with a basis $\{v_x\}_{x\in I}$. We denote by $\{\gamma_x\}_{x\in I}$ the dual basis of $\{v_x\}_{x\in I}$ for the free left A-module $\mathrm{Hom}_A(\Lambda, A)$. Also, setting $\gamma = \sum_{x\in I}\gamma_x$, we define a mapping $\phi : \Lambda \to \mathrm{Hom}_A(\Lambda, A), \lambda \mapsto \gamma\lambda$. Then Λ is an associative ring with $1 = \sum_{x\in I} v_x$ and contains A as a subring, and ϕ is an isomorphism of A-Λ-bimodules, i.e., Λ/A is a Frobenius extension of first kind. The main result of this talk is the following theorem.

Theorem 1. *Assume A_e is local, $A_xA_{x^{-1}} \subseteq \mathrm{rad}(A_e)$ for all $x \neq e$ and A is reflexive as a right A_e-module. Then the following are equivalent.*

(1) *$A \cong \mathrm{Hom}_{A_e}(A, A_e)$ as right A-modules.*

(2) *There exist a unique $s \in I$ and some $\alpha \in \mathrm{Hom}_{A_e}(A, A_e)$ such that*

$$\phi_{sx,x} : v_{sx}\Lambda \xrightarrow{\sim} \mathrm{Hom}_{A_e}(\Lambda v_x, A_e), \lambda \mapsto (\mu \mapsto \alpha(\gamma(\lambda\mu)))$$

for all $x \in I$.

(3) *There exist a unique $s \in I$ and some $\alpha_s \in \mathrm{Hom}_{A_e}(A_s, A_e)$ such that*

$$\psi_x : A_{sx} \xrightarrow{\sim} \mathrm{Hom}_{A_e}(A_{x^{-1}}, A_e), a \mapsto (b \mapsto \alpha_s(ab))$$

for all $x \in I$.

Formulating the ring structure of Λ constructed as avobe, we make the following.

Definition 2. A ring Λ together with a group homomorphism

$$\eta : I^{\mathrm{op}} \to \mathrm{Aut}(\Lambda), x \mapsto \eta_x$$

is said to be an I-bigraded ring, denoted by (Λ, η), if $1 = \sum_{x\in I} v_x$ with the v_x orthogonal idempotents and $\eta_y(v_x) = v_{xy}$ for all $x, y \in I$. A homomorphism $\varphi : (\Lambda, \eta) \to (\Lambda', \eta')$ is defined as a ring homomorphism $\varphi : \Lambda \to \Lambda'$ such that $\varphi(v_x) = v'_x$ and $\varphi\eta_x = \eta'_x\varphi$ for all $x \in I$.

We show that every I-bigraded ring is isomorphic to the I-bigraded ring Λ constructed above.

This talk is based on [1].

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Classical derived functor as fully faithful embeddings

Manuel Saorín Castaño

The original problem can be stated as follows. Suppose that A and B are algebras and let T be a $B-A-$bimodule. When is it true that the derived Hom functor $\mathrm{RHom}_A(T, ?) : \mathcal{D}A \longrightarrow \mathcal{D}B$ (resp. the derived tensor product $? \otimes_B^L T : \mathcal{D}B \longrightarrow \mathcal{D}A$) is fully faithful?. I will present necessary and sufficient conditions for this to happen, even in the more general context of derived categories of small dg categories. As a byproduct, when applied to ordinary algebras, we recover several results in the literature due to Bazzoni-Mantese-Tonolo, Yang, Chen-Xi, Angeleri-König-Liu, Bazzoni-Pavarin, etc. Another consequence of our results is a parametrization, for a small dg category $\mathcal{A}$, of all the co-smashing subcategories of $\mathcal{D}\mathcal{A}$ which contain the compact objects. The contents of the presentation appear in the joint paper with Pedro Nicolás, "Generalized tilting theory" (arxiv.org/abs/1208.2803).

