The 48th Symposium on Ring Theory

and Representation Theory

ABSTRACT

Nagoya University, Nagoya

September 7 – 10, 2015

Program

September 7 (Monday)

9:00–9:30 Obara Daiki (Tokyo University of Science)

On the Hochschild cohomology ring modulo nilpotence of the quiver algebra with quantum-like relations

9:40–10:10 Noritsugu Kameyama (Shinshu University), Mitsuo Hoshino (University of Tsukuba), Hirotaka KOGA (Tokyo Denki University)

Crossed products for matrix rings

10:20–10:50 Hideto Asashiba (Shizuoka University)

Cohen-Montgomery duality of bimodules and its applications to equivalences given by bimodules

11:00–11:50 Jan Schröer (University of Bonn)

Convolution algebras and enveloping algebras I

13:40–14:10 Yu Liu (Nagoya University)

Cotorsion pairs on triangulated and exact categories

14:20–14:50 Yuya Mizuno (Nagoya University), Kota Yamaura (University of Yamanashi)

Higher APR tilting preserve n-representation infiniteness

15:00–15:30 Izuru Mori (Shizuoka University)

2-dimensional quantum Beilinson algebras

15:50–16:40 Susumu Ariki (Osaka University)

Survey on representation theory of quiver Hecke algebras I

16:50–17:40 Steffen Koenig (Universität Stuttgart)

Some special cases of Morita-Tachikawa correspondence

September 8 (Tuesday)

9:00–9:30 Yuta Kozakai (Tokyo University of Science), Naoko Kunugi (Tokyo University of Science)

Construction of two-sided tilting complexes for Brauer tree algebras

9:40–10:10 Takahide Adachi (Nagoya University)

The classification of two-term tilting complexes for Brauer graph algebras

10:20–10:50 Hirotaka Koga (Tokyo Denki University), Mitsuo Hoshino (University of Tsukuba)

Derived equivalences and Gorenstein projective dimension

11:00–11:50 Jan Schröer (University of Bonn)

Convolution algebras and enveloping algebras II

13:40–14:10 Ken-ichi Yoshida (Nihon University), Shiro Goto (Meiji University), Naoki Taniguchi (Meiji University), Naoyuki Matsuoka (Meiji University)
Almost Gorenstein Rees algebras

14:20–14:50 Hiroki Matsui (Nagoya University)
The structure of preenvelopes with respect to maximal Cohen-Macaulay modules

15:00–15:30 Ryo Takahashi (Nagoya University)
Thick subcategories of derived categories of isolated singularities

15:50–16:40 Susumu Ariki (Osaka University)
Survey on representation theory of quiver Hecke algebras II

16:50–17:40 Watatani Yasuo (Kyushu University)
Quivers, operators on Hilbert spaces and operator algebras I

September 9 (Wednesday)

9:00–9:30 Satoshi Yamanaka (Okayama University)
On weakly separable extensions and weakly quasi-separable extensions

9:40–10:10 Akira Ueda (Shimane University), Hidetoshi Marubayashi (Tokushima Bunri University)
Examples of Ore extensions which are maximal orders whose based rings are not maximal orders

10:20–10:50 Yotsanan Meemark（Chulalongkorn University）
Cayley Graphs over a Finite Chain Ring and GCD-graphs

11:00–11:50 Watatani Yasuo (Kyushu University)
Quivers, operators on Hilbert spaces and operator algebras II

13:40–14:10 Ryo Kanda (Nagoya University)
Atom-molecule correspondence in Grothendieck categories

14:20–14:50 Ayako Itaba (Shizuoka University)
Finite condition (Fg) for self-injective Koszul algebras

15:00–15:30 Hiroyuki Minamoto (Osaka Prefecture University)
Higher products on Yoneda Ext algebras

15:50–16:40 Steffen Koenig（Universität Stuttgart）
Filtered categories and representations of boxes

16:50–17:40 Masahisa Sato (Yamanashi university)
Special Lecture: Report on the 7th China-Japan-Korea International Conference on Ring Theory

18:30– Conference dinner

September 10 (Thursday)

9:00–9:30 Sota Asai (Nagoya University)
The Grothendieck groups of mesh algebras

9:40–10:10 Hiramatsu Naoya (Kure National College of Technology)
Stable degenerations of Cohen-Macaulay modules over simple singularities of type (A_n)

10:20–10:50 Yuta Kimura (Nagoya University)
Tilting theory of preprojective algebras and c-sortable elements

11:10–11:40 Kenta Ueyama (Hirosaki University), Izuru Mori (Shizuoka University)
Tilting objects for noncommutative quotient singularities

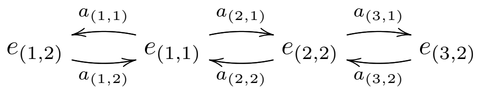
On the Hochschild cohomology ring modulo nilpotence of the quiver algebra with quantum-like relations

Daiki Obara

We consider the finite dimensional quiver algebra defined by n cycles and quantum-like relations. We determine the minimal projective bimodule resolution and the Hochschild cohomology ring modulo nilpotence of this algebra. In [1] and [2], we determined the minimal projective bimodule resolution([1]) and the Hochschild cohomology ring modulo nilpotence([2]) of the quiver algebra defined by 2 cycles and a quantum-like relation.

Let k be a field, $\mathrm{HH}^*(A)$ the Hochschild cohomology ring of a finite dimensional k-algebra A and $\mathcal{N}$ the ideal of $\mathrm{HH}^*(A)$ generated by all homogeneous nilpotent elements. In [4], using the Hochschild cohomology ring modulo nilpotence $\mathrm{HH}^*(A)/\mathcal{N}$ Snashall and Solberg defined a support variety of A-module. And, in [3], Snashall gave the question to as whether we can give necessary and sufficient conditions on a finite dimensional algebra A for $\mathrm{HH}^*(A)/\mathcal{N}$ to be finitely generated as an algebra.

In this talk, for simplicity, we consider the case of 3 cycles. Let $A_1 = kQ_1/I_1$ where Q_1 is the following quiver:

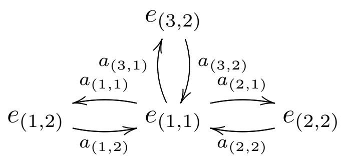


and I_1 is the ideal of kQ_1 generated by

$$X_i^{n_i} \text{ for } 1 \leq i \leq 3, X_1X_2 - X_2X_1, X_2X_3 - X_3X_2,$$
$$a_{(1,2)}a_{(2,1)}X_2^l a_{(3,1)}, a_{(3,2)}a_{(2,2)}X_2^l a_{(1,1)} \text{ for } 0 \leq l \leq n_2 - 1,$$

where $X_i = (a_{(i,1)} + a_{(i,2)})^2$ and $n_i \geq 2$.

Let $A_2 = kQ_2/I_2$ where Q_2 is the following quiver:



and I_2 is the ideal of kQ_2 generated by

$$X_i^{n_i} \text{ for } 1 \leq i \leq 3, X_1X_2 - X_2X_1, X_1X_3 - X_3X_1, X_2X_3, X_3X_2,$$

where $X_i = (a_{(i,1)} + a_{(i,2)})^2$ and $n_i \geq 2$. We introduce the minimal projective bimodule resolutions and the Hochschild cohomology rings modulo nilpotence of these algebras.

References

[1] D. Obara, *Hochschild cohomology of quiver algebras defined by two cycles and a quantum-like relation*, Comm. Algebra **40** (2012), 1724–1761, 136–142.

[2] D. Obara, *Hochschild cohomology of quiver algebras defined by two cycles and a quantum-like relation II*, Comm. Algebra **43** (2015), 3404–3446, 136–142.

[3] N. Snashall, *Support varieties and the Hochschild cohomology ring modulo nilpotence.* In Proceedings of the 41st Symposium on Ring Theory and Representation Theory, Shizuoka, Japan, Sept. 5–7, 2008; H. Fujita, Ed.; Symp. Ring Theory Represent. Theory Organ. Comm., Tsukuba, 2009, 68–82.

[4] N. Snashall and Ø. Solberg, *Support varieties and Hochschild cohomology rings*, Proc. London Math. Soc. **88** (2004), 705–732.

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Crossed products for matrix rings

Mitsuo Hoshino, Noritsugu Kameyama and Hirotaka Koga

In this talk, we provide a systematic way to define new multiplications on $\mathrm{M}_n(R)$, the ring of $n \times n$ full matrices with entries in R.

Fix a set of integers $I = \{0, 1, \dots, n-1\}$ with $n \geq 2$ arbitrary. To begin with, starting from an arbitrary ring R, we will construct an I-graded ring A so that A/R is a split Frobenius extension of second kind. Namely, we will define an appropriate multiplication on a free right R-module A with a basis $\{u_i\}_{i \in I}$ using the following two data: a certain pair (q, χ) of an integer q and a mapping $\chi : \mathbb{Z}_+ \to \mathbb{Z}$; a certain triple (σ, c, t) of $\sigma \in \mathrm{Aut}(R)$ and $c, t \in R$. Then we will define an appropriate multiplication on a free right A-module Λ with a basis $\{v_i\}_{i \in I}$ so that Λ/A is a Frobenius extension of first kind. To do so, we need the group structure of I. Since we have to distinguish the addition in I and that in $\mathbb{Z}_+$, we fix a cyclic permutation of I

$$\pi = \begin{pmatrix} 0 & 1 & \cdots & n-1 \\ 1 & 2 & \cdots & 0 \end{pmatrix}$$

and by the law of composition $I \times I \to I, (i, j) \mapsto \pi^j(i)$ we make I a cyclic group with 0 the unit element. Then, as a right R-module, Λ has a basis $\{e_{ij}\}_{i,j \in I}$ such that $e_{ij}e_{kl} = 0$ unless $j = k$, $e_{ij}e_{jk} = e_{ik}c_{ijk}$ with $c_{ijk} \in R$ for all $i, j, k \in R$ and $xe_{ij} = e_{ij}\sigma_{ij}(x)$ with $\sigma_{ij} \in \mathrm{Aut}(R)$ for all $x \in R$ and $i, j \in I$. Using the above two data, we will provide a concrete construction of such families $\{c_{ijk}\}_{i,j,k \in I}$ and $\{\sigma_{ij}\}_{i,j \in I}$.

This talk is based on [4].

References

[1] H. Abe and M. Hoshino, Frobenius extensions and tilting complexes, Algebras and Representation Theory **11**(3) (2008), 215–232.

[2] M. Hoshino, Strongly quasi-Frobenius rings, Comm. Algebra **28**(8)(2000), 3585–3599.

[3] M. Hoshino, N. Kameyama and H. Koga, Group-graded and group-bigraded rings, J. Algebra and Its Applications (to appear)

[4] M. Hoshino, N. Kameyama and H. Koga, Crossed products for matrix rings, preprint.

