The 50th Symposium on Ring Theory

and Representation Theory

ABSTRACT

University of Yamanashi, Kofu, Yamanashi

October 7–10, 2017

(September 6, 2017; Revised September 26, 2017)

Program

* Short talks

October 7 (Saturday)

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A necessary condition for two commutative noetherian rings to be singularly equivalent

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Localization functors in derived categories of commutative Noetherian rings

***10:30–10:40** Tokuji Araya (Okayama University of Science)
On the vanishing of self extensions over Cohen-Macaulay local rings

***10:40–10:50** Toshinori Kobayashi (University of Nagoya)
On delta invariants of certain ideals

***10:50–11:00** Minjae Kwon (Kyungpook National University)
The Krull dimension of composite power series rings over valuation rings

11:20–12:10 Robert Wisbauer (Heinrich Heine Universität Düsseldorf)
Around Azumaya rings - An overview of ring theory in the last decades

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On quiver Grassmannians and orbit closures for gen-finite modules

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The moduli of subalgebras of the full matrix ring of degree 3

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Two-sided tilting complexes and folded tree-to-star complexes

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Singularity categories and silting objects

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* Short talks

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d-representation-finite self-injective algebras

***10:30–10:40** Sota Asai (Nagoya University)
Bricks over preprojective algebras

***10:40–10:50** Toshiya Yurikusa (Nagoya University)
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***10:50–11:00** Osamu Iyama (Nagoya University)
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Strongly quasi-hereditary algebras and rejective subcategories

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Two constructions of Iwanaga-Gorenstein algebras

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Sigma-pure-injective modules for string algebras

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The Feit -Thompson conjecture is true

14:45–15:15 Gangyong Lee (Chungnam National University), Mauricio Medina-Bárcenas (Chungnam National University) and Khanh Tung Nguyen (Vietnam National University)
The number of partial matrix rings

15:30–16:00 Hideyuki Koie, Tomohiro Itagaki and Katsunori Sanada (Tokyo University of Science)
The ordinary quivers of Hochschild extension algebras for self-injective Nakayama algebras

16:15–16:45 Isao Kikumasa (Yamaguchi University), Kazutoshi Koike (Okinawa National College of Technology) and Kiyoichi Oshiro (Yamaguchi University)
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On finitely graded IG-algebas and the stable categories of their (graded) CM-modules

9:45–10:15 Laurent DEMONET (Nagoya University)
Lattice of torsion classes

10:30–11:00 Takahide Adachi (Osaka Prefecture University) and Mizuno Yuya (Shizuoka University)
How to capture t-structures by silting theory

11:20–12:10 William Crawley-Boevey (Universität Bielefeld)
Representations of equipped graphs

A necessary condition for two commutative noetherian rings to be singularly equivalent

Hiroki Matsui

Let R be a commutative noetherian ring. The *singularity category* of R is by definition the Verdier quotient

$$\mathsf{D_{sg}}(R) := \mathsf{D^b}(\mathsf{mod}R)/\mathsf{K^b}(\mathsf{proj}R).$$

where $\mathsf{D^b}(\mathsf{mod}R)$ stands for the bounded derived category of finitely generated R-modules, and $\mathsf{K^b}(\mathsf{proj}R)$ the bounded homotopy category of finitely generated projective R-modules. The singularity category has the structure of a triangulated category which has been introduced by Buchweitz [3] and connected to the Homological Mirror Symmetry Conjecture by Olrov [5].

For two commutative noetherian rings R and S, we say that R, S are *singularly equivalent* if their singularity categories $\mathsf{D_{sg}}(R), \mathsf{D_{sg}}(S)$ are equivalent as triangulated categories. It is well known that Morita equivalences and derived equivalences imply singularly equivalences. Complete characterization for Morita equivalences and derived equivalences are kown [4, 6], however singularly equivalences are quite difficult. Indeed, only a few examples of such equivalences are known.

The main purpose of this talk is give the following necessary condition for singularly equivalences:

Theorem 1. *Let R and S be local complete intersection rings which are locally hypersurfaces on the punctured spectra. If R and S are singularly equivalenct, then their singular loci $\mathsf{Sing}R$ and $\mathsf{Sing}S$ are homeomorphic.*

The key to prove this theorem is the *support theory* for triangulated categories. The support theory is developed by P. Balmer [1, 2] for "tensor" triangulated categories and is a powerful tool to show such a reconstruction theorem. Since singularity categories do not have tensor triangulated structure in general, we discuss the *support theory* "without" tensor structure.