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Complements and closed submodules relative to torsion theories

Yasuhiko Takehana

Throughout this paper R is a ring with a unit element, every right R-module is unital and Mod-R is the category of right R-modules. A subfunctor of the identity functor of Mod-R is called a preradical. For preradical σ, $\mathcal{T}_\sigma := \{M \in \text{Mod-}R | \sigma(M) = M\}$ is the class of σ-torsion right R-modules, and $\mathcal{F}_\sigma := \{M \in \text{Mod-}R | \sigma(M) = 0\}$ is the class of σ-torsion free right R-modules. A preradical t is called to be idempotent(a radical) if $t(t(M)) = t(M)(t(M/t(M)) = 0)$. It is well known that $(\mathcal{T}_t, \mathcal{F}_t)$ is a torsion theory for an idempotent radical t. A preradical t is called to be left exact if $t(N) = N \cap t(M)$ holds for any module M and its submodule N. For a preradical σ and a module M and its submodule N, N is called to be σ-dense submodule of M if $M/N \in \mathcal{T}_\sigma$. If N is an essential and σ-dense submodule of M, then N is called to be a σ-essential submodule of M(M is a σ-essential extension of N). For an idempotent radical σ a module M is called to be σ-injective if the functor $\text{Hom}_R(\_, M)$ preserves the exactness for any exact sequence $0 \to A \to B \to C \to 0$ with $C \in \mathcal{T}_\sigma$. We denote $E(M)$ the injective hull of a module M. For an idempotent radical σ, $E_\sigma(M)$ is called the σ-injective hull of a module M, where $E_\sigma(M)$ is defined by $E_\sigma(M)/M := \sigma(E(M)/M)$. Then even if σ is not left exact, $E_\sigma(M)$ is σ-injective and a σ-essential extension of M, is a maximal σ-essential extension of M and is a minimal σ-injective extension of M.

Let B be a submodule of a module M. It is said that B is a complement in M if there exists a submodule X of M such that B is maximal in $\{Y \subseteq M | Y \cap X = 0\}$. It is well known that B is a complement in M if and only if B is essentially closed in M. We call B is σ-essentially closed in M if B has no proper σ-essential extension in M. In this talk, we generalize this by using left exact radical σ and state the related results. Following proposition generalize Proposition 1.4 in [K. R. Goodearl , Ring theory, Dekker,1976].

Proposition 1. *Let σ be a left exact radical and B be a submodule of a module M. We denote $\overline{B}/B := \sigma(M/B)$. Then the following conditions from (1) to (9) are equivalent.*

(1) *B is essentially closed in $\overline{B}$.*

(2) *B is σ-essentially closed in M.*

(3) *B is a complement of a submodule in $\overline{B}$.*

(4) *If X is a complement of B in $\overline{B}$, then B is a complement of X in $\overline{B}$.*

(5) *It holds that $B = E_\sigma(B) \cap M$.*

(6) *If X is an essential submodule of $\overline{B}$ containing B, then X/B is essential in $\overline{B}/B$.*

(7) *It holds that $B = E(B) \cap \overline{B}$.*

(8) *There exists submodules M_1 and K of M such that $K \subseteq M_1$, $M/M_1 \in \mathcal{F}_\sigma$ and B is a complement of K in M_1.*

(9) *If X is a σ-essential submodule of M containing B, then X/B is σ-essential in M/B.*

Next we consider C_i modules relative to torsion theories. For C_i modules, see [S. H. Mohamed and B. J. Muller, Continuous and discrete modules, Cambridge Univ. Press, 1990]. We call a module M σ-quasi-injective if for any σ-dense submodule N of M, $\text{Hom}_R(\_, M)$ preserves the exactness of a short exact sequence $0 \to N \to M \to M/N \to 0$.