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Cohen-Montgomery duality for bimodules and its applications to equivalences given by bimodules

Hideto Asashiba

We fix a commutative ring $\Bbbk$ and a group G. To include infinite coverings of $\Bbbk$-algebras into consideration we usually regard $\Bbbk$-algebras as locally bounded $\Bbbk$-categories with finite objects, therefore we will work with small $\Bbbk$-categories. For small $\Bbbk$-categories R and S with G-actions we introduce *G-invariant* S-R-bimodules and their category denoted by $S\text{-Mod}^G\text{-}R$, and denote by R/G the orbit category of R by G, which is a small G-graded $\Bbbk$-category. For small G-graded $\Bbbk$-categories A and B we introduce *G-graded* B-A-bimodules and their category denoted by $B\text{-Mod}_G\text{-}A$, and denote by $A\#G$ the smash product of A and G, which is a small $\Bbbk$-category with G-action. Then the Cohen-Montgomery duality theorem [2, 1] says that we have equivalences $(R/G)\#G \simeq R$ and $(A\#G)/G \simeq A$, by which we identify these pairs. In the talk we introduce functors $(\text{-})/G : S\text{-Mod}^G\text{-}R \to (S/G)\text{-Mod}_G\text{-}(R/G)$ and $(\text{-})\#G : A\text{-Mod}_G\text{-}B \to (A\#G)\text{-Mod}^G\text{-}(B\#G)$, and show that they are equivalences and quasi-inverses to each other (by applying $A := R/G$, $R := A\#G$, etc.), have good properties with tensor products and preserve projectivity of bimodules. We apply this to equivalences given by bimodules such as Morita equivalences, stable equivalences of Morita type (and later standard derived equivalences) to have theorems such as for stable equivalences of Morita type:

Theorem 1. (1) *There exists a "G-invariant stable equivalence of Morita type" between R and S if and only if there exists a "G-graded stable equivalence of Morita type" between R/G and S/G.*

(2) *There exists a "G-graded stable equivalence of Morita type" between A and B if and only if there exists a "G-invariant stable equivalence of Morita type" between $A\#G$ and $B\#G$.*

Here we note that a G-invariant (resp. G-graded) stable equivalence of Morita type is defined to be a usual stable equivalence of Morita type with additional properties, and does not mean an equivalence between stable categories of G-invariant (resp. G-graded) modules.

References

[1] Asashiba, H.: *A generalization of Gabriel's Galois covering functors II: 2-categorical Cohen-Montgomery duality*, preprint arXiv: 0905.3884.

[2] Cohen, M. and Montgomery, S.: *Group-graded rings, smash products, and group actions*, Trans. Amer. Math. Soc. **282** (1984), 237–258.

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Convolution algebras and enveloping algebras I, II

Jan Schröer

To any finite-dimensional algebra A over the complex numbers one can associate convolution algebras defined by constructible functions on varieties of finite-dimensional A-modules. These convolution algebras turn out to be isomorphic to enveloping algebras of Lie algebras. We will explain this rather general theory (which was mainly developed by Schofield) and also provide numerous examples.

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Cotorsion pairs on triangulated and exact categories

YU LIU

The notion of cotorsion pair in triangulated and exact categories is a general framework to study important structures in representation theory. Recently the notion of hearts of cotorsion pairs was introduced in [8] and [6], and they are proved to be abelian categories, which were known for the heart of t-structure [2] and the quotient category by cluster tilting subcategory. We refer to [7] and [1] for more results on hearts of cotorsion pairs.

In this talk, we give an equivalence between hearts and the functor categories over cohearts. For the details of functor category, see [4, Definition 2.9].

For any cotorsion pair $(\mathcal{U}, \mathcal{V})$ on a triangulated category $\mathcal{T}$, we introduce the notion of *cohearts* of a cotorsion pair, denote by

$$\mathcal{C} = \mathcal{U}[-1] \cap {}^{\perp}\mathcal{U}.$$

This is a generalization of coheart of a co-t-structure, which plays an important role in [5]. We have the following theorem in triangulated category.

Theorem 1. *Let $(\mathcal{U}, \mathcal{V})$ be a cotorsion pair on a triangulated category $\mathcal{T}$. If $\mathcal{U}[-1] \subseteq \mathcal{C} * \mathcal{U}$, then the heart of $(\mathcal{U}, \mathcal{V})$ has enough projectives, and moreover it is equivalent to the functor category* $\mathrm{mod}\mathcal{C}$.

This generalizes [5, Theorem 3.4] which is for t-structure. One standard example of this theorem is the following: let A be a Noetherian ring with finite global dimension, then the standard t-structure of $\mathsf{D}^b(\mathrm{mod}A)$ has a heart $\mathrm{mod}A$ with co-heart $\mathrm{proj}A$, and we have an equivalence $\mathrm{mod}A \simeq \mathrm{mod}(\mathrm{proj}A)$ in this case.

For any cotorsion pair $(\mathcal{U}, \mathcal{V})$ on an exact category $\mathcal{E}$, we denote

$$\mathcal{C} = \mathcal{U} \cap {}^{\perp_1}\mathcal{U}$$

the *coheart* of $(\mathcal{U}, \mathcal{V})$. We have the following theorem in exact category.

Theorem 2. *Let $(\mathcal{U}, \mathcal{V})$ be a cotorsion pair on an exact category $\mathcal{E}$ with enough projectives and injectives, if for any any object $U \in \mathcal{U}$, there exists an exact sequence $0 \to U' \to C \to U \to 0$ where $U' \in \mathcal{U}$ and $C \in \mathcal{C}$, then the heart of $(\mathcal{U}, \mathcal{V})$ has enough projectives, and moreover it is equivalent to the functor category* $\mathrm{mod}(\mathcal{C}/\mathcal{P})$*, where $\mathcal{P}$ is the subcategory of projetive objects on $\mathcal{E}$.*

We also check that the condition $\mathcal{U}[-1] \subseteq \mathcal{C} * \mathcal{U}$ on triangulated category is satisfied in many cases, for example, when $\mathcal{U}$ is covariantly finite in a Krull-Schmidt triangulated category. The case for exact category is similar.

References

[1] N. Abe, H. Nakaoka, *General heart construction on a triangulated category (II): Associated cohomological functor*, Appl. Categ. Structures **20** (2012), no. 2, 162–174.

[2] A. A. Beilinson, J. Bernstein, P. Deligne, *Faisceaux pervers*, Analysis and topology on singular spaces, I (Luminy 1981), 5–171, Astérisque, **100**, Soc. Math. France, Pairs, 1982.

[3] A. Beligiannis, I. Reiten, *Homological and homotopical aspects of torsion theories*, Mem. Amer. Math. Soc. **188** (2007) no.883.

[4] O. Iyama, Y. Yoshino, *Mutation in triangulated categories and rigid Cohen-Macaulay modules*, Invent. Math. **172** (2008), no. 1, 117–168.

[5] S. Koenig, D. Yang, *Silting objects, simple-minded collections, t-structures and co-t-structures for finite-dimensional algebras*, Doc. Math. **19** (2014), 403–438.

[6] Y. Liu, *Hearts of twin cotorsion pairs on exact categories*, J. Algebra. **394** (2013), 245–284.

[7] ———, *Half exact functors associated with general hearts on exact categories*, arXiv: 1305. 1433.

[8] H. Nakaoka, *General heart construction on a triangulated category (I): unifying t-structures and cluster tilting subcategories*, Appl. Categ. Structures **19** (2011), no. 6, 879–899.

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Higher APR tilting preserve n-representation infiniteness

Yuya Mizuno, Kota Yamaura

This is based on our work [MY].

Hereditary algebras (algebras with global dimension at most one) are one of the most fundamental and important classes of algebras. As a generalization of hereditary algebras, n-hereditary algebras was recently introduced by [1] from the viewpoint of a higher analog of Auslander-Reiten theory. This class consists of two classes, called *n-representation finite* and *n-representation infinite* algebras, which are, respectively, generalization of hereditary representation finite and hereditary representation infinite algebras.

In this talk, we give a construction of a family of n-representation infinite algebras. The key tool is the notion of *m-APR tilting* modules, which is a generalization of classical reflection functors [APR, BGP]. We show that m-APR tilting modules preserve n-representation infiniteness for an arbitrary $1 \leq m \leq n$.

Moreover, we show that m-APR tilting modules lift to tilting modules over the corresponding $(n+1)$-preprojective algebras, which are $(n+1)$-Calabi-Yau algebras.

References

[APR] M. Auslander, M. I. Platzeck, I. Reiten, *Coxeter functors without diagrams*, Trans. Amer. Math. Soc. 250 (1979), 1–46.

[BGP] I. N. Bernstein, I. M. Gelfand, V. A. Ponomarev, *Coxeter functors and Gabriel's theorem*, Uspehi Mat. Nauk 28 (1973), no. 2(170), 19–33.

[MY] Y. Mizuno, K. Yamaura, *Higher APR tilting preserve n-representation infiniteness*, arXiv:1503.08475.

[HIO] M. Herschend, O. Iyama, S. Oppermann, *n-representation infinite algebras*, Adv. Math. 252 (2014), 292–342.

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2-dimensional quantum Beilinson algebras

Izuru Mori

A Fano algebra introduced by Minamoto is roughly speaking a finite dimensional algebra of finite global dimension which is derived equivalent to a (noncommutative) Fano variety [2]. Over such an algebra, a notion of regular module was introduced by Herschend, Iyama and Oppermann from the view point of representation theory of finite dimensional algebras [1]. In this talk, we will define the notions of Fano algebra and regular module, and then explicitly calculate moduli spaces of regular modules over typical examples of Fano algebras, namely, 2-dimensional quantum Beilinson algebras, using techniques of noncommutative algebraic geometry [3].

References

[1] M. Herschend, O. Iyama and S. Oppermann, *n-representation infinite algebras*, Adv. Math. **252** (2014), 292–342.
[2] H. Minamoto, *Ampleness of two-sided tilting complexes*, Int. Math. Res. Not. **1** (2012), 67–101.
[3] I. Mori, *Regular modules over 2-dimensional quantum Beilinson algebras of Type S*, Math. Z. **279** (2015), 1143–1174.

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Survey on representation theory of quiver Hecke algebras I, II

Susumu Ariki

The two talks aim at non-experts. In the first lecture, I will introduce quiver Hecke algebras and cyclotomic quiver Hecke algebras with focus on finite and affine Lie types. I begin with explaining how they naturally generalize the group algebra of the symmetric group, the finite and affine Hecke algebra, by mentioning Brundan and Kleshchev's theorem. Then I state Kang and Kashiwara's theorem on the categorification of integrable highest weight modules and Rouquier's categorification of Weyl group action. In the second lecture, I survey construction of irreducible modules by several authors, theory of standard and costandard modules by Syu Kato and Brundan-Kleshchev-McNamara. If time allows, I explain my work with Euiyong Park, where other types of Fock spaces appear.

References

[1] J. Brundan and A. Kleshchev, *Blocks of cyclotomic Hecke algebras and Khovanov-Lauda algebras*, Invent. Math. **178** (2009), 451–484.

[2] ———, *Graded decomposition numbers for cyclotomic Hecke algebras*, Adv. Math. **222** (2009), 1883–1942.

[3] S.-J. Kang and M. Kashiwara, *Categorification of highest weight modules via Khovanov-Lauda-Rouquier algebras*, Invent. Math. **190** (2012), 699–742.

[4] A. Mathas, *Cyclotomic quiver Hecke algebras of type A*, arxiv:1310.2142.

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Some special cases of
Morita Tachikawa correspondence

Steffen Koenig

Classical Morita-Tachikawa correspondence provide a bijection, up to Morita equivalence between pairs (Λ, M), where Λ is an artin algebra and M a generator-cogenerator, and artin algebras Γ of dominant dimension at least two. A generator-cogenerator is a finitely generated module M such that each indecomposable projective or injective Λ-module is isomorphic to a direct summand of M. The algebra Γ has dominant dimension at least two if (at least) the first two injective terms in its minimal injective resolution are also projective. This beautiful corresondence sends the pair (Λ, M) to the endomorphism ring $End_\Lambda(M)$.

A famous special case is Auslander's correspondence. Here Λ is of finite representation type and M contains each indecomposable module as a direct summand, up to isomorphism. The 'Auslander algebra' Γ then is characterised by having dominant dimension at least two and global dimension at most two. Auslander's correspondence has been extended by Iyama to 'higher Auslander correspondence'.

Recently, further special cases of Morita-Tachikawa correspondence have been studied: gendo-symmetric algebras [1, 2, 3], where Λ is symmetric, Morita algebras [5], where Λ is self-injective, and gendo-Gorenstein algebras [4], where Λ is Gorenstein. These algebras and some of their properties will be described in this lecture.

