SOURCE ALGEBRAS AND COHOMOLOGY OF BLOCK IDEALS OF FINTIE GROUP ALGEBRAS

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Abstract. Let B a block ideal of the group algebra of a finite group G over a field k with a defect group P. We shall give a criterion for a (kP, kP)-bimodule defined by a (P, P)-double coset to be isomorphic to a direct summand of the source algebra of the block B viewed as a (kP, kP)-bimodule.

Key Words: block ideal, defect group, source algebra, cohomology

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1. Block ideals, source algebras and cohomology rings

Throughout this note we let G denote a finite group and k an algebraically closed field of characteristic p dividing the order of G.

Let B be a block ideal of the group algebra kG; let P be a defect group of B. Let X be a source module of B, which is an indecomposable direct summand of B as $k[G\times P^{\mathrm{op}}]$-module having ΔP as vertex; X has a trivial source. The source module X can be written as $X = kGi$ with a source idempotent i. Let (P, b_P) be the Sylow B-subpair such that the Brauer construction $X(P)$ belongs to b_P; let $\mathcal{F}_{(P,b_P)}(B, X) = \{ (R, b_R) \mid (R, b_R) \subseteq (P, b_P) \}$ be the Brauer category associated with (P, b_P). Then the cohomology ring $H^*(G, B, X)$ of the block B with respect to X is defined to be the subring of the cohomology ring $H^*(P, k)$ of the defect group P consisting of $\mathcal{F}_{(P,b_P)}(B, X)$-stable elements (Linckelmann [4]).

The cohomology ring $H^*(G, B, X)$ is so tightly related to the source algebra $ikGi = X^* \otimes_B X$. Namely

Theorem 1 ([4, Theorem 5.1], [7, Theorem 1]). *Under the notation above an element $\zeta \in H^*(P, k)$ belongs to the cohomology ring $H^*(G, B, X)$ if and only if the diagonal embedding $\delta_P\zeta \in HH^*(kP)$ is $ikGi$-stable, where $ikGi$ is viewed as a (kP, kP)-bimodule.*

Upon this fact the author proposed in [7] a conjecture that the transfer map defined by the source algebra would describe the block cohomology. To be more precise we let $t_{ikGi} : HH^*(kP) \to HH^*(kP)$ be the transfer map defined by $ikGi$ as a (kP, kP)-bimodule. Then we can define a map $t : H^*(P, k) \to H^*(P, k)$ giving rise to the following

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commutative diagram

$$\begin{array}{ccc} H^*(P,k) & \xrightarrow{\delta_P} & HH^*(kP) \\ {\scriptstyle t}\big\downarrow & & \big\downarrow {\scriptstyle t_{ikGi}} \\ H^*(P,k) & \xrightarrow[\delta_P]{} & HH^*(kP) \end{array} .$$

Conjecture. *Under the notation above it would follow that*

$$H^*(G,B,X) = t\, H^*(P,k).$$

If we let

$$ikGi \simeq \bigoplus_{PxP} k[PxP]$$

be a direct sum decomposition of indecomposable (kP,kP)-bimodules, then the map t is described as follows:

$$t : H^*(P,k) \to H^*(P,k); \zeta \mapsto \sum_{PxP} \operatorname{tr}^P \operatorname{res}_{P\cap {}^xP} {}^x\zeta.$$

However we have had few knowledge for indecomposable direct summands of $ikGi$; we have for an element $x \in G$ outside the inertia group $N_G(P,b_P)$ of the Sylow subpair (P,b_P) almost no information for $k[PxP]$ to be isomorphic to a direct summand of $ikGi$, whereas the direct summands isomorphic to $k[Px]$ for $x \in N_G(P,b_P)$ is so well understood, as we can see in [8, Theorem 44.3].