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Localization functors in derived categories of commutative Noetherian rings

Tsutomu Nakamura

This talk is based on joint work with Y. Yoshino [3]. Let R be a commutative Noetherian ring. We denote by $\mathcal{D} = D(\mathrm{Mod}\ R)$ the derived category of all chain complexes of R-modules. In this talk, we introduce the notion of localization functors with cosupport in subsets of $\mathrm{Spec} R$. Recall that the cosupport of $X \in \mathcal{D}$ is defined by $\mathrm{cosupp}\, X = \{\, \mathfrak{p} \in \mathrm{Spec}\, R \mid \mathrm{RHom}_R(\kappa(\mathfrak{p}), X) \neq 0 \,\}$, where $\kappa(\mathfrak{p}) = R_\mathfrak{p}/\mathfrak{p}R_\mathfrak{p}$. For a subset of $\mathrm{Spec}\, R$, we write $\mathcal{C}^W = \{\, Y \in \mathcal{D} \mid \mathrm{cosupp}\, Y \subset W \,\}$, which is a colocalizing subcategory of $\mathcal{D}$. By some argument of localization theory of triangulated categories, there exists a left adjoint funtor $\lambda^W : \mathcal{D} \to \mathcal{C}^W$ to the inclusion functor $j^W : \mathcal{C}^W \hookrightarrow \mathcal{D}$, see [2]. We define the localization functor with cosupport in W as the functor λ^W. This notion is a common generalization of classical localizations $(-) \otimes_R S^{-1}R$ with respect to multiplicatively closed subsets S of R and left derived functors $\mathrm{L}\Lambda^{V(\mathfrak{a})}$ of $\mathfrak{a}$-adic completion functors $\Lambda^{V(\mathfrak{a})}$ for ideals $\mathfrak{a}$ of R.

In this talk, we will report several properties of λ^W, including the two results bellow.

Theorem 1. *Let W be a subset of $\mathrm{Spec}\, R$. Assume that there is no inclusion relation between two distinct prime ideals in W. Then λ^W is isomorphic to $\prod_{\mathfrak{p} \in W} \mathrm{L}\Lambda^{V(\mathfrak{p})}(- \otimes_R R_\mathfrak{p})$.*

In the following theorem, we denote by η^W the natural morphism $\mathrm{id}_\mathcal{D} \to \lambda^W$ induced by the adjointness property of (λ^W, j^W), and use the fact that $\lambda^U \lambda^W = \lambda^U$ if $U \subset W$.

Theorem 2. *Let W be a subset of $\mathrm{Spec}\, R$ and X be a chian complex in $\mathcal{D}$. Assume that W_0 is a subset of W with $\overline{W_0}^s \cap W = W$, where $\overline{W_0}^s$ is the specialization-closure of W_0. We set $W_0 = W \setminus W_1$. Then there is a triangle of the following form;*

$$\lambda^W X \xrightarrow{\ f\ } \lambda^{W_1} X \oplus \lambda^{W_0} X \xrightarrow{\ g\ } \lambda^{W_1}\lambda^{W_0} X \longrightarrow \lambda^W X[1],$$

where

$$f = \begin{pmatrix} \eta^{W_1}(\lambda^W X) \\ \eta^{W_0}(\lambda^W X) \end{pmatrix}, \quad g = \begin{pmatrix} \lambda^{W_1}(\eta^{W_0}(X)) & (-1) \cdot \eta^{W_1}(\lambda^{W_0} X) \end{pmatrix}.$$

Thanks to these theorems, we can compute λ^W by the induction on the maximum length of chains of prime ideals in W. Moreover, the second one says that $\lambda^W X$ is the homotopy pullback of the two morphisms $\lambda^{W_1}(\eta^{W_0}(X)) : \lambda^{W_1} X \to \lambda^{W_1}\lambda^{W_0} X$ and $\eta^{W_1}(\lambda^{W_0} X) : \lambda^{W_0} X \to \lambda^{W_1}\lambda^{W_0} X$.

As an application of the results, we are able to obtain a simpler proof of a classical theorem in [4] by Raynaud and Gruson, which states that the projective dimension of a flat R-module is at most the Krull dimension of R. Furthermore, time permitting, we will give an explicit way to compute λ^W by using the notion of Čech complexes, which enables us to construct pure injective resolutions (cf. [1]) of flat or finitely generated R-modules in a functorial manner.

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On the vanishing of self extensions over Cohen-Macaulay local rings

Tokuji Araya

This talk is based on joint work with O. Celikbas, A. Sadeghi and R. Takahashi.