References

[1] M. Fang, S. Koenig, *Schur functors and dominant dimension*, Trans. Amer. Math. Soc. **363** (2011), 1555–1576.
[2] ——— *Endomorphism algebras of generators over symmetric algebras*, J. Algebra **332** (2011), 428–433.
[3] ——— *Gendo-symmetric algebras, canonical comultiplication, bar cocomplex and dominant dimension*, to appear in Trans. Amer. Math. Soc.
[4] N. Gao, S. Koenig, *Grade, dominant dimension and Gorenstein algebras*, J. Algebra **427** (2015), 118–141.
[5] O. Kerner, K. Yamagata, *Morita algebras*, J. Algebra **382** (2013). 185–202.

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Construction of two-sided tilting complexes for Brauer tree algebras

Yuta Kozakai, Naoko Kunugi

Let Γ be a Brauer tree with e edges and multiplicity m of the exceptional vertex, and A a Brauer tree algebra over k associated to Γ. In [2], Rickard showed that A is derived equivalent to the algebra $B(e,m)$, by constructing a one-sided tilting complex T over A with endomorphism ring $B(e,m)$, where $B(e,m)$ is a Brauer tree algebra over k with respect to a "star" with e edges and exceptional vertex with multiplicity m in the center (or equivalently is a self-injective Nakayama algebra over k with e simple modules and nilpotency degree of the radical being $m+1$). In the construction, the complex T has a length of distance between the exceptional vertex and the furthest edge from the exceptional vertex.

We try to construct a two-sided tilting complex C over $A \otimes_k B(e,m)^{op}$ with the same length as that of the one-sided tilting complex T constructed by Rickard in [2]. The complex C should be one-sided tilting complex when restricted to A, so it is expected to coinside with one-sided tilting complex T.

We could construct such a two-sided complex if Γ is in the shape of "line" with e edges and with exceptional vertex with multiplicity m in the endpoint of the line. The construction is as follows: By [1] and [3], we have that two derived equivalent self-injective algebras are stably equivalent of Morita type. Therefore, there exists an $A \otimes_k B(e,m)^{op}$-module M inducing a stably equivalent of Morita type between A and $B(e,m)$. We construct a two-sided tilting complex over $A \otimes_k B(e,m)^{op}$ inducing a derived equivalence between A and $B(e,m)$ by deleting some direct summand of the module in each term of the minimal projective resolution of M.

Also, given any Brauer tree algebra, we shall be able to construct such a two-sided tilting complex similarly.

References

[1] B.Keller, D,Vossieck, Sous les catégories dérivées. (French) [Beneath the derived categories] C. R. Acad. Sci. Paris Sér. I Math. **305** (1987), no. 6, 225–228.

[2] J.Rickard, Derived categories and stable equivalence. J. Pure Appl. Algebra **61** (1989), no. 3, 303–317.

[3] J.Rickard, Derived equivalences as derived functors. J. London Math. Soc. (2) **43** (1991), no. 1, 37–48.

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The classification of two-term tilting complexes for Brauer graph algebras

Takahide Adachi

This talk is based on joint work with Aaron Chan and Takuma Aihara.

Let Λ be a finite dimensional algebra over an algebraically closed field and $\mathcal{T} := \mathcal{K}^b(\mathrm{proj}\Lambda)$ the homotopy category of bounded complexes of finitely generated projective Λ-modules. A complex T in $\mathcal{T}$ is said to be *pretilting* if $\mathrm{Hom}_{\mathcal{T}}(T, T[i]) = 0$ for any integer $i \neq 0$. A pretilting complex T in $\mathcal{T}$ is said to be *tilting* if $\mathcal{T} = \mathrm{thick}T$, where thick$T$ denotes by the smallest thick subcategory of $\mathcal{T}$ containing T. A (pre)tilting complex T is said to be *two-term* if it is of the form $(0 \to T^{-1} \to T^0 \to 0)$.

The study of derived categories has been one of the central themes in representation theory. ¿From Morita theoretic perspective, tilting complexes play an important role because the endomorphism algebras are derived equivalent to the original algebra [2]. It is well-known that derived equivalences preserve many homological properties. Thus it is important to classify tilting complexes for a given algebra.

Our aim of this talk is to give a classification of two-term tilting complexes for Brauer graph algebras. A Brauer graph algebra $\Lambda_{G,m}$ is a symmetric special biserial algebra defined by a ribbon graph G with a multiplicity (function) m for the vertices. It is well-known that, for a special biserial algebra, the 0-th cohomology of any indecomposable two-term pretilting complex is always a string module (see, for example, [3]). Inspired from this property, we can define the notion of "(half-)walk" on a ribbon graph. Then, we give a bijection between the set of isomorphism classes of indecomposable two-term pretilting complexes for a Brauer graph algebra $\Lambda_{G,m}$ and the set of special walks, called admissible walks, on the ribbon graph G. By using the bijection, we give a combinatorial description of two-term tilting complexes.

Theorem 1. [1] *Let $\Lambda_{G,m}$ be the Brauer graph algebra of a ribbon graph G with a multiplicity m for the vertices. Then there is a bijection between the set of isomorphism classes of basic two-term tilting complexes for $\Lambda_{G,m}$ and the set of complete admissible collections of admissible walks on G.*

Consequently, we obtain the following corollary.

Corollary 2. [1] *Let G be a ribbon graph and m, n be multiplicities for the vertices of G. Then there is a natural bijection between the set of isomorphism classes of basic two-term tilting complexes for $\Lambda_{G,m}$ and that for $\Lambda_{G,n}$.*

Moreover, we can easily calculate the endomorphism algebra of a two-term tilting complexes from the combinatorial description.

References

[1] T. Adachi, T. Aihara, A. Chan, *Tilting Brauer graph algebras I: Classification of two-term tilting complexes*, arXiv:1504.04827.

[2] J. Rickard, *Morita theory for derived categories*, J. London Math. Soc. **39**, 1989, 436–456.

[3] B. Wald, J. Waschbüsch, *Tame biserial algebras*, J. Algebra **95** (1985), no. 2, 480–500.

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Derived equivalences and Gorenstein projective dimension

Mitsuo Hoshino and Hirotaka Koga

Let $\mathcal{A}, \mathcal{B}$ be abelian categories with enough projectives. We denote by $\mathcal{P}_\mathcal{A}$ the full subcategory of $\mathcal{A}$ consisting of all projective objects in $\mathcal{A}$ and by $\mathcal{D}^{\mathrm{b}}(\mathcal{A})$ the derived category of complexes over $\mathcal{A}$ with bounded cohomology. In this note, we introduce the notion of complexes of finite Gorenstein projective dimension and show that a derived equivalence $F : \mathcal{D}^{\mathrm{b}}(\mathcal{A}) \xrightarrow{\sim} \mathcal{D}^{\mathrm{b}}(\mathcal{B})$ induces an equivalence between the full triangulated subcategories consisting of complexes of finite Gorenstein projective dimension provided that F satisfies the following condition: there exists $a > 0$ such that $\mathrm{Hom}_{D^{\mathrm{b}}(\mathcal{B})}(FP, Q[i]) = 0 = \mathrm{Hom}_{D^{\mathrm{b}}(\mathcal{B})}(Q, FP[i])$ for all $P \in \mathcal{P}_\mathcal{A}$ and $Q \in \mathcal{P}_\mathcal{B}$ unless $-a \le i \le a$.

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ALMOST GORENSTEIN REES ALGEBRAS

Ken-ichi Yoshida, Shiro Goto, Naoyuki Matsuoka, Naoki Taniguchi

Almost Gorenstein rings are newcomers, which form a class of Cohen-Macaulay rings that are not necessarily Gorenstein, but still good hopefully next to Gorenstein rings. The notion of almost Gorenstein rings was introduced by V. Barucci and R. Fröberg [1] in 1997 in the case where the local rings are of dimension one and analytically unramified. After then, the notion was extended to arbitrary Cohen-Macaulay rings by S. Goto, N. Matsuoka, and T.T. Phuong [2] and S. Goto, R. Takahashi and N. Taniguchi [3]. Let us recall the definition here: see [3].

Definition 1 (**Almost Gorenstein local ring**)**.** Let $(R, \mathfrak{m})$ be a Cohen-Macaulay local ring with the canonical module K_R. Then R is called an *almost Gorenstein local ring* if there exists an exact sequence

$$0 \to R \to K_R \to C \to 0$$

of R-modules such that $\mu_R(C) = e^0_{\mathfrak{m}}(C)$, where $\mu_R(C)$ (resp. $e^0_{\mathfrak{m}}(C)$) denotes the number of generators for C (resp. the multiplicity of C with respect to $\mathfrak{m}$).

Definition 2 (**Almost Gorenstein graded ring**)**.** Let $R = \oplus_{n\geq 0} R_n$ be a Cohen-Macaulay graded ring over a local ring R_0 with the graded canonical module K_R. Then R is called an *almost Gorenstein graded ring* if there exists an exact sequence

$$0 \to R \to K_R(-a(R)) \to C \to 0$$

of graded R-modules such that $\mu_R(C) = e^0_{\mathfrak{M}}(C)$, where $\mathfrak{M}$ stands for the unique graded maximal ideal of R and $a(R)$ denotes the a-invariant of R.

Several concrete examples of almost Gorenstein local/graded rings can be found in [3]; e.g. two-dimensional rational singularities. Although Gorenstein Rees algebras are rather rare, it seems that there are many examples of almost Gorenstein Rees algebras. However, we have a few examples of those algebras. So we pose the following question.

Question. Let R be a Gorenstein local ring, and let $I \subset R$ be an ideal. When is the Rees algebra $\mathrm{R}(I) = R[It] \subseteq R[t]$ almost Gorenstein?

1.1. p_g**-ideals.** The following theorem is the main theorem in this talk, which gives many examples of normal almost Gorenstein graded rings.

In what follows, let $(R, \mathfrak{m})$ be a two-dimensional Gorenstein normal local ring, and $I \subset R$ be an $\mathfrak{m}$-primary integrally closed ideal of R. Put $\mathcal{R} = \mathrm{R}(I)$ and $\mathfrak{M}$ denotes the unique graded maximal ideal of $\mathcal{R}$.

Theorem 3 (see [6])**.** *Suppose that I is a p_g-ideal* (*see* [8, 9]), *that is, $\mathrm{R}(I)$ is a Cohen-Macaulay normal domain. Then $\mathcal{R} = \mathrm{R}(I)$ is an almost Gorenstein graded ring.*

Corollary 4. *Let R be a two-dimensional Gorenstein rational singularity. Then $\mathrm{R}(I)$ is an almost Gorenstein graded ring for any $\mathfrak{m}$-primary integrally closed ideal I of R.*

Example 5 (see [5])**.** Let $(R, \mathfrak{m})$ be a two-dimensional regular local ring. Then $\mathrm{R}(\mathfrak{m}^n)$ is an almost Gorenstein graded ring for every integer $n \geq 1$.

1.2. **Parameter ideals.** In general, even if $\mathrm{R}(I)_{\mathfrak{M}}$ is an almost Gorenstein local ring, $\mathrm{R}(I)$ is not a necessarily almost Gorenstein graded ring. In fact, it seems that almost Gorenstein graded Rees algebras are rather rare. See below.

Theorem 6 (see [4])**.** *Let R be a Gorenstein local ring, and let $a_1, \ldots, a_r$ $(r \geq 3)$ be a subsystem of parameters for R and set $Q = (a_1, \ldots, a_r)R$. Then the following conditions are equivalent.*

(1) *$\mathrm{R}(Q)$ is an almost Gorenstein graded ring.*
(2) *R is a regular local ring and $a_1, \ldots, a_r$ form a part of a regular system of parameters for R.*

On the other hand, there are many examples of almost Gorenstein local rings which is a localization of Rees algebras.