The aim in this note is to give a criterion for a (kP,kP)-bimodule $k[PgP]$ to be isomorphic to a direct summand of $ikGi$. Here we fix a notation; for a double coset PgP we let

$$t_{PgP} : H^*(P,k) \to H^*(P,k); \zeta \mapsto \operatorname{tr}^P \operatorname{res}_{P\cap {}^gP} {}^g\zeta.$$

Theorem 2. *Let $(R,b_R),(S,b_S) \subseteq (P,b_P)$; assume that $C_P(R)$ is a defect group of b_R or $C_P(S)$ is a defect group of b_S. For $g \in G$ with ${}^g(R,b_R) = (S,b_S)$ if the map*

$$t_g : H^*(P,k) \to H^*(P,k); \zeta \mapsto \operatorname{tr}^P \operatorname{res}_S {}^g\zeta$$

does not vanish, then the following hold:

(1) *$S = P \cap {}^gP$; hence $t_g = t_{PgP}$,*
(2) *the (kP,kP)-bimodule $k[PgP]$ is isomorphic to a direct summand of $ikGi$,*
(3) *a (kP,kP)-bimodule $k[Pg'P]$ is isomorphic to $k[PgP]$ if and only if $Pg'P = PcgP$ for some $c \in C_G(S)$.*

Note in the above that the blocks b_R and b_S are considered as blocks in $kC_G(R)$ and $kC_G(S)$, respectively. We prove the theorem above in Section 2.

In Kawai–Sasaki [1] we calculated cohomology rings of 2-blocks of tame representation type and of blocks with defect groups isomorphic to wreathed 2-groups of rank 2. There we constructed transfer maps on the cohomology rings of defect groups; the images of these maps are just the cohomology rings of the blocks. In Section 3 we shall apply Theorem 2 to show that our transfer maps are defined by direct summands of the source algebras of block ideals of tame representation type.

Proof of Theorem 2. We first show in this section the following proposition.

Proposition 3. *Let $P \leqslant G$ be an arbitrary p-subgroup. The (kP, kP)-bimodules $k[PxP]$ and $k[PyP]$, where $x, y \in G$, are isomorphic if and only if $PyP = PcxP$ for some $c \in C_G(P \cap {}^xP)$ with the property that $P \cap {}^xP = P \cap {}^{cx}P$. In this case $P \cap {}^xP$ and $P \cap {}^yP$ are conjucate in P and the transfer maps t_{PxP} and t_{DyD} coincide.*

Proof. Proof. The (kP, kP)-bimodule $k[PxP]$ as a $k[P \times P^{\rm op}]$-module has ${}^{(x,1)}\Delta({}^{x^{-1}}P \cap P)$ as vertex; and we see that

$$k[PxP] = k[P \times P^{\rm op}] \otimes_{k[{}^{(x,1)}\Delta({}^{x^{-1}}P \cap P)]} k.$$

Hence we have that

$$\begin{aligned} k[PxP] \simeq k[PyP] \Longleftrightarrow \exists\, &(a,b) \in P \times P^{\rm op} \text{ s.t.} \\ &{}^{(x,1)}\Delta({}^{x^{-1}}P \cap P) = {}^{(a,b)}\Big({}^{(y,1)}\Delta({}^{y^{-1}}P \cap P)\Big). \end{aligned}$$

The last equation is equivalent to the following equation:

$$\{\, ({}^xs, s^{-1}) \mid s \in {}^{x^{-1}}P \cap P \,\} = \{\, ({}^{ay}t, b{\cdot}t^{-1}{\cdot}b^{-1}) \mid t \in {}^{y^{-1}}P \cap P \,\}.$$

Here, the multiplication in right component of pairs is in the oposite group $P^{\rm op}$ so that, rewriting it by using the multiplication in P, we obtain that $b{\cdot}t^{-1}{\cdot}b^{-1} = b^{-1}t^{-1}b$. Namely we see for an arbitrary $s \in {}^{x^{-1}}P \cap P$ that there exists a unique element $t \in {}^{y^{-1}}P \cap P$ such that

$${}^xs = {}^{ay}t, \quad s^{-1} = b^{-1}t^{-1}b.$$

The second equation above implies that $t = {}^bs$. Substitute this to the first one to obtain

$${}^xs = {}^{ayb}s.$$

This equation holds for an arbitrary $s \in {}^{x^{-1}}P \cap P$; hence there exists an element $c \in C_G(P \cap {}^xP)$ such that

$$ayb = cx.$$

Note that $P \cap {}^xP = {}^{xb^{-1}}({}^{y^{-1}}P \cap P)$, since $s = {}^{b^{-1}}t$. Then we have that

$$\begin{aligned} P \cap {}^{cx}P &= P \cap {}^{ayb}P = P \cap {}^{ay}P && (\because b \in P) \\ &= {}^a(P \cap {}^yP) && (\because a \in P) \\ &= {}^{cxb^{-1}y^{-1}}(P \cap {}^yP) = {}^{cxb^{-1}}({}^{y^{-1}}P \cap P) = {}^c(P \cap {}^xP) \\ &= P \cap {}^xP. && (\because c \in C_G(P \cap {}^xP)) \end{aligned}$$