Throughout R denotes a commutative Noetherian local ring and $\operatorname{mod} R$ denotes the category of all finitely generated R-modules.

The celebrated Auslander-Reiten Conjecture, on the vanishing of self extensions of a module, is one of the long-standing conjectures in ring theory.

Conjecture (Auslander-Reiten [2]). Let $M \in \operatorname{mod} R$. If $\operatorname{Ext}_R^i(M, M) = \operatorname{Ext}_R^i(M, R) = 0$ for all $i \geq 1$, then M is free.

Although it is still open, there are several results in the literature that establish the conjecture over Gorenstein rings under certain conditions.

For a nonnegative integer n and an R-module M, we set $X^n(R) = \{\mathfrak{p} \in \operatorname{Spec}(R) | \operatorname{ht}(\mathfrak{p}) \leq n\}$ and say M is *locally free on* $X^n(R)$ if $M_\mathfrak{p}$ is a free $R_\mathfrak{p}$-module for each prime ideal $\mathfrak{p} \in X^n(R)$.

Theorem 1 (Araya [1]). *Let R be a Gorenstein local ring of dimension $d \geq 2$ and let $M \in \operatorname{mod} R$. Then M is free provided that the following hold:*

(1) *M is locally free on $X^{d-1}(R)$.*
(2) *M is maximal Cohen-Macaulay.*
(3) $\operatorname{Ext}_R^{d-1}(M, M) = 0$.

Theorem 2 (Ono and Yoshino [3]). *Let R be a Gorenstein local ring of dimension $d \geq 3$ and let $M \in \operatorname{mod} R$ be a module. Then M is free provided that the following hold:*

(1) *M is locally free on $X^{d-2}(R)$.*
(2) *M is maximal Cohen-Macaulay.*
(3) $\operatorname{Ext}_R^{d-2}(M, M) = \operatorname{Ext}_R^{d-1}(M, M) = 0$.

By looking at these theorems, there is a natural conjecture.

Conjecture. Let n be a positive integer. Let R be a Gorenstein local ring of dimension $d > n$ and let $M \in \operatorname{mod} R$. Then M is free provided that the following hold:

(1) M is locally free on $X^n(R)$.
(2) M is maximal Cohen-Macaulay.
(3) $\operatorname{Ext}_R^i(M, M) = 0$ for all $n \leq i \leq d - 1$.

In this talk, we will give an affirmative answer of this conjecture. Moreover, we will show an extension of this over Cohen-Macaulay local rings that admit canonical modules. In particular, our main result recovers theorems of Araya, and Ono and Yoshino simultaneously.

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On delta invariants of certain ideals

Toshinori Kobayashi

Let $(R, \mathfrak{m}, k)$ be a Cohen-Macaulay local ring with a canonical module. The Auslander δ-invariant $\delta_R(M)$ for a finitely generated R-module M is defined to be the rank of maximal free summand of the minimal Cohen-Macaulay approximation of M. For an integer $n \geq 0$, the n-th δ-invariant is defined by Auslander, Ding and Solberg [1] as $\delta_R^n(M) = \delta_R(\Omega_R^n M)$, where $\Omega_R^n M$ denotes the n-th syzygy module of M in the minimal free resolution. On these invariants, combining the Auslander's result (see [1, Corollary 5.7]) and Yoshino's one [4], we can see the following theorem.

Theorem 1 (Auslander, Yoshino)**.** *Let $d > 0$ be the Krull dimension of R. Consider the following conditions.*

(a) *R is a regular local ring.*
(b) *There exists $n \geq 0$ such that $\delta^n(R/\mathfrak{m}) > 0$.*
(c) *There exist $n > 0$ and $l > 0$ such that $\delta^n(R/\mathfrak{m}^l) > 0$.*

Then, the implications (a) $\Leftrightarrow$ (b) $\Rightarrow$ (c) *hold. The implication* (c) $\Rightarrow$ (a) *holds if* $\operatorname{depth} \mathsf{gr}_{\mathfrak{m}}(R) \geq d - 1$.

Here we denote by $\mathsf{gr}_I(R)$ the associated graded ring of R with respect to an ideal I of R. In this talk, I will give some necessary and sufficient conditions for an ideal to be a parameter ideal of R in terms of the δ-invarinants. More precisely I will explain the following result.

