Two-sided tilting complexes and folded tree-to-star complexes

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Motivation and Background

Fact on Derived Equivalences (Rickard)

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- Γ and Λ are derived equivalent.

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T : one-sided tilting complex over Γ

$\Lambda := \mathrm{End}_{D^b(\Gamma)}(T)^{op}$

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Then there exists a two-sided tilting complex C of Γ-Λ-bimodules satisfying $C \cong T$ in $D^b(\Gamma)$.

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(1) Can we construct two-sided tilting complex C of Γ-Λ-bimodules satisfying $C \cong T$ in $D^b(\Gamma)$ by using concrete bimodules?

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(1) Can we construct two-sided tilting complex C of Γ-Λ-bimodules satisfying $C \cong T$ in $D^b(\Gamma)$ by using concrete bimodules?

(2) Given an operation on a one-sided tilting complex T which produces another one-sided tilting complex T', then can we get an operation on the two-sided tilting complex C which produces a two-sided tilting complex C' satisfying $C' \cong T'$ in $D^b(\Gamma)$?

$$\begin{array}{ccc} {}_{\Gamma}C_{\Lambda} & \xrightarrow{\exists ?\,\text{an operation}} & {}_{\Gamma}C'_{\Lambda} \\ \text{restriction to } \Gamma \downarrow & & \downarrow \text{restriction to } \Gamma \\ {}_{\Gamma}T & \xrightarrow{\text{an operation}} & {}_{\Gamma}T' \end{array}$$

Previous work

Classification of Brauer tree algebras (Rickard)

Up to derived equivalence, a Brauer tree algebra is determined by the number of edges and multiplicity of Brauer tree.

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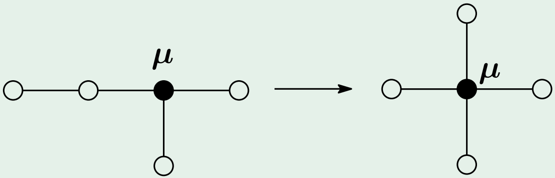
The proof of this result is done by constructing a tree-to-star complex, that is a one-sided tilting complex with endomorphism ring Brauer star algebra.

Example



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Theorem (Rickard-Schaps)

T : Rickard tree-to-star complex
T' : complex obtained by applying foldings to T several times
Then T' is a tree-to-star complex again.

$$T = T_0 \xrightarrow{\text{folding}} T_1 \xrightarrow{\text{folding}} \cdots \xrightarrow{\text{folding}} T_n = T'$$

Main Result

A : Brauer tree algebra with e edges and multiplicity μ
B : Brauer star algebra with e edges and multiplicity μ
T : Rickard tree-to-star complex

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Theorem (K-Kunugi), (K)

(1) We can construct a two-sided tilting complex C of A-B-bimodules satisfying $C_{\downarrow A} \cong T$ from a minimal projective resolution of M as an A-B-bimodule.

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(2) We can realize foldings as operations of two-sided tilting complexes of A-B-bimodules.

$$\begin{array}{ccccccc} C = C_0 & \longrightarrow & C_1 & \longrightarrow & \cdots & \longrightarrow & C_n \\ \downarrow & & \downarrow & & & & \downarrow \\ T = T_0 & \xrightarrow{\text{folding}} & T_1 & \xrightarrow{\text{folding}} & \cdots & \xrightarrow{\text{folding}} & T_n \end{array}$$

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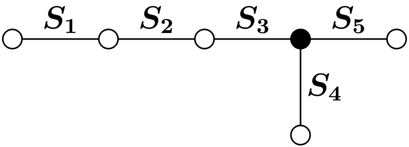

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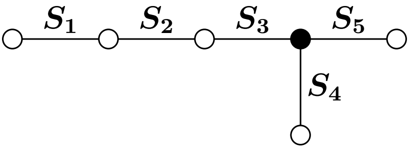

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$$T = (3P(S_3) \oplus P(S_4) \oplus P(S_5) \to 2P(S_2) \to P(S_1))$$