Suppose conversely for an element $c \in C_G(P \cap {}^xP)$ that $PyP = PcxP$ with $P \cap {}^xP = P \cap {}^{cx}P$. Then we have

$$\begin{aligned} {}^{(cx)^{-1}}P \cap P &= {}^{(cx)^{-1}}(P \cap {}^{cx}P) = {}^{(cx)^{-1}}(P \cap {}^xP) = {}^{x^{-1}c^{-1}x}({}^{x^{-1}}P \cap P) \\ &= {}^{x^{-1}}P \cap P. \qquad (\because x^{-1}c^{-1}x \in C_G({}^{x^{-1}}P \cap P)) \end{aligned}$$

Then the indecomposable $k[P \times P^{\mathrm{op}}]$-module $k[PcxP]$ has vertex

$$\begin{aligned} {}^{(cx,1)}\Delta({}^{(cx)^{-1}}P \cap P) &= {}^{(cx,1)}\Delta({}^{x^{-1}}P \cap P) = \{\, ({}^{cx}s, s^{-1}) \mid s \in {}^{x^{-1}}P \cap P \,\} \\ &= \{\, ({}^{x}s, s^{-1}) \mid s \in {}^{x^{-1}}P \cap P \,\} = {}^{(x,1)}\Delta({}^{x^{-1}}P \cap P). \end{aligned}$$

Hence we see that $k[PcxP] \simeq k[PxP]$, as desired.

Also wee see, under the condition above, since we can write $cx = ayb$ with suitable a, $b \in P$, that

$$P \cap {}^{x}P = P \cap {}^{cx}P = P \cap {}^{ayb}P = P \cap {}^{ay}P = {}^{a}(P \cap {}^{y}P),$$

hence clearly the last assertion holds. □

Proof of Theorem 2. Because $S \leqslant P \cap {}^{g}P$ we see for $\zeta \in H^*(P, k)$ that

$$\begin{aligned} \mathrm{tr}^P \mathrm{res}_S {}^{g}\zeta &= \mathrm{tr}^P \mathrm{tr}^{P \cap {}^{g}P} \mathrm{res}_S \mathrm{res}_{P \cap {}^{g}P} {}^{g}\zeta \\ &= |\, P \cap {}^{g}P : S \,|\, \mathrm{tr}^P \mathrm{res}_{P \cap {}^{g}P} {}^{g}\zeta. \end{aligned}$$

Hence if $P \cap {}^{g}P > S$, then the map t_g vanishes. Thus we have that $S = P \cap {}^{g}P$.

If $C_P(R)$ is a defect group of b_R, then we see by [3, Lemma 3.3 (iv)] that the (kS, kR)-bimodule $k[gR] = k[Sg]$ is isomorphic with a direct summand of $ikGi$. If on the other hand $C_P(S)$ is a defect group of b_S, then an argument similar to that in the proof of [3, Lemma 3.3 (iv)] tells us that the (kS, kR)-bimodule $k[Sg]$ is isomorphic to a direct summand of $ikGi$. Namely in both cases $k[Sg]$ is isomorphic to a direct summand of $ikGi$ as (kS, kR)-bimodule.