Theorem 7 (see [4]). *Suppose that R is a Gorenstein local ring of dimension $d \geq 3$, and let $a_1, \dots, a_r$ ($r \geq 3$) be a subsystem of parameters for R and set $Q = (a_1, \dots, a_r)R$. Then the following conditions are equivalent.*

(1) *$R(Q)_{\mathfrak{M}}$ is an almost Gorenstein local ring.*
(2) *R is a regular local ring.*

Example 8 (see [4]). Let R be a Cohen-Macaulay local ring of dimension 2. If $R(Q)_{\mathfrak{M}}$ is an almost Gorenstein local ring for a parameter ideal $Q \subset R$, then R is a Gorenstein local ring.

We do not know whether a similar result holds true in the case where $d \geq 3$.

1.3. **Socle ideals.** Let R be a regular local ring with $d = \dim R$ and infinite residue field. Let Q be a parameter ideal of R such that $Q \neq \mathfrak{m}$, and let $I = Q \colon \mathfrak{m}$. Put $\mathcal{R} = R(I)$ and $\mathfrak{M}$ denotes the unique graded maximal ideal of $\mathcal{R}$.

Theorem 9 (see [4]). *Suppose $d \geq 3$. Then the following conditions are equivalent.*

(1) *$\mathcal{R}$ is an almost Gorenstein graded ring.*
(2) *Either $I = \mathfrak{m}$, or $d = 3$ and $I = (x) + \mathfrak{m}^2$ for some $x \in \mathfrak{m} \setminus \mathfrak{m}^2$.*

Example 10 (see [5]). Suppose that $d = 2$ and $Q \subset \mathfrak{m}^3$. Then $\mathcal{R}_{\mathfrak{M}}$ is *not* an almost Gorenstein local ring.

References

[1] V. Barucci and R.Fröberg, *One-dimensional almost Gorenstein rings*, J. Algebra **188** (1997), 418–442.
[2] S. Goto, N. Matsuoka and T. T. Phuong, *Almost Gorenstein rings*, J. Algebra, **379** (2013), 355–381.
[3] S. Goto, R. Takahashi and N. Taniguchi, *Almost Gorenstein rings -towards a theory of higher dimension*, J. Pure Appl. Algebra, **219** (2015), 2666–2712.
[4] S. Goto, N. Matsuoka, N. Taniguchi and K.Yoshida, *The almost Gorenstein Rees algebras of parameters*, Preprint 2015.
[5] ——— *The almost Gorenstein Rees algebras over regular local ring*, Preprint 2015.
[6] S. Goto, N. Taniguchi and K. Yoshida, *The almost Gorenstein Rees algebras of p_g-ideals*, in preparation.
[7] J. K. Verma, *Joint reductions and Rees algebras*, Math. Proc. Camb. Phil. Soc. **109** (1991), 335–342.
[8] T. Okuma, K.-i. Watanabe and K. Yoshida, *Good ideals and p_g-ideals in two-dimensional normal singularities*, Preprint 2014.
[9] ——— *A characterization of p_g-ideals in two-dimensional normal singularities*, in preparation.

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The structure of preenvelopes with respect to maximal Cohen-Macaulay modules

Hiroki Matsui

Throughout this article, we assume that $(R, \mathfrak{m}, k)$ is a d-dimensional Cohen-Macaulay local ring with canonical module ω. All R-modules are assumed to be finitely generated. Denote by $\mathsf{mod}R$ the category of finitely generated R-modules and by MCM the full subcategory of $\mathsf{mod}R$ consisting of maximal Cohen-Macaulay R-modules.

We define $(-)^\dagger := \mathsf{Hom}_R(-, \omega)$ and $\delta_M : M \to M^{\dagger\dagger}$ as a natural homomorphism for an R-module M. Note that if M is maximal Cohen-Macaulay, δ_M is an isomorphism.

Let $\mathcal{X}$ be a full subcategory of $\mathsf{mod}R$. The notion of $\mathcal{X}$-(pre)covers and $\mathcal{X}$-(pre)envelopes have been playing an important role in the representation theory of algebras; see [2, 3, 4, 7] for instance. For $\mathcal{X} = \mathsf{MCM}$, a celebrated theorem due to Auslander and Buchweitz [1] says that for any R-module M, there exists a short exact sequence

$$0 \to Y \xrightarrow{f} X \xrightarrow{\pi} M \to 0$$

where X is maximal Cohen-Macaulay and Y has finite injective dimension. The map π is called a maximal Cohen-Macaulay approximation of M. Then π is an MCM-precover of M, and is an MCM-cover if Y and X have no non-zero common direct summand via f. If R is Henselian, every R-module has an MCM-cover; see [6, 8].

In this talk we mainly study the MCM-envelope, and the MCM-preenvelope which is called special. An R-homomorphism $\mu : M \to X$ is a special MCM-preenvelope if it is an MCM-preenvelope and satisfies $\mathsf{Ext}^1_R(\mathsf{Coker}\mu, \mathsf{MCM}) = 0$. A result of Holm [5, Theorem A] states that every R-module has a special MCM-preenvelope, and if R is Henselian, every R-module has an MCM-envelope.

It is natural to ask when a given homomorphism is a special MCM-preenvelope or an MCM-envelope, and we give an answer to this question by using the kernels and cokernels. pplying this result, we give another characterization of special MCM-preenvelopes in terms of the existence of certain complexes. Furthermore, we can also prove the result on the structure of minimal MCM-coresolutions and this result recovers a main theorem of Holm [5, Theorem C] in the Henselian case.

References

[1] M. Auslander and R.-O. Buchweitz, The homological theory of maximal Cohen-Macaulay approximations, Colloque en l'honneur de Pierre Samuel (Orsay, 1987), *Mém. Soc. Math. France (N.S.)*, No. 38 (1989), 5–37.

[2] M. Auslander and I. Reiten, Applications of contravariantly finite subcategories, *Adv. Math.* **86** (1991), 111–152.

[3] M. Auslander and S. O. Smalø, Almost split sequences in subcategories, *J. Algebra* **69** (1980), no. 2, 426–454.

[4] M. Auslander, S. O. Smalø, Preprojective modules over Artin algebras, *J. Algebra* **66** (1) (1980), 61–122.

[5] H. Holm, Approximations by maximal Cohen-Macaulay modules, *Pacific J. Math* (to appear).

[6] R. Takahashi, On the category of modules of Gorenstein dimension zero, *Math. Z.* **251** (2005), no. 2, 249–256.

[7] R. Takahashi, Contravariantly finite resolving subcategories over commutative rings, *Amer. J. Math.* **133** (2011), no. 2, 417–436.

[8] Y. Yoshino, Cohen-Macaulay approximations (Japanese), *Proceedings of the 4th Symposium on Representation Theory of Algebras, Izu, Japan*, (1993), pp. 119–138.

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Thick subcategories of derived categories of isolated singularities

Ryo Takahashi

A *thick* subcategory of a triangulated category is by definition a full triangulated subcategory closed under direct summands. The notion of a thick subcategory has been introduced by Verdier [14] by the name of *épaisse* subcategory to develop the theory of Verdier localizations.

Classifying thick subcategories is one of the most important problems shared by homotopy theory, ring theory, algebraic geometry and representation theory. It was first done by Devinatz, Hopkins and Smith [3, 7] in the 1980s; they classified the thick subcategories of the triangulated category of compact objects in the p-local stable homotopy category. Later on, as an analog of the Devinatz–Hopkins–Smith theorem for commutative rings, Hopkins and Neeman [6, 8] classified the thick subcategories of the derived category of perfect complexes over a commutative noetherian ring, and it was extended to a quasi-compact quasi-separated scheme by Thomason [13]. As an analog of the Hopkins–Neeman theorem for finite groups, Benson, Carlson and Rickard [1] classified the thick subcategories of the stable category of finite dimensional representations of a finite p-group. It was extended to a finite group scheme by Friedlander and Pevtsova [5] and further generalized to the derived category of a finite group by Benson, Iyengar and Krause [2].

Several years ago the speaker [10, 11] classified the thick subcategories of the singularity category of a local hypersurface, and recently it was generalized to the derived category of a complete intersection by Stevenson [9]. On the other hand, it turned out in the work of Dwyer, Greenlees and Iyengar [4] that every thick subcategory of the derived category of a complete intersection is *standard*, that is, it contains a nonzero perfect complex. In this talk, we consider classifying standard thick subcategories of the derived category of an *isolated singularity*, i.e., a commutative noetherian local ring that is locally regular on the punctured spectrum.

References

[1] D. J. Benson; J. F. Carlson; J. Rickard, Thick subcategories of the stable module category, *Fund. Math.* **153** (1997), no. 1, 59–80.

[2] D. J. Benson; S. B. Iyengar; H. Krause, Stratifying modular representations of finite groups, *Ann. of Math. (2)* **174** (2011), no. 3, 1643–1684.

[3] E. S. Devinatz; M. J. Hopkins; J. H. Smith, Nilpotence and stable homotopy theory, I, *Ann. of Math. (2)* **128** (1988), no. 2, 207–241.

[4] W. Dwyer; J. P. C. Greenlees; S. Iyengar, Finiteness in derived categories of local rings, *Comment. Math. Helv.* **81** (2006), no. 2, 383–432.

[5] E. M. Friedlander; J. Pevtsova, Π-supports for modules for finite group schemes, *Duke Math. J.* **139** (2007), no. 2, 317–368.

[6] M. J. Hopkins, Global methods in homotopy theory, *Homotopy theory (Durham, 1985)*, 73–96, London Math. Soc. Lecture Note Ser., 117, *Cambridge Univ. Press, Cambridge*, 1987.

[7] M. J. Hopkins; J. H. Smith, Nilpotence and stable homotopy theory, II, *Ann. of Math. (2)* **148** (1998), no. 1, 1–49.

[8] A. Neeman, The chromatic tower for $D(R)$, With an appendix by Marcel Bökstedt, *Topology* **31** (1992), no. 3, 519–532.

[9] G. Stevenson, Subcategories of singularity categories via tensor actions, *Compos. Math.* **150** (2014), no. 2, 229–272.

[10] R. Takahashi, Classifying thick subcategories of the stable category of Cohen-Macaulay modules, *Adv. Math.* **225** (2010), no. 4, 2076–2116.

[11] R. Takahashi, Thick subcategories over Gorenstein local rings that are locally hypersurfaces on the punctured spectra, *J. Math. Soc. Japan* **65** (2013), no. 2, 357–374.

[12] R. Takahashi, Thick subcategories of derived categories of isolated singularities, in preparation.

[13] R. W. Thomason, The classification of triangulated subcategories, *Compos. Math.* **105** (1997), no. 1, 1–27.

[14] J.-L. Verdier, Catégories dérivées, état 0, SGA $4\frac{1}{2}$, 262–308, Lecture Notes in Mathematics, 569, *Springer-Verlag, Berlin-New York*, 1977.

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QUIVERS, OPERATORS on HILBERT SPACES and OPERATOR ALGEBRAS I,II

Watatani, Yasuo

One of the aims of the theory of representations of finite dimensional algebras is to describe how a finite number of linear transformations can act simultaneously on a finite dimensional vector space. In my talk, we consider bounded linear operators on a infinite-dimensional Hilbert space instead of linear transformations on a finite dimensional vector space. We describe similarities and differences between them.

Quivers are related with operator algebras in the following (at least) three different ways.

(1)(Cuntz-Krieger algebras [1])

Strongly connected quiver generates an important classe of purely infinite simple C^*-algebras, called Cuntz-Krieger algebras, and they are classified by their K-groups. The vertices are represented by orthogonal subspaces and the arrows are represented by partial isometries with the orthognal ranges.