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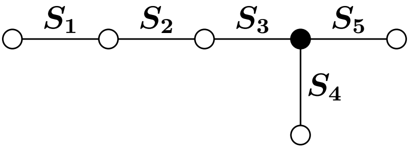

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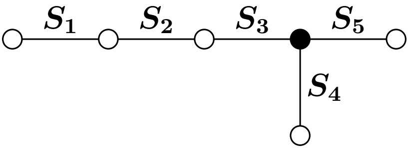

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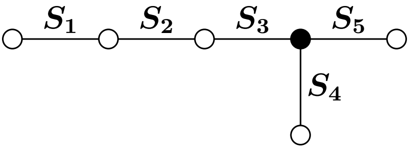

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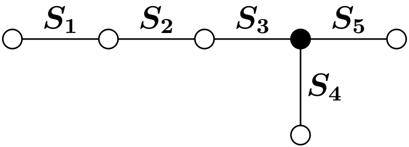

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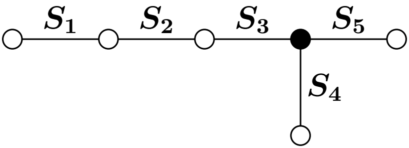

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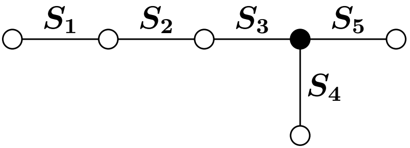

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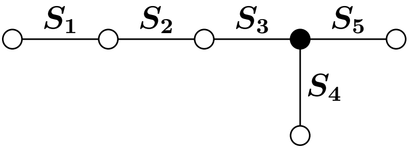

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We will construct a two-sided tilting complex C_i satisfying $C_i \cong T_i$ in $D^b(A)$ for each $i \in \{\emptyset, 1, 2\}$.

Example

V_i : simple B-module satisfying $S_i \cong \mathbf{top}\,(M \otimes_B V_i)$.
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Example

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$\downarrow$ folding

$$T_1 = (2P(S_2) \to P(S_1) \oplus 3P(S_3) \oplus P(S_4) \oplus P(S_5))$$

$$C_1 : \quad \begin{array}{c} \\ \\ P(S_4) \otimes P(V_3)^* \\ \oplus \\ P(S_5) \otimes P(V_4)^* \\ \oplus \\ P(S_3) \otimes P(V_5)^* \end{array} \quad \to \quad \begin{array}{c} P(S_2) \otimes P(V_2)^* \\ \oplus \\ P(S_3) \otimes P(V_3)^* \\ \oplus \\ P(S_4) \otimes P(V_4)^* \\ \oplus \\ P(S_5) \otimes P(V_5)^* \end{array} \quad \to {}_A M_B$$

Example

Remark

Since $C_1 = (P(S_2) \otimes P(V_2)^* \to M)$ is isomorphic to $(P(S_2) \otimes P(V_2)^* \to I(M) \twoheadrightarrow \Omega^{-1}M)$ in $D^b(A)$, we can write C_1 as follows:

$$C_1 : \quad P(S_2) \otimes P(V_2)^* \to \begin{array}{c} P(S_1) \otimes P(V_1)^* \\ \oplus \\ P(S_3) \otimes P(V_2)^* \\ \oplus \\ P(S_2) \otimes P(V_3)^* \\ \oplus \\ P(S_4) \otimes P(V_4)^* \\ \oplus \\ P(S_5) \otimes P(V_5)^* \end{array} \to {}_A\Omega^{-1}M_B$$

Example

Remark

Since $C_1 = (P(S_2) \otimes P(V_2)^* \to M)$ is isomorphic to $(P(S_2) \otimes P(V_2)^* \to I(M) \twoheadrightarrow \Omega^{-1}M)$ in $D^b(A)$, we can write C_1 as follows:

$$C_1 : \quad P(S_2) \otimes P(V_2)^* \to \begin{matrix} P(S_1) \otimes P(V_1)^* \\ \oplus \\ P(S_3) \otimes P(V_2)^* \\ \oplus \\ P(S_2) \otimes P(V_3)^* \\ \oplus \\ P(S_4) \otimes P(V_4)^* \\ \oplus \\ P(S_5) \otimes P(V_5)^* \end{matrix} \to {}_A\Omega^{-1}M_B$$