As in the proof of [7, Theorem 1] we can take an indecomposable direct summand $k[PxP]$ of $ikGi$ such that

$$k[Sg] \;\big|\; k[PxP]$$

as (kS, kR)-bimodules; then we can write $g = caxb$ using suitable elements $a, b \in P$ and an element $c \in C_G(S)$. Since $S = P \cap {}^{caxb}P \leqslant {}^{cax}P$, we have that ${}^{a^{-1}}S = {}^{a^{-1}c^{-1}}S \leqslant {}^{x}P$ so that ${}^{a^{-1}}S \leqslant P \cap {}^{x}P$. Therefore it follows that

$$\begin{aligned} \mathrm{res}_S {}^{g}\zeta &= \mathrm{res}_S {}^{caxb}\zeta = \mathrm{res}_S {}^{cax}\zeta \\ &= {}^{c}\mathrm{res}_S {}^{ax}\zeta = \mathrm{res}_S {}^{ax}\zeta && (\because c \in C_G(S)) \\ &= {}^{a}\mathrm{res}_{{}^{a^{-1}}S} {}^{x}\zeta \\ &= {}^{a}\mathrm{res}_{{}^{a^{-1}}S} \mathrm{res}_{P \cap {}^{x}P} {}^{x}\zeta && (\because {}^{a^{-1}}S \leqslant P \cap {}^{x}P) \end{aligned}$$

so that

$$\begin{aligned} \mathrm{tr}^P \mathrm{res}_S {}^{g}\zeta &= \mathrm{tr}^P {}^{a}\mathrm{res}_{{}^{a^{-1}}S} \mathrm{res}_{P \cap {}^{x}P} {}^{x}\zeta \\ &= \mathrm{tr}^P \mathrm{tr}^{P \cap {}^{x}P} \mathrm{res}_{{}^{a^{-1}}S} \mathrm{res}_{P \cap {}^{x}P} {}^{x}\zeta && (\because a \in P) \\ &= \mathrm{tr}^P \,|\, P \cap {}^{x}P : {}^{a^{-1}}S \,|\, \mathrm{res}_{P \cap {}^{x}P} {}^{x}\zeta \\ &= |\, P \cap {}^{x}P : {}^{a^{-1}}S \,|\, \mathrm{tr}^P \mathrm{res}_{P \cap {}^{x}P} {}^{x}\zeta. \end{aligned}$$

This implies that

$${}^{a^{-1}}S = P \cap {}^{x}P, \quad \mathrm{tr}^P \mathrm{res}_S {}^{g}\zeta = \mathrm{tr}^P \mathrm{res}_{P \cap {}^{x}P} {}^{x}\zeta.$$

Thus we see that

$$S = {}^{a}(P \cap {}^{x}P) = P \cap {}^{ax}P = P \cap {}^{c^{-1}g}P.$$

Because $c \in C_G(S)$, we can apply Proposition 3 to $k[PgP]$ and $k[Pc^{-1}gP]$ to conclude that $k[Pc^{-1}gP] \simeq k[PgP]$. Moreover, since $Pc^{-1}gP = PaxbP = PxP$, we have that $k[PgP] \mid ikGi$.

Finally we see for $c \in C_G(S)$ that ${}^{cg}(R, b_R) = (S, b_S)$ and that

$$\mathrm{tr}^P \mathrm{res}_S {}^{cg}\zeta = \mathrm{tr}^P {}^{c}\mathrm{res}_S {}^{g}\zeta = \mathrm{tr}^P \mathrm{res}_S {}^{g}\zeta \quad \text{for } \zeta \in H^*(P, k).$$

Hence we have that $P \cap {}^{cg}P = S$. Again Proposition 3 says that $k[PcgP] \simeq k[PgP]$. The "only if" part of our assertion (3) is obvious by Proposition 3. □

3. Tame 2-blocks

Linckelmann [5] says that the family

$$\mathcal{F} = \{ (S, b_S) \subseteq (P, b_P) \mid (S, b_S) \text{ is extremal and essential} \} \cup \{ (P, b_P) \}$$

is a conjugation family.