Theorem 2. *Let $(R, \mathfrak{m})$ be a Cohen-Macaulay local ring with a canonical module ω, having infinite residue field k and Krull dimension $d > 0$. Let I be an $\mathfrak{m}$-primary ideal of R such that I/I^2 is a free R/I-module. Consider the following conditions.*

(a) *$\delta(R/I) > 0$.*
(b) *I is a parameter ideal of R.*
(c) *There exists $n \geq 0$ such that $\delta^n(R/I) > 0$.*
(d) *There exist $n > 0$ and $l > 0$ such that $\delta^n(R/I^l) > 0$.*

Then, the implications (a) $\Rightarrow$ (b) $\Leftrightarrow$ (c) $\Rightarrow$ (d) *hold. The implication* (d) $\Rightarrow$ (c) *holds if* $\operatorname{depth} \mathsf{gr}_I(R) \geq d - 1$ *and I^i/I^{i+1} is a free R/I-module for any $i > 0$. The implication* (b) $\Rightarrow$ (a) *holds if $I \subset \mathsf{tr}(\omega)$.*

Here $\mathsf{tr}(\omega)$ is the trace ideal of ω. that is, the image of the natural homomorphism $\omega \otimes_R \mathsf{Hom}_R(\omega, R) \to R$ mapping $x \otimes f$ to $f(x)$ for $x \in \omega$ and $f \in \mathsf{Hom}_R(\omega, R)$.

There are some examples of $\mathfrak{m}$-primary ideals I which satisfy the conditions in Theorem 2, that is, I^i/I^{i+1} is a free R/I-module and $\mathsf{gr}_I(R) \geq d - 1$. One of them is the maximal ideal $\mathfrak{m}$ in the case where $\mathsf{gr}_{\mathfrak{m}}(R)$ is Cohen-Macaulay (for example, R is a hypersurface or a localization of a homogeneous graded Cohen-Macaulay ring.) Thus, Theorem 2 recovers Theorem 1 by letting $I = \mathfrak{m}$.

Other interesting examples are Ulrich ideals. These ideals are defined in [2] and many examples of Ulrich ideals are given in [2] and [3]. As an application of Theorem 2, I will give an uniform upper bound for orders of Ulrich ideals of R when R has Gorenstein punctured spectrum.

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The Krull dimension of composite power series rings over valuation rings

Minjae Kwon

In this talk, we study the Krull dimension of composite power series rings $V + XI[\![X]\!]$ where V is valuation rings and I is a nonzero ideal of V. We introduce the concepts a prime chain ordering and a phi function. Using these concepts, I will show how to construct chain of prime ideals with length $2^{\aleph_1}$. If base ring V is a nondiscrete valuation ring, then we can construct the chain of prime ideals with length $2^{\aleph_1}$ in $V + XI[\![X]\!]$. Hence we have the result the Krull dimension of $V + XI[\![X]\!] \geq 2^{\aleph_1}$. If the case base ring V is a discrete valuation ring with rank n, then we have the result the Krull dimension of $V + XI[\![X]\!] = n + 1$.

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Around Azumaya rings - An overview of ring theory in the last decades

Robert Wisbauer

The classical Wedderburn-Artin structure theorem, states that an artin ring modulo its prime radical is a finite product of matrix rings over division rings. Since the beginning of the last century, the generalisation of this theorem has served as motivation for further development of abstract algebra. The category theory emerging in the middle of the last century gave new impulse for research in this field. Although it was initially taken up only hesitantly, it is now an indispensable part of the ongoing research - not only in algebra.

In this approach also coalgebraic structures took their place, opening new ways to characterise (and generalise) separable, Azumaya and Frobenius algebras. Over fields, all these examples are finite dimensional, whereas Azumaya rings, which include all simple algebras, need not be so. Thus they need a somewhat different treatment.

Düsseldorf

On quiver Grassmannians and orbit closures for gen-finite modules

Julia Sauter

This is joint work in progress with Matthew Pressland, generalizing work with Crawley-Boevey [3], which in turn generalized [1, 2, 4].

One calls a module *gen-finite* if there are only finitely many isomorphism classes of indecomposable modules which are generated by the module.

Given a cogenerator for an algebra, we will show that its endomorphism ring admits a natural tilting module. We will show how to use this to construct desingularizations of quiver Grassmannians and orbit closures for gen-finite modules.

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The moduli of subalgebras of the full matrix ring of degree 3

Kazunori Nakamoto and Takeshi Torii

Let k be an algebraically closed field. We say that k-subalgebras A and B of $\mathrm{M}_3(k)$ are equivalent if $P^{-1}AP = B$ for some $P \in \mathrm{GL}_3(k)$. There are 26 equivalence classes of k-subalgebras of $\mathrm{M}_3(k)$. More generally, let us consider subalgebras of $\mathrm{M}_3(R)$ over an arbitrary commutative ring R (or scheme X).