Lemma 2. *Let σ be a left exact radical. Then A is σ-quasi-injective if and only if $f(A) \subseteq A$ for any $f \in Hom_R(E_\sigma(A), E_\sigma(A))$*

Lemma 3. *Let σ be a left exact radical. If A is σ-quasi-injective and $E_\sigma(A) = M \oplus N$, then $A = (M \cap A) \oplus (N \cap A)$.*

Proposition 4. *Let σ be a left exact radical. Any σ-quasi-injective module M satisfies the following two conditions.*

(σ-C_1) *Every σ-dense submodule of M is σ-essential in a summand of M(or σ-CS module, or σ-extending module)*

(σ-C_2) *If a σ-dense submodule A of M is isomorphic to a summand of M, then A is a summand of M. (or σ-direct injective)*

Ideals and torsion theories

Hiroyuki MINAMOTO

We introduce ideal theoretic conditions on an ideal $\mathcal{I}$ of an abelian category $\mathcal{A}$, which are show to be equivalent to the condition that the ideal is associated to a torsion class (resp. pre-torsion class, Serre subcategory) of $\mathcal{A}$. We also discuss an ideal which is associated to a radical (a sub functor of the identity functor $\mathrm{id}_{\mathcal{A}}$ which has a special property) of $\mathcal{A}$.

This work came out from an attempt to obtain a formalism of the argument which is given in the proof of the following theorem on 2-representation infinite (2-RI) algebras ([1]): Let Λ be a 2-RI algebra and τ_2, τ_2^- be 2-Auslander-Reiten translations. A Λ-module M is called θ-minimal if the canonical morphism $M \to \tau_2\tau_2^- M$ is injective.

Theorem 1. *Let $M \neq 0$ be a θ-minimal indecomposable Λ-module. Assume that $\mathrm{Hom}_\Lambda(M, \Lambda) = 0$. (e.g., M is a non-projective 2-preprojective module or 2-regular module.) Then the Auslander-Reiten component Γ_M containing M is of type $\mathbb{Z}\mathrm{A}_\infty$ unless it contains projective module or injective modules.*

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Defining relations of 3-dimensional quadratic AS-regular algebras

Gahee Kim

M.Artin により 1990 年代に創設された非可換代数幾何学の主要な研究対象の一つとして Artin-Schelter[1] によって導入された AS-regular algebra がある. 実際, 非可換代数幾何学は Artin-Tate-Van den Bergh [2] による 3 次元 AS-regular algebra の分類から始まった分野である. この AS-regular algebra の研究は現在でも盛んに行われている [4],[5]. 本研究の目的は, 3 次元 quadratic AS-regular algebra を代数的に分類することである. 以下標数 0 の代数的閉体 k を固定する.

Definition 1. [1] ネーター連結次数付き代数 A が次を満たすとき d 次元 AS-regular algebra であるという.

- $\mathrm{gldim} A = d < \infty$
- $\mathrm{Ext}^i_A(k, A) \cong \begin{cases} k & (i = d), \\ 0 & (i \neq d). \end{cases}$

例えば, 0 次元 AS-regular algebra は k のみであり, 1 次元 AS-regular algebra は $k[x]$ と同型である.

次数 1 の元で生成された 3 次元 AS-regular algebra の分類は geometric pair を使った幾何的な手法で Artin-Tate-Van den Bergh [2] により完成されている.

ここでは, $\mathbb{P}^n$ の閉部分スキーム E と $\sigma \in \mathrm{Aut}_k E$ からなる幾何の組 (E, σ) と一対一に対応する algebra $\mathcal{A}(E, \sigma)$ を geometric algebra と言う (正確な定義は [3] を参照). このとき, 任意の 3 次元 quadratic AS-regular algebra は geometric algebra であり, 先に述べたようにこの 3 次元 quadratic AS-regular algebra の場合の幾何の組 (E, σ) による幾何的な分類は完成されていて [2], このときの E の可能性としては,$\mathbb{P}^2$ か $\mathbb{P}^2$ 内の 3 次曲線で,

(1)$\mathbb{P}^2$
(2) 三角形を作る三本の直線
(3) 一点で交わる三本の直線
(4) 二点で交わる二次曲線と直線
(5) 一点で交わる二次曲線と直線
(6)cuspidal cubic curve
(7)nodal cubic curve
(8)elliptic curve
(9) 三重直線
(10) 二重直線と直線の和

のいずれかである.