For a subpair $(S, b_S) \subseteq (P, b_P)$ we consider the following stability condition:

$$\mathrm{S}(S, b_S) \qquad \mathrm{res}_S {}^{g}\zeta = \mathrm{res}_S \zeta \quad \forall g \in N_G(S, b_S).$$

Then we see

$$H^*(G, B, X) = \{ \zeta \in H^*(P, k) \mid \zeta \text{ satisfies } \mathrm{S}(S, b_S) \text{ for an arbitrary } (S, b_S) \in \mathcal{F} \}.$$

In the rest of the note we let $p = 2$ and assume that the block B is of tame representation type; the defect group P is one of the followings:

(1) dihedral 2-group

$$D_n = \langle x, y \mid x^{2^{n-1}} = y^2 = 1, yxy^{-1} = x^{-1} \rangle, \ n \geqslant 3;$$

(2) generalized quaternion 2-group

$$Q_n = \langle x, y \mid x^{2^{n-2}} = y^2 = z, z^2 = 1, yxy^{-1} = x^{-1} \rangle, \ n \geqslant 3;$$

(3) semidihedral 2-group

$$SD_n = \langle x, y \mid x^{2^{n-1}} = y^2 = 1, yxy^{-1} = x^{-1+2^{n-2}} \rangle, \ n \geqslant 4.$$

The following would be well known.

Proposition 4. (1) *If $P = D_n$ $(n \geqslant 3)$, then*

$$\{ (E, b_E) \subseteq (P, b_P) \mid E \simeq \textit{a four-group}, N_G(E, b_E)/C_G(E) \simeq \mathrm{GL}(2,2) \} \cup \{ (P, b_P) \}$$

is a conjugation family.

(2) *If $P = SD_n$ $(n \geqslant 4)$, then*

$$\begin{aligned} &\{ (E, b_E) \subseteq (P, b_P) \mid E \simeq \textit{a four-group}, N_G(E, b_E)/C_G(E) \simeq \mathrm{GL}(2,2) \} \\ &\cup \{ (V, b_V) \subseteq (P, b_P) \mid V \simeq \textit{a quaternion group}, N_G(V, b_V)/VC_G(V) \simeq \mathrm{GL}(2,2) \} \\ &\cup \{ (P, b_P) \} \end{aligned}$$

is a conjugation family.

(3) *If* $P = Q_n$ $(n \geqslant 4)$*, then*

$\{ (V, b_V) \subseteq (P, b_P) \mid V \simeq$ *a quaternion group*, $N_G(V, b_V)/VC_G(V) \simeq \mathrm{GL}(2,2) \}$
$\cup \{ (P, b_P) \}$

is a conjugation family.

3.1. **Blocks with semidihedral defect groups.** In this subsection we let $P = SD_n$ $(n \geqslant 4)$; let

$$E = \langle x^{2^{n-2}}, y \rangle, \quad V = \langle x^{2^{n-3}}, xy \rangle.$$

Here we state the cohomology ring $H^*(P, k)$ of $P = SD_n = \langle x, y \mid x^{2^{n-1}} = y^2 = 1, yxy^{-1} = x^{-1+2^{n-2}} \rangle$, $n \geqslant 4$. Let $\xi = x^*$, $\eta = y^* \in H^1(P, k)$. Let $\alpha \in H^2(\langle x \rangle, k)$ be the standard element. Let $\nu = \mathrm{norm}^P \alpha \in H^4(P, k)$. Choose an element $\theta \in H^3(P, k)$ appropriately; we can describe as follows:

$$H^*(P, k) = k[\xi, \eta, \theta, \nu]/(\xi^2 - \xi\eta, \xi^3, \xi\theta, \theta^2 - \eta^6 - \eta^2\nu - \xi^2\nu).$$

We constructed in Kawai–Sasaki [1] a transfer map from $H^*(P, k)$ to $H^*(G, B, X)$.

From now on we assume that $N_G(E, b_E)/C_G(E) \simeq \mathrm{GL}(2,2)$ and $N_G(V, b_V)/VC_G(V) \simeq \mathrm{GL}(2,2)$. Let ω and ω' are automorphisms of E and V of order three, respectively. Then the cohomology ring of the block is described as follows:

$$H^*(G, B, X) = \{ \zeta \in H^*(P, k) \mid \mathrm{res}_E \zeta = \mathrm{res}_E {}^{\omega}\zeta, \mathrm{res}_V \zeta = \mathrm{res}_V {}^{\omega'}\zeta \}.$$

We let $g_0 \in N_G(E, b_E)$ and $g_1 \in N_G(V, b_V)$ induce the automorphisms $\omega \in \mathrm{Aut}\, E$ and $\omega' \in \mathrm{Aut}\, V$, respectively:

(1) $\langle x^{2^{n-3}}, g_0 \rangle C_G(E)/C_G(E) = N_G(E, b_E)/C_G(E) \simeq \mathrm{GL}(2,2)$,
(2) $\langle x^{2^{n-4}}, g_1 \rangle VC_G(V)/VC_G(V) = N_G(V, b_V)/VC_G(V) \simeq \mathrm{GL}(2,2)$.

