Infinite sequences of Frobenius extensions

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A/R : a ring extension A of R

Definition 1 (Nakayama-Tsuzuku, 1960).
A/R : a Frobenius extension of first kind
$\overset{\text{def}}{\Longleftrightarrow}$

(F1) A_R : f.g. proj.

(F2) ${}_R A_A \cong_R \mathrm{Hom}_R(A, R)_A$

Theorem 2. $\forall$ Frobenius extension of first kind A/R

$\exists$ a sequence of ring extensions

$$A_0 = R \subset A_1 = A \subset \cdots \subset A_n \subset \cdots$$

s.t. A_{i+1}/A_i : Frobenius extension of first kind for all $i \geq 0$.

In fact, $A_{i+1} = \mathrm{End}_{A_{i-1}}(A_i)$.

Theorem 3. R : any ring

P_R : f.g. proj.

$Q := \mathrm{Hom}_R(P, R)$

$\Lambda := \mathrm{End}_R(P)$

$A \subset \Lambda$ subring s.t.

(1) Q_A : f.g. proj.

(2) ${}_A P_R \cong_A \mathrm{Hom}_A(Q, A)_R$

$\Rightarrow \Lambda / A$: Frobenius extension of first kind.

Proof. $\boldsymbol{\Lambda = \mathrm{Hom}_R(P, P) \cong P \otimes_R \mathrm{Hom}_R(P, R) \cong P \otimes_R Q}$ **as** $\boldsymbol{\Lambda}$**-bimodules.**

$\boldsymbol{\therefore \Lambda_A \cong P \otimes_R Q_A}$ **: f.g. proj.**

$\boldsymbol{\mathrm{Hom}_A(\Lambda, A) \cong \mathrm{Hom}_A(P \otimes_R Q, A) \cong \mathrm{Hom}_R(P, \mathrm{Hom}_A(Q, A)) \cong \mathrm{Hom}_R(P, P) \cong \Lambda}$ **as** $\boldsymbol{(A, \Lambda)}$**-bimodules.** □