任意の 3 次元 quadratic AS-regular algebra は

$$k\langle x_1, x_2, x_3\rangle/(f_1, f_2, f_3) \quad , \quad (\deg\, x_i = 1, f_j : \text{斉次 2 次多項式})$$

で表されるが, 本研究では, 上に分類されている各 E に対して, geometric pair (E, σ) から導かれる 3 次元 quadratic AS-regular algebra $A = \mathcal{A}(E, \sigma)$ がどのような関係式を持つかを調べ, それらが

(1) いつ次数付き代数として同型になるか,

(2) いつ次数付き森田同値になるか.

を関係式のパラメータを用いて分類することを目標としている.

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Quantum planes and iterated Ore Extensions

Ryo Akiyama

Throughout my talk, we fix an algebraically closed field k of characteristic 0. An AS-regular algebra defined below is one of the main objects of study in noncommutative algebraic geometry.

Definition 1 ([1]). A noetherian connected graded algebra A is called a d-dimensional AS-regular algebra if

- $\mathrm{gldim} A = d < \infty$,
- $\mathrm{Ext}^i_A(k, A) \cong \begin{cases} k & i = d \\ 0 & i \neq d. \end{cases}$

Iterated Ore extensions of k in the title are known as examples of AS-regular algebras. They are defined as follows:

Definition 2 ([2]). Let A be a connected graded algebra, σ a graded algebra automorphism of A and δ a graded σ-derivation (i.e., $\delta : A(-m) \to A$, for some $m \in \mathbb{N}$, is a graded linear map such that $\delta(ab) = \delta(a)b + \sigma(a)\delta(b)$ for all $a, b \in A$). Then σ, δ uniquely determine a connected graded algebra S satisfying

- $S = A[z]$ with $\deg(z) = m$, as a graded left A-module, and
- for any $a \in A$, $za = \sigma(a)z + \delta(a)$.

The algebra S is denoted by $A[z; \sigma, \delta]$ and is called the graded Ore extension of A associated to σ and δ. Then we define an n-iterated graded Ore extension of k by

$$k[z_1; \sigma, \delta][z_2; \sigma, \delta] \cdots [z_n; \sigma, \delta].$$

For example, 2-dimensional AS-regular algebras are 2-iterated graded Ore extensions of k ([3]). In this talk, we will try to answer the question which 3-dimensional quadratic AS-regular algebras (quantum projective planes) are 3-iterated graded Ore extensions of k. We obtain a classification of quantum affine planes by using 3-iterated graded Ore extensions of k.

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The symmetry, period and Calabi-Yau dimension of m-fold mesh algebras

Manuel Saorín Castaño

A m-fold mesh algebra Λ, as introduced by Erdmann and Skowronski, is defined as the orbit category of the stable translation quiver $\mathbb{Z}\Delta$, where Δ is a Dynkin quiver of type $\mathbf{A}_n$, $\mathbf{D}_n$ or $\mathbf{E}_r$ $(r = 6, 7, 8)$, under the action of a weakly admissible group of automorphisms. The class of m-fold mesh algebras properly contains that of stable Auslander algebras of standard selfinjective algebras of finite representation type. In this presentation, we will give the full list of all the m-fold mesh algebras which are symmetric and weakly symmetric. Due to a result of Brenner-Butler-King, the m-fold mesh algebras are known to be periodic, that is, for any such algebra Λ, there is an integer $m > 0$ such that the m-th syzygy of Λ as a bimodule is isomorphic to Λ. The smallest of these m is called the (Ω-)period of Λ. In the presentation, we will give the precise formula for that period. Also, we will give necessary and sufficient conditions of the stable category $\Lambda - \underline{mod}$ to be Calabi-Yau, in the sense of Kontsevich, calculating explicitly the Calabi-Yau dimension of this triangulated category. This completes and extends earlier work in this direction by Erdmann-Skowronski, Dugas and Ivanov-Volkov. The contents of the presentation appear in the joint paper with Estefanía Andreu Juan, "The symmetry, period and Calabi-Yau dimension of finite dimensional mesh algebras" (arxiv.org/abs/1304.0586).