(2)(Principal graphs for subfactors [5], [4],[8])

The category of bimodules for a subfactor forms a principal graph (a certain bitartite graph) and a good invariant in subfactor theoory. In particular, irreucible hyperfinite subfactors with Jones index less than four have Dynkin diagrams A,D and E. The vertices are constructed by irreducible bimodules and arrows are constructed by bimodule homomorphisms.

(3)(Hilbert representations of quivers [2], [3])

A Hilbert representation of a quiver is to associate Hilbert spaces for the vertices and bounded operators for arrows. Jordan blocks correspond to strongly irreducible operators. The invaraint subspace problem is one of the famous unsolved problem in functional anaysis and rephrased by the existance of simple Hilbert representation of a quiver.

We study operator algebras instead of finite dimensional algebras. We have two important classes of operator algebras, that is, C^*-algebras and von Neumann algebras. A C^*-algebra is a operator-norm-topology closed *-subalgebra of the algebra $B(H)$ of bounded operators on a Hilbert space H. A von Neumann algebra is a weak-operator-topology closed *-subalgebra of $B(H)$. C^*-algebras are regarded as quantized (locally) compact Housdorff spaces. Von Neumann algebras are regarded as quantized measure spaces.

We can associated C^*-algebras for topological dynamical systems and von Neumann algebras for measurable dynamical systems. In the last part of my talk, I will show our study on C^*-algebras associated complex dynamical systems ([6]) and self-similar dynamical systems ([7]). These results are based on joint works with Masatoshi Enomoto and Tsuyoshi Kajiwara.

In order to "feel" the difference between purely algebraic setting and functional analytic setting, let us consider the following typical examples: Let L_1 be one-loop quiver, that is, L_1 is a quiver with one vertex $\{1\}$ and 1-loop $\{\alpha\}$ such that $s(\alpha) = r(\alpha) = 1$. Consider two infinite-dimensional spaces the polynomial ring $\mathbb{C}[z]$ and the Hardy space $H^2(\mathbb{T})$. Then $\mathbb{C}[z]$ is dense in $H^2(\mathbb{T})$ with respect to the Hilbert space norm.

Define a purely algebraic representation (V, T) of L_1 by $V_1 = \mathbb{C}[z]$ and the multiplication operator T_α by z. That is, $T_\alpha h(z) = zh(z)$ for a polynomial $h(z) = \sum_n a_n z^n$. Since $End(V, T) \cong \mathbb{C}[z]$ have no idempotents, the purely algebraic representation (V, T) is indecomposable.

Next we define a Hilbert representation (H, S) by $H_1 = H^2(\mathbb{T})$ and $S_\alpha = T_z$ the Toeplitz operator with the symbol z. Then $S_\alpha = T_z$ is the multiplication operator by z on $H^2(\mathbb{T})$ and is identified with the unilateral shift. Then

$$\begin{aligned} End(H,S) &\cong \{A \in B(H^2(\mathbb{T})) \mid AT_z = T_zA\} \\ &= \{T_\phi \in B(H^2(\mathbb{T})) \mid \phi \in H^\infty(\mathbb{T})\} \end{aligned}$$

is the algebra of analytic Toeplitz operators and isomorphic to $H^\infty(\mathbb{T})$. By the F. and M. Riesz Theorem, if $f \in H^2(\mathbb{T})$ has the zero set of positive measure, then $f = 0$. Since $H^\infty(\mathbb{T}) = H^2(\mathbb{T}) \cap L^\infty(\mathbb{T})$, $H^\infty(\mathbb{T})$ has no non-trivial idempotents. Thus there exists no non-trivial idempotents which commutes with T_z and Hilbert space (H, S) is indecomposable. In this sense, the analytical aspect of Hardy space is quite important in our setting.

Any subrepresentation of the purely algebraic representation (V, T) is given by the restriction to an ideal $J = p(z)\mathbb{C}[z]$ for some polynomial $p(z)$. Any subrepresentation of the Hilbert representation (H, S) is given by an invariant subspace of the shift operator T_z. Beurling theorem shows that any subrepresentation of (H, S) is given by the restriction to an invariant subspace $M = \varphi H^2(\mathbb{T})$ for some inner function φ. For example, if an ideal J is defined by

$$J = \{f(z) \in \mathbb{C}[z] \mid f(\lambda_1) = \cdots = f(\lambda_n) = 0\}$$

for some distinct numbers $\lambda_1, \dots \lambda_n \in \mathbb{C}$, then the corresponding polynomial $p(z)$ is given by $p(z) = (z - \lambda_1)\dots(z - \lambda_n)$. The case of Hardy space is much more analytic. We shall identify $H^2(\mathbb{T})$ with a subspace $H^2(\mathbb{D})$ of analytic functions on the open unit disc $\mathbb{D}$. If an invariant subspace M is defined by

$$M = \{f \in H^2(\mathbb{D}) \mid \ f(\lambda_1) = \cdots = f(\lambda_n) = 0\}$$

for some distinct numbers $\lambda_1, \dots \lambda_n \in \mathbb{D}$, then the corresponding inner function φ is given by a finite Blaschke product

$$\varphi(z) = \frac{(z - \lambda_1)}{1 - \overline{\lambda_1} z} \cdots \frac{(z - \lambda_n)}{1 - \overline{\lambda_n} z}.$$

Here we cannot use the notion of degree like polynomials and we must manage to treat orthogonality to find such an inner function φ.

References

[1] J. Cuntz and W. Krieger, *A class of C^*-algebras and topological Markov chains,* Invent. Math. **56** (1980), 251–268.

[2] M. Enomoto and Y. Watatani, *Relative position of four subspaces in a Hilbert space,* Adv. Math. **201** (2006), 263–317.

[3] M. Enomoto and Y. Watatani, *Indecomposable representations of quivers on infinite-dimensional Hilbert spaces,* J. Funct. Anal. **256** (2009), 959–991.

[4] D. Evans and Y. Kawahigashi, *Quantum Symmetries on Operator Algebras,* 1998, Oxford Science Publications.

[5] V. Jones, *Index for subfactors,*Invent. Math. **72** (1983), 1–25. .

[6] T. Kajiwara T. and Y. Watatani, *C^*-algebras associated with complex dynamical systems,* Indiana Math. J. **54** (2005), 755–778.

[7] T. Kajiwara and Y. Watatani, *C^*-algebras associated with self-similar sets,* J. Operator Theory **56** (2006), 225-247.

[8] Y. Watatani, *Index for C^*-subalgebras,* Mem. Amer. Math. Soc.,**424** (1990).

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On weakly separable extensions and weakly quasi-separable extensions

Satoshi Yamanaka

Let A/B be a ring extension with common identity 1, M an A-A-bimodule, and x, y arbitrary elements in A. An additive map δ is called a *B-derivation* of A to M if $\delta(xy) = \delta(x)y + x\delta(y)$ and $\delta(\alpha) = 0$ for any $\alpha \in B$. Moreover, δ is called *central* if $\delta(x)y = y\delta(x)$, and δ is called *inner* if $\delta(x) = mx - xm$ for some fixed element $m \in M$. We say that a ring extension A/B is *separable* if the A-A-homomorphism of $A \otimes_B A$ onto A defined by $a \otimes b \mapsto ab$ splits. It is well known that A/B is separable if and only if for any A-A-bimodule M, every B-derivation of A to M is inner. In [4], H. Komatsu showed that A/B is *quasi-separable* if and only if for any A-A-bimodule M, every central B-derivation of A to M is zero. Recently in [1], N. Hamaguchi and A. Nakajima generalized separable extensions and quasi-separable extensions as follows:

Definition 1. ([1, Definition 2.1]) (1) A/B is called *weakly separable* if every B-derivation of A to A is inner.

(2) A/B is called *weakly quasi-separable* if every central B-derivation of A to A is zero.

Let B be a ring, ρ an automorphism of B, and D a ρ-derivation of B. $B[X;\rho,D]$ will mean the skew polynomial ring in which the multiplication is given by $\alpha X = X\rho(\alpha) + D(\alpha)$ for any $\alpha \in B$. If $D = 0$ this written as $B[X;\rho]$; and if $\rho = 1$, as $B[X;D]$. By $B[X;\rho,D]_{(0)}$ we denote the set of all monic polynomials g in $B[X;\rho,D]$ such that $gB[X;\rho,D] = B[X;\rho,D]g$. For $f \in B[X;\rho,D]_{(0)}$, f is called *separable* (resp. *weakly separable*, *weakly quasi-separable*) in $B[X;\rho,D]$ if the residue ring $B[X;\rho,D]/fB[X;\rho,D]$ is separable (resp. weakly separable, weakly quasi-separable) over B.

In [9], the author showed the difference between the separability and the weakly separability in $B[X;\rho]$ and $B[X;D]$. In this talk, we shall show some results concerning weakly separable polynomials and weakly quasi-separable polynomials in skew polynomial rings.

References

1. N. Hamaguchi and A. Nakajima, On generalizations of separable polynomials over rings, *Hokkaido Math. J.,* **42** 2013, no. 1, 53–68.
2. S. Ikehata, On separable polynomials and Frobenius polynomials in skew polynomial rings, *Math. J. Okayama Univ.,* **22** 1980, 115–129.
3. S. Ikehata, Azumaya algebras and skew polynomial rings, *Math, J. Okayama Univ.,* **23** 1981, 19–32.
4. H. Komatsu, Quasi-separable extensions of noncommutative rings, *Comm. Algebra,* **29** (3) 2001, 1011–1019.
5. Y. Miyashita, On a skew polynomial ring, *J. Math. Soc. Japan,* **31** 1979, no. 2, 317–330.
6. T. Nagahara, On separable polynomials over a commutative ring II, *Math. J. Okayama Univ.,* **15** 1972, 149–162.
7. S. Yamanaka and S. Ikehata, An alternative proof of Miyasita's theorem in a skew polynomial rings, *Int. J. Algebra,* **21** 2012, 1011–1023
8. S. Yamanaka and S. Ikehata, On Galois polynomials of degree p in skew polynomial rings of derivation type, *Southeast Bull. Math.,* **37** 2013, 625–634.
9. S. Yamanaka, On weakly separable polynomials and weakly quasi-separable polynomials over rings, *Math. J. Okayama Univ.,* **58** 2016, to appear.

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Exapmles of Ore extensions which are maximal orders whose based rings are not maximal orders

H. Marubayashi and A. Ueda

Let R be a prime Goldie ring and (σ, δ) be a skew derivation on R. It is well known that if R is a maximal order, then the Ore extension $R[x; \sigma, \delta]$ is a maximal order. It was a long standing open question that the convserse is true or not in case $\sigma \neq 1$ and $\delta \neq 0$.

In this talk, we give an example of non-maximal order R with a skew derivation (σ, δ) on R ($\sigma \neq 1, \delta \neq 0$) such that $R[x; \sigma, \delta]$ is a maximal order as follows (here, by a *skew derivation* (σ, δ) on a ring R, we mean that σ is an automorphism of R and δ is a left σ-derivation on R.)

Let D be a hereditary Noetherian prime ring (an HNP ring for short) with quotient ring K satisfying the following:

(a) there is a cycle $\mathfrak{m}_1, \ldots, \mathfrak{m}_n$ $(n \geq 2)$ such that $\mathfrak{p} = \mathfrak{m}_1 \cap \cdots \cap \mathfrak{m}_n = aD = Da$ for some $a \in D$, and
(b) any maximal ideal $\mathfrak{n}$ different from $\mathfrak{m}_i$ $(1 \leq i \leq n)$ is invertible.

Examples of an HNP ring D satisfying the conditions (a) and (b) are found in [1] and [2]. The simplest example is $D = \begin{pmatrix} \mathbb{Z} & p\mathbb{Z} \\ \mathbb{Z} & \mathbb{Z} \end{pmatrix}$, where $\mathbb{Z}$ is the ring of integers and p is a prime number.