Example

$$T_1 = (2P(S_2) \to P(S_1) \oplus 3P(S_3) \oplus P(S_4) \oplus P(S_5))$$

$\downarrow$ folding

$$T_2 = (P(S_1) \oplus 3P(S_3) \oplus P(S_4) \oplus P(S_5) \to 2P(S_2))$$

$$C_1 : \quad P(S_2) \otimes P(V_2)^* \to \begin{array}{c} P(S_1) \otimes P(V_1)^* \\ \oplus \\ P(S_3) \otimes P(V_2)^* \\ \oplus \\ P(S_2) \otimes P(V_3)^* \\ \oplus \\ P(S_4) \otimes P(V_4)^* \\ \oplus \\ P(S_5) \otimes P(V_5)^* \end{array} \to {}_A\Omega^{-1}M_B$$

Example

$$T_1 = (2P(S_2) \to P(S_1) \oplus 3P(S_3) \oplus P(S_4) \oplus P(S_5))$$

$\downarrow$ folding

$$T_2 = (P(S_1) \oplus 3P(S_3) \oplus P(S_4) \oplus P(S_5) \to 2P(S_2))$$

$$C_1 : \quad P(S_2) \otimes P(V_2)^* \to \begin{matrix} P(S_1) \otimes P(V_1)^* \\ \oplus \\ P(S_3) \otimes P(V_2)^* \\ \oplus \\ P(S_2) \otimes P(V_3)^* \\ \oplus \\ P(S_4) \otimes P(V_4)^* \\ \oplus \\ P(S_5) \otimes P(V_5)^* \end{matrix} \to {}_A\Omega^{-1} M_B$$

Example

$$T_1 = (2P(S_2) \to P(S_1) \oplus 3P(S_3) \oplus P(S_4) \oplus P(S_5))$$

$\downarrow$ folding

$$T_2 = (P(S_1) \oplus 3P(S_3) \oplus P(S_4) \oplus P(S_5) \to 2P(S_2))$$

$$C_1 : \quad P(S_2) \otimes P(V_2)^* \to \begin{array}{c} P(S_1) \otimes P(V_1)^* \\ \oplus \\ P(S_3) \otimes P(V_2)^* \\ \oplus \\ P(S_2) \otimes P(V_3)^* \\ \oplus \\ P(S_4) \otimes P(V_4)^* \\ \oplus \\ P(S_5) \otimes P(V_5)^* \end{array} \to {}_A\Omega^{-1}M_B$$

Example

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$$T_2 = (P(S_1) \oplus 3P(S_3) \oplus P(S_4) \oplus P(S_5) \to 2P(S_2))$$

$$C_1 : \quad P(S_2) \otimes P(V_2)^* \to \begin{matrix} P(S_1) \otimes P(V_1)^* \\ \oplus \\ P(S_3) \otimes P(V_2)^* \\ \oplus \\ P(S_2) \otimes P(V_3)^* \\ \oplus \\ P(S_4) \otimes P(V_4)^* \\ \oplus \\ P(S_5) \otimes P(V_5)^* \end{matrix} \to {}_A\Omega^{-1}M_B$$

Example

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$$T_2 = (P(S_1) \oplus 3P(S_3) \oplus P(S_4) \oplus P(S_5) \to 2P(S_2))$$

$$C_2 : \quad \begin{array}{ccccc} & & P(S_1) \otimes P(V_1)^* & & \\ & & \oplus & & \\ P(S_2) \otimes P(V_2)^* & & P(S_3) \otimes P(V_2)^* & & \\ & & \oplus & & \\ & \to & P(S_2) \otimes P(V_3)^* & \to & {}_A\Omega^{-1} M_B \\ & & \oplus & & \\ & & P(S_4) \otimes P(V_4)^* & & \\ & & \oplus & & \\ & & P(S_5) \otimes P(V_5)^* & & \end{array}$$