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Mutation and mutation quivers of symmetric special biserial algebras

Takuma Aihara

The notion of mutation, which is an operation to make a new object from a given one, plays crucial roles in representation theory of algebras. Three kinds of mutation are well-known: quiver-mutation [6], cluster tilting mutation [5, 7] and silting/tilting mutation [1]. From Morita theoretic viewpoint, the class of tilting complexes is one of the most important classes, that is, tilting complexes induce derived equivalences which preserve many homological properties. We focus on tilting complexes and tilting mutation. In the case of finite dimensional algebras, we know two famous tilting modules and complexes: Auslander-Platzeck-Reiten tilting modules [3] and Okuyama-Rickard tilting complexes [9, 10]. These are special cases of tilting mutation. Now it is a crucial problem to find the combinatorial structure of tilting complexes. For example, we know reflection of quivers [4] and mutation/flip of Brauer trees [2, 8].

In this talk we wrestle with this problem for symmetric special biserial algebras, where there exist a one-to-one correspondence between the following two classes:

(1) Symmetric special biserial algebras;
(2) Special quivers with cycle-decomposition (*SB quivers*);

Introducing mutation of SB quivers, which can be regarded as an analogue of derived equivalences associated with Bernstein-Gelfand-Ponomarev reflection of quivers [4] and Fomin-Zelevinski quiver-mutation [6], we explicitly give the combinatorics description of tilting mutation of symmetric special biserial algebras. The main result of this talk is the following:

Theorem 1. *The following two operations are compatible:*

- *Tilting mutation of symmetric special biserial algebras;*
- *Mutation of SB quivers;*

In classification of derived equivalent classes of algebras, 'reduction' theorem plays a crucial role. It means to give one of the 'simplest' algebras which are derived equivalent to a given algebra. We give 'reduction' theorem for symmetric special biserial algebras, by applying the main theorem.

Theorem 2. *Any symmetric special biserial algebra is derived equivalent to a symmetric special biserial algebra with the same number of vertices and the same multiplicities* (>1) *of the cycles, and having two cycles* C, C' *such that every vertex belongs to either 'only* C*', '*C *and* C'*' or '*C *and a loop'.*

Moreover, we will observe the behavior of the tilting quiver of a symmetric special biserial algebra.

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Support tau-tilting modules and preprojective algebras

Yuya Mizuno

The aim of this talk is to study *support τ-tilting modules* over preprojective algebras.

Preprojective algebras first appeared in the work of Gelfand-Ponomarev [2]. Since then, they have been one of the important objects in many branches of mathematics and studied from various viewpoints.

On the other hand, the notion of support τ-tilting modules was recently introduced in [1], which is defined as follows.

Definition 1. Let Λ be a finite dimensional algebra, τ the Auslander-Reiten translation and X a Λ-module.

(a) We call X *τ-tilting* if $\mathrm{Hom}_\Lambda(X, \tau X) = 0$ and $|X| = |\Lambda|$, where $|X|$ denotes the number of non-isomorphic indecomposable direct summands of X.

(b) We call X *support τ-tilting* if there exists an idempotent e of Λ such that X is a τ-tilting $(\Lambda/\langle e\rangle)$-module.

We can show that (classical) tilting modules are τ-tilting modules, so that this concept gives a generalization of tilting modules. Moreover, it is shown that there are deep connections between support τ-tilting modules, torsion classes, silting complexes and cluster tilting objects. Support τ-tilting modules also have nicer mutation theory than tilting modules' one and admit interesting combinatorial descriptions. Furthermore, it is useful to study support τ-tilting modules for giving derived equivalences. It is therefore fruitful to investigate these remarkable modules.