We define a skew derivation (σ, δ) on D by $\sigma(r) = ara^{-1}$ and $\delta(r) = 0$ for all $r \in D$. Let $R = D[t]$ be the polynomial ring over D in an indeterminate t. Then (σ, δ) on D is extended to a skew derivation on R by $\sigma(t) = t$ and $\delta(t) = a$. A fractional R-ideal $\mathfrak{a}$ is called *σ-invariant* if $\sigma(\mathfrak{a}) = \mathfrak{a}$ and is called *δ-stable* if $\delta(\mathfrak{a}) \subseteq \mathfrak{a}$. A σ-invariant and δ-stable fractional R-ideal is said to be *(σ, δ)-stable.*

Note that a maximal ideal of $K[t]$ is either $tK[t]$ or $\omega K[t]$ for some central element ω of $K[t]$ and so any prime v-ideal $\mathfrak{b}$ of R with $\mathfrak{b} \cap D = (0)$ is either $\mathfrak{b} = tR$ or $\mathfrak{b} = \omega K[t] \cap R$.

Let $S = R[x; \sigma, \delta]$ be an Ore extension in an indeterminate x. Then, under the above notations,

Proposition 1. *$P = \mathfrak{p}[t][x; \sigma, \delta]$, $N = \mathfrak{n}[t][x; \sigma, \delta]$ and $B = \mathfrak{b}[x; \sigma, \delta]$ (in case $\delta(\omega) = 0$) and $C = \mathfrak{b}^p[x; \sigma, \delta]$ (in case $\delta(\omega) \neq 0$ and char $K = p \neq 0$) are prime invertible ideals of S.*

Using Proposition 1, we can prove

Lemma 2. *Let A be an ideal of S such that $A = A_v$.*
(1) If $\mathfrak{a} = A \cap R \neq (0)$, then $\mathfrak{a}$ is a (σ, δ)-stable invertible ideal of R and $A = \mathfrak{a}[x; \sigma, \delta]$.
(2) If $A \cap R = (0)$, then A is v-invertible.

From Lemma 2, we obtain the following theorem.

Theorem 3. *$S = R[x; \sigma, \delta]$ is a maximal order and R is not a maximal order.*

References

1. M. Harada, *Structure of hereditary orders over local rings*, J. Math Osaka City Univ. **14** (1963), 1–22
2. G. Q. Abbasi, S. Kobayashi, H. Marubayashi and A. Ueda, *Noncommutative unique factorization rings*, Comm. in Algebra, **19(1)**, (1991), 167–198.

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Cayley Graphs over a Finite Chain Ring and GCD-graphs

Yotsanan Meemark

The study of ring-theoretic graph has become an exciting research topic in the last two decades. This includes unitary Cayley graphs, integral circulant graphs, zero-divisor graphs and gcd-graphs. Mostly, the work is on determining its eigenvalues (which are real) and computing the sum of absolute values of the eigenvalues, called the energy of a graph. The energy is a graph parameter introduced by Gutman (see [2] and [1] for a good survey) arising from the Hückel molecular orbital approximation for the total π-electron energy. Nowadays, the energy of graph is studied for purely mathematical interest.

Let D be a unique factorization domain (UFD) and $c \in D$ a nonzero nonunit element. Assume that the commutative ring $D/(c)$ is finite. For a set $\mathcal{C}$ of proper divisors of c, we define the *gcd-graph*, $D_c(\mathcal{C})$, to be a graph whose vertex set is the quotient ring $D/(c)$ and edge set is

$$\{\{x + (c), y + (c)\} : x, y \in D \text{ and } \gcd(x - y, c) \in D^{\times}\mathcal{C}\}.$$

This gcd-graph on a quotient ring of a unique factorization domain (UFD) introduced in [3] generalizes a gcd-graph or an integral circulant graph (i.e., its adjacency matrix is circulant and all eigenvalues are integers) defined over $\mathbb{Z}_n, n \geq 2$, (see [4, 8]). An integral circulant graph can also be considered as an extension of a unitary Cayley graph.

Since the number of set of divisors $\mathcal{C}$ of $c = p_1^{s_1} \ldots p_k^{s_k}$ can be very large, the energy of gcd-graph (over a $D/(c)$ or $\mathbb{Z}_n$) still not thoroughly studied. As usual, the common thing that ones would think of when they are working on a UFD is prime powers. We shall give the energy of gcd-graphs with the divisor set $\mathcal{C}$, where $\mathcal{C}$ consists of certain prime powers, by studying the energy of the Cayley graph over the finite ring $D/(p_i^{s_i})$. When $D = \mathbb{Z}$, this graph is the integral circulant graph with prime power order studied by Sander and Sander in [7]. They derived a closed formula for its energy and worked on minimal and maximal energies for a fixed prime power p^s and varying divisor sets. While we have tried to extend their results to a Cayley graph over finite commutative rings, we come across a kind of rings which has a nice property on ideals, called a finite chain ring. The structure of this ring has been well studied, see [5, 6]. It is a finite local ring and generalizes the ring $D/(p^s)$ and the Galois ring $\mathbb{Z}_{p^s}[x]/(f(x))$, where $f(x)$ is a monic polynomial in $\mathbb{Z}_{p^s}[x]$ and the canonical reduction $\bar{f}(x)$ in $\mathbb{Z}_p[x]$ is irreducible. This explains why we focus our work on a finite chain ring.

In this work, we determine the spectrum and obtain the energy of a Cayley graph over a finite chain ring extending the idea of integral circulant graphs with prime power order presented in [7] in which they compute the energy via a sum of Ramanujan sums as studied in [4]. Our approach here is to examine all eigenvalues with multiplicities and then obtain the sum of their absolute values directly similar to [3]. We also gives further results on a gcd graph on a quotient ring of a unique factorization domain using tensor products.

References

1. R. A. Brualdi, *Energy of a Graph*, http://www.public.iastate.edu/lhogben/energyB.pdf.
2. I. Gutman, *The Energy of a Graph: Old and New Results, Algebraic Combinatorics and Applications*, Springer, Berlin, 2001.
3. D. Kiani, M.M.H Aghaei, Y. Meemark and B. Suntornpoch, Energy of unitary Cayley graphs and gcd-graphs, *Linear Algebra Appl.*, **435** (6) (2011), 1336–1343.
4. W. Klotz and T. Sander, Some properties of unitary Cayley graphs, *The Electronic J. Comb.*, **14** (2007), #R45.
5. B. R. McDonald, *Finite Rings with Identity*, Marcel Dekker, New York, 1974.
6. G.H. Norton and A. Sălăgean, On the structure of linear and cyclic codes over finite chain rings, *Appl. Algebra Eng. Comm. Comput.*, **10 (6)** (2000), 489–506.
7. J.W. Sander and T. Sander, The energy of integral circulant graphs with prime power order, *Appl. Anal. Discrete Math.*, **5** (2011), 22–36.
8. W. So, Integral circulant graphs, *Discrete Math.*, **306** (2006), 153–158.

Atom-molecule correspondence in Grothendieck categories

Ryo Kanda

The notion of Grothendieck category is a generalization of the category of modules over a ring. In this talk, we investigate two kinds of spectra of a Grothendieck category, the atom spectrum and the molecule spectrum, and obtain a new result on right noetherian rings.

For a Grothendieck category $\mathcal{A}$, the *atom spectrum* $\operatorname{ASpec}\mathcal{A}$ is the set of equivalence classes of monoform objects (see [2] and [3]), and the *molecule spectrum* $\operatorname{MSpec}\mathcal{A}$ is the set of equivalence classes of prime objects. For a right noetherian ring Λ, the category $\operatorname{Mod}\Lambda$ of right Λ-modules is a Grothendieck category. In this case, the atom spectrum $\operatorname{ASpec}(\operatorname{Mod}\Lambda)$ is in bijection with the set of isomorphism classes of indecomposable injective right Λ-modules, while the molecule spectrum $\operatorname{MSpec}(\operatorname{Mod}\Lambda)$ is in bijection with the set of two-sided prime ideals of Λ. Therefore $\operatorname{ASpec}\mathcal{A}$ and $\operatorname{MSpec}\mathcal{A}$ are regarded as the "one-sided spectrum" and the "two-sided spectrum" of the Grothendieck category $\mathcal{A}$, respectively.

The aim of this talk is to relate these spectra with each other. For a Grothendieck category $\mathcal{A}$ with a certain condition, we show that there exist canonical maps

$$\varphi\colon \operatorname{ASpec}\mathcal{A} \to \operatorname{MSpec}\mathcal{A} \quad\text{and}\quad \psi\colon \operatorname{MSpec}\mathcal{A} \to \operatorname{ASpec}\mathcal{A}$$

such that $\varphi\psi = \mathrm{id}$. These are Grothendieck-categorical generalizations of the maps given by Gabriel [1] for an arbitrary right noetherian ring. If the ring is commutative, the maps φ and ψ are bijective, and the atom spectrum and the molecule spectrum will be identified.

A key to further analysis is partial orders on the atom spectrum and the molecule spectrum. Both of them are generalizations of the inclusion relation between prime ideals of a commutative ring. We show that the maps φ and ψ are poset homomorphisms and that they induce a strong connection between the atom spectrum and the molecule spectrum.

Theorem 1. *Let $\mathcal{A}$ be a Grothendieck category having a noetherian generator and satisfying the Ab4* condition. Then the maps φ and ψ induces a bijection between the minimal elements of the atom spectrum* $\operatorname{ASpec}\mathcal{A}$ *and the minimal elements of the molecule spectrum* $\operatorname{MSpec}\mathcal{A}$.

This result can be applied to an arbitrary right noetherian ring and reveals a connection between indecomposable injective modules and two-sided prime ideals.

References

1. P. Gabriel, *Des catégories abéliennes*, Bull. Soc. Math. France **90** (1962), 323–448.
2. R. Kanda, *Classifying Serre subcategories via atom spectrum*, Adv. Math. **231** (2012), no. 3–4, 1572–1588.
3. ———, *Specialization orders on atom spectra of Grothendieck categories*, J. Pure Appl. Algebra, **219** (2015), no. 11, 4907–4952.

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Finite condition (Fg) for self-injective Koszul algebras

Ayako Itaba

For a finite-dimensional algebra Λ over an algebraically closed field k, Erdmann, Holloway, Taillefer, Snashall and Solberg ([2]) introduced certain finiteness conditions, denoted by (Fg), and showed that if Λ satisfies the finite condition (Fg), then the support varieties have many properties analogous to those for finite group algebras.

Definition 1. ([2]) A finite-dimensional algebra Λ satisfies the finite condition (Fg) if the following two conditions (Fg1) and (Fg2) hold:

- **(Fg1):** There is a graded subalgebra H of the Hochschild cohomology ring $\mathrm{HH}^*(\Lambda)$ of Λ such that H is a commutative Noetherian ring with $H^0 = \mathrm{HH}^0(\Lambda)(= \mathrm{Z}(\Lambda))$.
- **(Fg2):** The Ext algebra of Λ

$$\mathrm{E}(\Lambda) := \mathrm{Ext}^*_\Lambda(\Lambda/\mathrm{rad}\,\Lambda, \Lambda/\mathrm{rad}\,\Lambda) = \bigoplus_{r=0}^{\infty} \mathrm{Ext}^r_\Lambda(\Lambda/\mathrm{rad}\,\Lambda, \Lambda/\mathrm{rad}\,\Lambda)$$

  is a finitely generated H-module.

Here, the Hochschild cohomology ring of Λ is defined to be a graded ring

$$\mathrm{HH}^*(\Lambda) := \mathrm{Ext}^*_{\Lambda^\mathrm{e}}(\Lambda, \Lambda) = \bigoplus_{r=0}^{\infty} \mathrm{Ext}^r_{\Lambda^\mathrm{e}}(\Lambda, \Lambda),$$

where $\Lambda^\mathrm{e} := \Lambda \otimes_k \Lambda^\mathrm{op}$ is the enveloping algebra of Λ.