Definition 5. We let

$$\mathrm{Tr}_P^B : H^*(P, k) \to H^*(P, k); \zeta \mapsto \zeta + \mathrm{tr}^P \mathrm{res}_E {}^{g_0}\zeta + \mathrm{tr}^P \mathrm{res}_V {}^{g_1}\zeta.$$

Theorem 6. *The image of* Tr_P^B *above coincides with the cohomology ring* $H^*(G, B, X)$*.*

Since (E, b_E), $(V, b_V) \subseteq (P, b_P)$ are extremal, we see that $C_P(E)$ and $C_P(V)$ are defect groups of b_E and b_V, respectively. Theorem 2 together with the facts that the maps $[\zeta \mapsto \mathrm{tr}^P \mathrm{res}_E {}^{g_0}\zeta]$ and $[\zeta \mapsto \mathrm{tr}^P \mathrm{res}_V {}^{g_1}\zeta]$ do not vanish implies that both of (kP, kP)-bimodules $k[Pg_0P]$ and $k[Pg_1P]$ are isomorphic to direct summands of $ikGi$; we obtain the following theorem.

Theorem 7. *Let* $M = kP \oplus k[Pg_0P] \oplus k[Pg_1P]$*. Then*

(1) $M \mid ikGi$*;*
(2) *the map* Tr_P^B *is induced by the transfer map* $t_M : HH(kP) \to HH(kP)$*;*
(3) *an element* $\zeta \in H^*(P, k)$ *belongs to* $H^*(G, B, X)$ *if and only if* $\delta_P\zeta \in HH^*(kP)$ *is* M*-stable.*

In the other cases of the iniertia quotients, we have similar results.

Suppose that the (kP, kP)-bimodules $k[PgP]$ is isomorphic to a direct summand of (kP, kP)-bimodule $ikGi$.

Let $R = {}^{g^{-1}}P \cap P$ and $S = P \cap {}^{g}P$ and let (R, b_R), $(S, b_S) \subseteq (P, b_P)$. Then Külshammer–Okuyama–Watanabe [2, Proposition 5] says that

$$^{g}(R, b_R) = (S, b_S) \subseteq (P, b_P).$$

The Brauer category $\mathcal{F}_{(P,b_P)}(B, X)$ is well understood so that the posibilities of fusions above are completely described and the transfer maps t_{PgP} are also determined.

Hence we obtain the following.

Theorem 8. *The source algebra ikGi induces, as a (kP, kP)-bimodule, the transfer map $t_{ikGi} : HH^*(kP) \to HH^*(kP)$ whose restriction to the cohomology ring $H^*(P, k)$ maps $\zeta \in H^*(P, k)$ as follows:*

$$\zeta \mapsto \zeta + l_0 \operatorname{tr}^P \operatorname{res}_E {}^{g_0}\zeta + l_1 \operatorname{tr}^P \operatorname{res}_V {}^{g_1}\zeta.$$

Here $l_0, l_1 \in \mathbf{Z}$.

3.2. **Blocks with dihedral or quaternion defect groups.** In the case of $P = D_n$ ($n \geqslant 3$), let us take four-groups

$$E_0 = \langle x^{2^{n-2}}, y \rangle, \quad E_1 = \langle x^{2^{n-2}}, xy \rangle.$$

In the case of $P = Q_n$ ($n \geqslant 4$, let us take quaternion groups

$$V_0 = \langle x^{2^{n-3}}, y \rangle, \quad V_1 = \langle x^{2^{n-3}}, xy \rangle.$$

Then we can construct the transfer maps $\operatorname{Tr}_P^B : H^(P, k) \to H^(P, k)$ whose images are $H^*(G, B, X)$s, by similar constructions in semidihedral case.

We have also results corresponding to Theorems 7 and 8.

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