In this talk, we investigate support τ-tilting modules over preprojective algebras using a connection with the Coxeter groups. We also discuss a relationship between support τ-tilting modules and certain categories of a module category of path algebras.

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Classifying τ-tilting modules over Nakayama algebras

Takahide Adachi

In this talk, we give a combinatorial description of τ-tilting modules over Nakayama algebras. Recently, to improve a difficulty of mutation of tilting modules ([2, 3] for details), the notion of τ-tilting modules was introduced in [1].

Definition 1. Let Λ be a basic finite dimensional algebra.

(1) A Λ-module M is called *tilting module* if $\mathrm{pd}M \leq 1$, $\mathrm{Ext}^1_\Lambda(M,M) = 0$ and $|M| = |\Lambda|$, where $|X|$ is the number of non-isomorphic indecomposable summand of a Λ-module X.

(2) A Λ-module M is called *τ-tilting module* if $\mathrm{Hom}_\Lambda(M,\tau M) = 0$ and $|M| = |\Lambda|$, where τ is the Auslander-Reiten translation.

(3) A Λ-module M is called *support τ-tilting module* if there exists an idempotent e of Λ such that M is a τ-tilting $(\Lambda/\langle e\rangle)$-module. In particular, if $e \neq 0$, M is called *proper* support τ-tilting module.

τ-tilting Λ-modules are a generalization of tilting Λ-modules. Indeed, we can easily check that any tilting Λ-module is τ-tilting. Moreover, if Λ is hereditary, every τ-tilting Λ-module is tilting.

Before we give the main result, we recall the following well-known result.

Theorem 2. *Let A be a path algebra of Dynkin quiver of type A_n with linear orientation. There are bijections:*

(a) *The set of isomorphism classes of tilting A-modules.*

(b) *The set of triangulations of an $(n+2)$-gon.*

(c) $\{(a_1,\cdots,a_n) \in \mathbb{Z}^n_{\geq 0} \mid \sum_{i=1}^n a_i = n,\ \sum_{i=1}^j a_i \leq j \ \ (^\forall j \in \{1,2,\cdots,n\})\}$.

Since the path algebra A is hereditary and Nakayama, Theorem 2 gives a combinatorial description of τ-tilting modules over the Nakayama algebra. We give an analog of Theorem 2 for τ-tilting modules over Nakayama algebras. Namely, the main result of this talk is the following theorem.

Theorem 3. *Let Λ be a Nakayama algebra with $n = |\Lambda|$. Assume that the length of each indecomposable projective Λ-module is at least n. There are bijections:*

(a) *The set of isomorphism classes of τ-tilting Λ-modules.*

(b) *The set of isomorphism classes of proper support τ-tilting Λ-modules.*

(c) *The set of triangulations of an n-gon with a puncture.*

(d) *The set of non-negative integer sequences $(a_1,a_2,\cdots,a_n)$ with $\sum_{i=1}^n a_i = n$.*

(If time allows, we give more general result for any Nakayama algebras.)

As a result, we determine the number of support τ-tilting Λ-modules.

Corollary 4. *The number of isomorphism classes of support τ-tilting Λ-modules is equal to $\binom{2n}{n}$.*

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On the poset of pre-projective tilting modules over path algebras

Ryoichi Kase

To classify tilting modules is an important problem of the representation theory of finite dimensional algebras . Theory of tilting-mutation introduced by Riedtmann and Schofield is one of the approach to this problem. Riedtmann and Schofield defined the tilting quiver related with tilting-mutation. Happel and Unger defined the partial order on the set of (isomorphic classes of) basic tilting modules and showed that tilting quiver is coincided with Hasse quiver of this poset.