Let A be a graded algebra finitely generated in degree 1 over an algebraically closed field k. In [3], Mori defined a co-point module over A which is a dual notion of point module introduced by Artin, Tate and Van den Bergh ([1]) in terms of Koszul duality. A co-point module is parameterized by a subset E of a projective space. If M_p is a co-point module corresponding to a point $p \in E$, then ΩM_p is also a co-point module. Therefore, there exists a map $\sigma : E \longrightarrow E$ such that $\Omega M_p = M_{\sigma(p)}$. This pair (E, σ) is called a cogeometric pair and, when E is a projective scheme and σ is an automorphism of E, A is called cogeometric ([3]). For a relationship between a cogeometric pair (E, σ) and the finite condition (Fg), the following conjecture is proposed by Mori.

Conjecture Let A be a cogeometric self-injective Koszul algebra such that the complexity of $A/\mathrm{rad}\,A$ is finite. Then A satisfies the condition (Fg) if and only if the order of σ is finite.

In this talk, we show the following two theorems about the above Mori's conjecture.

Theorem 2. *If A is cogeometric and satisfies the condition* (Fg)*, then the order of σ is finite.*

Theorem 3. *If A satisfies* $(\mathrm{rad}\,A)^3 = 0$*, then A satisfies the condition* (Fg) *if and only if the order of σ is finite.*

References

1. M. Artin, J. Tate and M. Van den Bergh, Some algebras associated to automorphisms of elliptic curves, The Grothendieck Festschrift, vol. 1, Progress in Mathematics vol. 86 (Brikhäuser, Basel, 1990) 33–85.
2. K. Erdmann, M. Holloway, R. Taillefer, N. Snashall and Ø. Solberg, Support varieties for selfinjective algebras, K-Theory **33** (2004), no. 1, 67–87.
3. I. Mori, Co-point modules over Koszul algebras, J. London Math. Soc. **74** (2006), 639–656.

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Higher products on Yoneda Ext algebras.

Hiroyuki Minamoto

We would like to recall a basic fact of Homological algebra. The n-th extension group $\operatorname{Ext}^n_A(N, M)$ the n-th derived functor of Hom functor has a description

$$\operatorname{Ext}^n_A(N, M) = \{0 \to M \to X_1 \to X_2 \to \cdots \to X_n \to N \to 0\} / (\text{equivalence})$$

and that under this description, the multiplication on the Ext algebra $\operatorname{Ext}_A(M, M) = \bigoplus_{n \geq 0} \operatorname{Ext}^n_A(M, M)$ corresponds to splicing exact sequences which represents corresponding elements.

Since the Ext algebra $\operatorname{Ext}^n_A(M, M)$ is the cohomology algebra of the endomorphism dg-algebra $\mathbf{R}\operatorname{Hom}_A(P, P)$ for a projective (or injective) resolution P of M, it has more structure than merely a graded associative multiplication. These are A_∞-structure and (higher, matric) Massey products, which are not 2-ary operations but multi-ary operations and are collectively called higher products. Such structure was found in topology and has been studied in many area. Recently, higher products have been becoming to play important role in representation theory (see e.g., [3]).

Now we meet a simple question that under the above description of Ext algebra by exact sequences, what operations for exact sequences correspond to higher products. It is not so obvious at the first sight. For example, presence of triple product tells us that there exists a way to construct an exact sequence $0 \to M \to X \to Y \to M \to 0$ from three short exact sequences $0 \to M \to U \to M \to 0, 0 \to M \to V \to M \to 0, 0 \to M \to W \to M \to 0$.

In this talk, an answer is given for Massey products. Namely we show that Massey products on a Ext algebra can be computed in terms of exact sequences. As an application we provide a proof and generalization of the result due to Gugenheim-May and Keller which states that in a suitable situation the Ext algebra $E = \operatorname{Ext}_A(S, S)$ of the direct sum S of all simple modules is generated by the degree 1-part E^1 as an algebra with higher products.

Theorem 1 ([1, Corollary 5.17],[2, 2.2.1.(b)])**.** *Let A be a locally finite non-negatively graded algebra over a field and S be a direct sum of all simple modules. Then the extension algebra $\operatorname{Ext}_A(S, S)$ is generated by $\operatorname{Ext}^0_A(S, S)$ and $\operatorname{Ext}^1_A(S, S)$ using Massey products.*

In our proof we see that this proposition is a trivial consequence of the elementary fact that every finite length module has composition series. This theorem has the following corollary.

Corollary 2. *A connected locally finite non-negatively graded algebra A is Koszul if and only if all Massey products on $\operatorname{Ext}^n_A(S, S)$ vanish.*

We can also prove the following theorem.

Theorem 3. *Let R be a Noetherian algebra with the center $Z = Z(R)$ and $\mathfrak{p}$ a maximal ideal of Z. We set $\kappa(\mathfrak{p}) := R \otimes_Z Z_{\mathfrak{p}}/\mathfrak{p}Z_{\mathfrak{p}}$. Then the Ext algebra $E = \operatorname{Ext}_R(\kappa(\mathfrak{p}), \kappa(\mathfrak{p}))$ is generated by E^0 and E^1 using Massey products.*

References

1. Gugenheim, V. K. A. M.; May, J. Peter On the theory and applications of differential torsion products. Memoirs of the American Mathematical Society, No. 142. American Mathematical Society, Providence, R.I., 1974. ix+94 pp.
2. Keller, Bernhard A -infinity algebras in representation theory. Representations of algebra. Vol. I, II, 74-86, Beijing Norm. Univ. Press, Beijing, 2002.
3. Koenig, Steffen; Kulshammer, Julian; Ovsienko, Sergiy Quasi-hereditary algebras, exact Borel subalgebras, A ∞ -categories and boxes. Adv. Math. 262 (2014), 546-592.

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Filtered categories and representations of boxes

Steffen Koenig

Standard objects occur in algebraic Lie theory, for instance as Verma or Weyl modules, and in algebraic geometry in exceptional collections. By Dlab and Ringel's standardisation theorem, all of these examples fit into the axiomatic setup of quasi-hereditary algebras, which are defined by existence and properties of standard modules. The relevant category both for applications and for much of the structure theory of quasi-hereditary algebras is the category with objects having finite filtrations by standard objects. This 'filtered category' is exact, but rarely abelian.

The main result of [1] describes the filtered category as category of representations of a box, where a box is a generalisation of an algebra. In this lecture, this result and its proof will be explained and some applications will be mentioned (PBW theorem for quasi-hereditary algebras, Ringel duality, ...).

References

1. S. Koenig, J. Külshammer, S. Ovsienko *Quasi-hereditary algebras, exact Borel subalgebras, A_∞-categories and boxes*, Adv. Math. **262** (2014), 546–592.

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Report on the 7th China-Japan-Korea International Conference on Ring Theory

Masahisa Sato

This lecture is a report on the 7th China-Japan-Korea International Conference on Ring Theory held in Zhejiang University of Technology and Zhejiang University in Hangzhou (杭州にある浙江工科大学および浙江大学) from July 1st to 7th.

The topics in this lecture is the following.

(1) About what kind place Hangzhou is:
Introduction to Universities and Impression West Lake

(2) On the history of China-Japan-Korea International Conference on Ring Theory in the twenty four years for young Japanese ring theorists with mathematics and mathematicians in China

(3) Trend topics in the 7th China-Japan-Korea International Conference on Ring Theory
This includes about derived and triangulated categories, Gorenstein projective modules, derivations.
Also outline of some invited lectures will be introduced.

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THE GROTHENDIECK GROUPS OF MESH ALGEBRAS

Sota Asai

The representation theory of finite-dimensional algebras investigates relationships between two finite-dimensional algebras A and B. One of the important relationships is *derived equivalence*, that is, the derived categories $D^{\mathrm{b}}(\mathrm{mod}\, A)$ and $D^{\mathrm{b}}(\mathrm{mod}\, B)$ are equivalent as triangulated categories.

We assume A and B are self-injective. We can consider another important relationship, *stable equivalence* for two finite-dimensional self-injective algebras, that is $\underline{\mathrm{mod}}\, A \cong \underline{\mathrm{mod}}\, B$ as triangulated categories. Rickard showed if A and B are derived equivalent, then A and B are stable equivalent in [5] and [6].

Invariants under these equivalences have been actively investigated. One of them is *Grothendieck groups*. For a finite-dimensional algebra A, the Grothendieck group $K_0(D^{\mathrm{b}}(\mathrm{mod}\, A))$ is a free abelian group with a basis given by the non-isomorphic simple A-modules.

If A is self-injective, $K_0(\underline{\mathrm{mod}}\, A)$, the Grothendieck group of the stable category, can be also considered. For the calculation, the category equivalence $\underline{\mathrm{mod}}\, A \cong D^{\mathrm{b}}(\mathrm{mod}\, A)/K^{\mathrm{b}}(\mathrm{proj}\, A)$ is crucial; namely $K_0(\underline{\mathrm{mod}}\, A)$ is the quotient of the free abelian group $K_0(D^{\mathrm{b}}(\mathrm{mod}\, A))$ by the subgroup generated by $[P_1], \ldots, [P_m]$, where $P_1, \ldots, P_m$ are the non-isomorphic indecomposable projective A-modules, and $[P_i]$ denotes the image of P_i in $K_0(D^{\mathrm{b}}(\mathrm{mod}\, A))$. $K_0(\underline{\mathrm{mod}}\, A)$ is not generally a free abelian group, and is useful for the classification of self-injective algebras by stable equivalence.

In this talk, I am going to deal with *mesh algebras* as a class of self-injective algebras. It is a special case of more general concept, *mesh categories*.

The *Auslander-Reiten quiver* (AR quiver) of a derived category is another invariant for derived equivalence (see [3] and [4] for detail), and in many important cases, a derived category can be recovered from its AR quiver as a mesh category, for example, derived categories of path algebras of Dynkin quivers [3].

Riedtmann introduced the mesh category $K(Q)$ for a *translation quiver* Q, and it plays an important role in the reprensation theory. Hence, we can study algebras via mesh categories.

Especially, I am interested in finite-dimensional mesh algebras of *stable translation quivers* and their stable equivalence. The quiver has a form $\boldsymbol{Z}\Delta/G$, where Δ is a Dynkin diagram, $\boldsymbol{Z}\Delta$ is the infinite translation quiver and G is a subgroup of $\mathrm{Aut}(\boldsymbol{Z}\Delta)$, by Riedtmann's structure theorem [7]. There are 10 types of such quivers,

$\boldsymbol{Z}A_n/\langle\tau^k\rangle$, $\boldsymbol{Z}A_n/\langle\tau^k\psi\rangle$ (n: odd, $\psi^2 = \mathrm{id}$), $\boldsymbol{Z}A_n/\langle\tau^k\varphi\rangle$ (n: even, $\varphi^2 = \tau^{-1}$),
$\boldsymbol{Z}D_n/\langle\tau^k\rangle$ ($n \geq 4$), $\boldsymbol{Z}D_n/\langle\tau^k\psi\rangle$ ($n \geq 4$, $\psi^2 = \mathrm{id}$), $\boldsymbol{Z}D_4/\langle\tau^k\chi\rangle$ ($\chi^3 = \mathrm{id}$),
$\boldsymbol{Z}E_6/\langle\tau^k\rangle$, $\boldsymbol{Z}E_6/\langle\tau^k\psi\rangle$ ($\psi^2 = \mathrm{id}$), $\boldsymbol{Z}E_7/\langle\tau^k\rangle$, $\boldsymbol{Z}E_8/\langle\tau^k\rangle$.

I am trying to classify these mesh algebras by derived equivalence and stable equivalence, using Grothendieck groups as one of the invariants. The calculation of the Grothendieck groups is hard, but paying attention to the projective resolution of each mesh algebra A as A-A-bimodule (see [2]) make it much easier. In this talk, I am going to introduce these results and explain how they are deduced briefly, mainly for the most basic type $\boldsymbol{Z}A_n\langle\tau^k\rangle$. This talk is based on [1] and the progress after that.

References

1. S. Asai, *The Grothendieck groups of mesh algebras: type A_n*, arXiv:1505.06983.
2. A. Dugas, *Resolutions of mesh algebras: periodicity and Calabi-Yau dimensions*, Math. Z., 271 (2012), 1151–1184.
3. D. Happel, *On the derived category of a finite-dimensional algebra*, Comment. Math. Helv., 62 (1987), 339–389.
4. D. Happel, *Triangulated Categories in the Representation Theory of Finite Dimensional Algebras*, London Mathematical Society Lecture Note Series, 119 (1988), Cambridge University Press.
5. J. Rickard, *Derived categories and stable equivalence*, Journal of Pure and Applied Algebra, 61 (1989), Issue 3, 303–317.
6. J. Rickard, *Morita Theory for Derived Categories*, J. London Math. Soc., s2-39 (3) (1989) , 436–456.
7. C. Riedtmann, *Algebren, Darstellungsköcher, Überlagerungen und zurück* (German), Comment. Math. Helv. 55 (1980), no. 2, 199–224.

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Stable degenerations of Cohen-Macaulay modules over simple singularities of type (A_n)

Naoya Hiramatsu

The concept of degenerations of modules introduced in representation theory for studying the structure of the module variety over a finite dimensional algebra. Classically Bongartz [1] investigated the degeneration problem of modules over an artinian algebra in relation with the Auslander-Reiten quiver. In [8], Zwara gave a complete description of degenerations of modules over representation finite algebras by using some order relations for modules known as the hom order, the degeneration order and the extension order. Now a theory of degenerations is considered for not only module categories, but derived categories [4] or stable categories [7], more generally, triangulated categories [5].

Let R be a commutative Gorenstein local k-algebra which is not necessary finite dimensional. Yoshino [7] introduced a notion of the stable analogue of degenerations of (maximal) Cohen-Macaulay R-module in the stable category $\underline{\mathrm{CM}}(R)$. The notion of the stable degenerations is closely related to the ordinary degenerations. In fact, the author [3] give a complete description of degenerations of Cohen-Macaulay modules over a ring of even dimensional simple singularity of type (A_n) by using the description of stable degenerations over it. Hence it is also important for the study of degeneration problem to investigate the description of stable degenerations.

The purpose of this paper is to describe stable degenerations of Cohen-Macaulay modules over simple singularities of type (A_n).

$$k[[x_0, x_1, x_2, \cdots, x_d]]/(x_0^{n+1} + x_1^2 + x_2^2 + \cdots + x_d^2).$$

First we consider an order relation on $\underline{\mathrm{CM}}(R)$ which is the stable analogue of the hom order.

Theorem 1. *Let R be of finite representation type and M and N be Cohen-Macaulay R-modules. Suppose that* $\dim \underline{\mathrm{Hom}}_R(X, M) = \dim \underline{\mathrm{Hom}}_R(X, N)$ *for each* $X \in \underline{\mathrm{CM}}(R)$. *Then* $\underline{M} \oplus \underline{\Omega M} \cong \underline{N} \oplus \underline{\Omega N}$.

As a corrollary, we can show that the stable hom order $\leq_{\underline{hom}}$ is a partial order on $\underline{\mathrm{CM}}(R)$ if n is of an odd odd dimension.

By using the stable analogue of the argument over finite dimensional algebras in [8], we can describe stable degenerations of Cohen-Macaulay modules over the ring.

Theorem 2. *Let R be a simple singularity of type (A_n). For Cohen-Macaulay R-modules M and N with* $[\underline{M}] = [\underline{N}]$ *in* $K_0(\underline{\mathrm{CM}}(R))$, *the following statements hold.*

(1) *If R is of odd dimension then $\underline{M} \leq_{st} \underline{N}$ if and only if $\underline{M} \leq_{\underline{hom}} \underline{N}$.*
(2) *If R is of even dimension then $\underline{M} \leq_{st} \underline{N}$ if and only if $\underline{M} \leq_{tri} \underline{N}$.*

References

1. K. Bongartz, *On degenerations and extensions of finite-dimensional modules.* Adv. Math. **121** (1996), 245–287.
2. N. Hiramatsu, *On stable degenerations of Cohen-Macaulay modules over simple singularities of type (A_n)*, arXiv:1501.05027.
3. N. Hiramatsu and Y. Yoshino, *Examples of degenerations of Cohen-Macaulay modules*, Proc. Amer. Math. Soc. **141** (2013), no. 7, 2275–2288.
4. B. Jensen, X, Su and A. Zimmermann, *Degenerations for derived categories.* J. Pure Appl. Algebra **198** (2005), no. 1-3, 281–295.
5. M. Saorin and A. Zimmermann, *An axiomatic approach for degenerations in triangulated categories.* Preprint.
6. Y. Yoshino, *Cohen-Macaulay Modules over Cohen-Macaulay Rings*, London Mathematical Society Lecture Note Series **146**. Cambridge University Press, Cambridge, 1990. viii+177 pp.
7. ———, *Stable degenerations of Cohen-Macaulay modules*, J. Algebra **332** (2011), 500–521.
8. G. Zwara, *Degenerations for modules over representation-finite algebras.* Proc. Amer. Math. Soc. **127** (1999), 1313–1322.

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Tilting theory of preprojective algebras and c-sortable elements

Yuta Kimura

Let Q be a finite acyclic quiver. The preprojective algebra Π of Q has been introduced by Gelfand-Ponomarev to study representation theory of the path algebra of Q. Preprojective algebras play important roles in many areas of mathematics. One of them is that preprojective algebras provide 2-Calabi-Yau triangulated categories with cluster tilting objects which have been studied in the view point of categorification of cluster algebras.

Let W be the Coxeter group of Q. In [4], Buan-Iyama-Reiten-Scott constructed an algebra Π_w for an element w in W, where Π_w is a factor algebra of Π by an ideal determined by w, and they showed that Π_w is an Iwanaga-Gorenstein algebra of dimension at most one, that is, injective dimensions of Π_w as both a left module and a right module are at most one. Therefore the category of submodules of finitely generated free Π_w-modules $\mathsf{Sub}\,\Pi_w$ is a Frobenius category. They proved that the stable category $\underline{\mathsf{Sub}}\,\Pi_w$ of $\mathsf{Sub}\,\Pi_w$ is a 2-Calabi-Yau triangulated category and there exist cluster tilting objects in $\underline{\mathsf{Sub}}\,\Pi_w$.

There exists a connection between $\underline{\mathsf{Sub}}\,\Pi_w$ and generalized cluster categories introduced by Amiot [1]. In [2], the authors showed that for any element w in W, there exists an algebra A_w of global dimension at most two such that $\underline{\mathsf{Sub}}\,\Pi_w$ is triangle equivalent to the generalized cluster category of A_w.

Recently, the speaker showed the derived category version of the result of Amiot-Reiten-Todorov when w is a c-sortable element. For this purpose, we regard the preprojective algebra Π as a $\mathbb{Z}$-graded algebra. The grading of Π induces a grading of Π_w. Since Π_w is an Iwanaga-Gorenstein algebra of dimension at most one, the category of graded submodules of finitely generated free graded Π_w-modules $\mathsf{Sub}^{\mathbb{Z}}\Pi_w$ is also a Frobenius category. The speaker showed the following theorem. Let $\underline{\mathsf{Sub}}^{\mathbb{Z}}\Pi_w$ be the stable category of $\mathsf{Sub}^{\mathbb{Z}}\Pi_w$.

Theorem 1. *If w is a c-sortable element, then the category $\underline{\mathsf{Sub}}^{\mathbb{Z}}\Pi_w$ has a tilting object M_w.*

By Keller's theorem, $\underline{\mathsf{Sub}}^{\mathbb{Z}}\Pi_w$ is triangle equivalent to the bounded homotopy category of finitely generated projective B_w-modules, where B_w is an endomorphism algebra of M_w in $\underline{\mathsf{Sub}}^{\mathbb{Z}}\Pi_w$. In this pint of view, it is important to study B_w, in particular the global dimension of B_w.

In this talk, we give a simple description of B_w by using the path algebra kQ of Q, and by using it, we see that the global dimension of B_w is finite. By [3], for any c-sortable element w in W, there exists a unique titling kQ-module T_w such that the category of submodules of finite direct sums of T_w has an additive generator X_w. Then we have the following result.

Theorem 2. *The following holds.*

(a) *There exists an isomorphism of algebras $B_w \simeq \mathsf{End}_{kQ}(X_w)/[T_w]$.*
(b) *The global dimension of $\mathsf{End}_{kQ}(X_w)/[T_w]$ is at most two.*

As a corollary, we have a desired result.

Corollary 3. *We have a triangle equivalence $\underline{\mathsf{Sub}}^{\mathbb{Z}}\Pi_w \simeq \mathsf{D}^{\mathsf{b}}(\mathsf{mod}\,B_w)$.*

References

1. C. Amiot, *Cluster categories for algebras of global dimension 2 and quivers with potential*, Ann. Inst. Fourier (Grenoble) 59 (2009), no. 6, 2525-2590.
2. C. Amiot, I. Reiten, G. Todorov, *The ubiquity of the generalized cluster categories*, Adv. Math. 226 (2011), no. 4, 3813-3849.
3. C. Amiot, O. Iyama, I. Reiten, G. Todorov, *Preprojective algebras and c-sortable words*, Proc. Lond. Math. Soc. (3) 104 (2012), no. 3, 513-539.
4. A. Buan, O. Iyama, I. Reiten, J. Scott, *Cluster structures for 2-Calabi-Yau categories and unipotent groups*, Compos. Math. 145 (2009), no. 4, 1035-1079.

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Tilting objects for noncommutative quotient singularities

Kenta Ueyama

Tilting objects play a key role in the study of triangulated categories. They often enable us to realize abstract triangulated categories as concrete derived categories of modules over algebras. One of the remarkable results on the existence of tilting objects has been obtained by Iyama and Takahashi.

Theorem 1 ([1, Theorem 2.7, Corollary 2.10]). *Let $S = k[x_1, \ldots, x_d]$ be a polynomial algebra over an algebraically closed field k of characteristic 0 such that $\deg x_i = 1$ and $d \geq 2$. Let G be a finite subgroup of $\mathrm{SL}(d, k)$ acting linearly on S. If the fixed subalgebra S^G is an isolated singularity, then the stable category $\underline{\mathsf{CM}}^{\mathbb{Z}}(S^G)$ of graded maximal Cohen-Macaulay modules has a tilting object.*

As a consequence, there exists a finite dimensional algebra Γ of finite global dimension such that

$$\underline{\mathsf{CM}}^{\mathbb{Z}}(S^G) \cong \mathsf{D}^{\mathsf{b}}(\mathsf{mod}\,\Gamma).$$

Since AS-regular algebras are noncommutative analogues of polynomial rings, it is natural to ask if a version of Iyama and Takahashi's theorem holds for AS-regular algebras. In this talk, we show that the answer to this question is positive, that is, an AS-regular version of Iyama and Takahashi's theorem holds.

This is a joint work with Izuru Mori [2].

References

1. O. Iyama and R. Takahashi, Tilting and cluster tilting for quotient singularities, *Math. Ann.* **356** (2013), 1065–1105.
2. I. Mori and K. Ueyama, Stable categories of graded maximal Cohen-Macaulay modules over noncommutative quotient singularities, preprint, `arXiv:1507.06377`.

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