In this talk we consider the poset of pre-projective tilting modules over path algebras. First we give an equivalent condition for that the poset of pre-projective tilting modules $\mathcal{T}_{\mathrm{p}}(Q)$ over a path algebra kQ is a distributive lattice:

Theorem 1. *Let Q be a finite connected acyclic quiver which is non Dynkin. Then $\mathcal{T}_p(Q)$ is a distributive lattice if and only if Q satisfies the following condition* (C)*;*

(C) for any vertex x of Q, there are at least two arrows starting or ending at x.

We define a poset $\mathcal{P}(Q)$ as follows:

• $\mathcal{P}(Q) = \{\text{indecomposable pre-projective modules over } kQ\}/\simeq$.

• $X > Y \overset{\text{def}}{\Leftrightarrow}$ there is a path from X to Y in Auslander-Reiten quiver of kQ.

The second main result of this talk is the following:

Theorem 2. *If Q satisfies the condition* (C)*, then there is a poset isomorphism*

$$\mathcal{T}_{\mathrm{p}}(Q) \simeq \mathcal{I}(\mathcal{P}(Q)) \setminus \{\emptyset\},$$

where $\mathcal{I}(\mathcal{P}(Q))$ is the ideal poset of $\mathcal{P}(Q)$.

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The dimension formula of the cyclic homology of truncated quiver algebras over a field of positive characteristic

Tomohiro Itagaki and Katsunori Sanada

Let K be a field and Δ a finite quiver. We fix a positive integer $m \geq 2$. The truncated quiver algebra A is defined by $K\Delta/R_\Delta^m$, where R_Δ^m is the two-sided ideal of $K\Delta$ generated by the all paths of length m. In this talk, we show the dimension formula of the cyclic homology $HC_n(A)$ of A over an arbitrary field.

In [4], Sköldberg gives a minimal left A^e-projective resolution and computes the Hochschild homology of A over a commutative ring. He also gives the projective resolution of quadratic monomial algebras and computes their Hochschild homology. On the other hand, Cibils gives a useful projective resolution for more general algebras in [1]. In [5], Sköldberg gives the cyclic homology of quadratic monomial algebras by computing the E^2-term of a spectral sequence determined by the mixed complex due to Cibils in [2]. On the other hand, Taillefer [6] gives a dimension formula of the cyclic homology of truncated quiver algebras over a field of characteristic zero by means of [3, Theorem 4.1.13].

We compute the cyclic homology $HC_n(A)$ of A by means of a spectral sequence, and we have the following theorem.

Theorem 1. *Suppose that $m \geq 2$ and $A = K\Delta/R_\Delta^m$. Then the dimension formula of the cyclic homology of A is given by, for $c \geq 0$,*

$$\begin{aligned}\dim_K HC_{2c}(A) = \#\Delta_0 &+ \sum_{e=1}^{m-1} a_{cm+e} + \sum_{c'=0}^{c-1}\sum_{e=1}^{m-1} \sum_{\substack{r>0 \\ \text{s.t. } r\zeta | c'm+e}} b_r \\ &+ \sum_{c'=1}^{c} \sum_{\substack{r>0 \\ \text{s.t. } r|c'm, \\ \gcd(m,r)\zeta|m}} b_r + \sum_{c'=1}^{c} \sum_{\substack{r>0 \\ \text{s.t.} r\zeta|\gcd(m,r)c'}} (\gcd(m,r)-1)b_r,\end{aligned}$$

$$\begin{aligned}\dim_K HC_{2c+1}(A) = &\sum_{\substack{r>0 \\ \text{s.t. } r|(c+1)m}} (\gcd(m,r)-1)b_r + \sum_{c'=0}^{c}\sum_{e=1}^{m-1} \sum_{\substack{r>0 \\ \text{s.t. } r\zeta|c'm+e}} b_r \\ &+ \sum_{c'=1}^{c+1} \sum_{\substack{r>0 \\ \text{s.t. } r|c'm, \\ \gcd(m,r)\zeta|m}} b_r + \sum_{c'=1}^{c} \sum_{\substack{r>0 \\ \text{s.t. } r\zeta|\gcd(m,r)c'}} (\gcd(m,r)-1)b_r.\end{aligned}$$

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