Definition 1. We say that a subsheaf $\mathcal{A}$ of $\mathcal{O}_X$-algebras of $\mathrm{M}_n(\mathcal{O}_X)$ is a *mold* of degree n on a scheme X if $\mathrm{M}_n(\mathcal{O}_X)/\mathcal{A}$ is a locally free sheaf. We denote by $\mathrm{rank}\mathcal{A}$ the rank of $\mathcal{A}$ as a locally free sheaf.

Proposition 2. *The following cotravariant functor is representable by a closed subscheme of the Grassmann scheme* $\mathrm{Grass}(d, n^2)$*:*

$$\begin{array}{cccl} \mathrm{Mold}_{n,d} & : & (\mathbf{Sch})^{op} & \to & (\mathbf{Sets}) \\ & & X & \mapsto & \left\{ \mathcal{A} \mid \ \mathcal{A} \textit{ is a rank } d \textit{ mold of degree } n \textit{ on } X \ \right\}. \end{array}$$

Let us consider the moduli $\mathrm{Mold}_{3,d}$ of rank d molds of degree 3. For $d = 1, 6, 7, 8, 9$, we have:

Example 3. Let $n = 3$. If $d = 1$ or $d \geq 6$, then

$$\begin{array}{rcl} \mathrm{Mold}_{3,1} & = & \mathrm{Spec}\mathbb{Z}, \\ \mathrm{Mold}_{3,6} & = & \mathrm{Flag} := \mathrm{GL}_3/\{(a_{ij}) \in \mathrm{GL}_3 \mid a_{ij} = 0 \text{ for } i > j\}, \\ \mathrm{Mold}_{3,7} & = & \mathbb{P}^2_{\mathbb{Z}} \coprod \mathbb{P}^2_{\mathbb{Z}}, \\ \mathrm{Mold}_{3,8} & = & \emptyset, \\ \mathrm{Mold}_{3,9} & = & \mathrm{Spec}\mathbb{Z}. \end{array}$$

In this talk, we describe the moduli $\mathrm{Mold}_{3,d}$ of rank d molds of degree 3 for $d = 2$ and 3.

Theorem 4. *When $d = 2$, we have* $\mathrm{Mold}_{3,2} \cong \mathbb{P}^2_{\mathbb{Z}} \times \mathbb{P}^2_{\mathbb{Z}}$.

Theorem 5. *When $d = 3$, we have an irreducible decomposition* $\mathrm{Mold}_{3,3} = \overline{\mathrm{Mold}^{\mathrm{reg}}_{3,3}} \cup \overline{\mathrm{Mold}^{\mathrm{S}_2}_{3,3}} \cup \overline{\mathrm{Mold}^{\mathrm{S}_3}_{3,3}}$*, where the relative dimensions of* $\overline{\mathrm{Mold}^{\mathrm{reg}}_{3,3}}$*,* $\overline{\mathrm{Mold}^{\mathrm{S}_2}_{3,3}}$*, and* $\overline{\mathrm{Mold}^{\mathrm{S}_3}_{3,3}}$ *over $\mathbb{Z}$ are 6, 4, and 4, respectively. Moreover, both* $\mathrm{Mold}^{\mathrm{S}_5}_{3,3} := \overline{\mathrm{Mold}^{\mathrm{reg}}_{3,3}} \cap \overline{\mathrm{Mold}^{\mathrm{S}_2}_{3,3}}$ *and* $\mathrm{Mold}^{\mathrm{S}_4}_{3,3} := \overline{\mathrm{Mold}^{\mathrm{reg}}_{3,3}} \cap \overline{\mathrm{Mold}^{\mathrm{S}_3}_{3,3}}$ *have relative dimension 2 over $\mathbb{Z}$, and* $\overline{\mathrm{Mold}^{\mathrm{S}_2}_{3,3}} \cap \overline{\mathrm{Mold}^{\mathrm{S}_3}_{3,3}} = \emptyset$.

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Two-sided tilting complexes and folded tree-to-star complexes

Yuta Kozakai

Let Γ be a Brauer tree with e edges and multiplicity m of the exceptional vertex, and A a Brauer tree algebra associated to Γ and B a Brauer tree algebra associated to a Brauer star with e edges and exceptional vertex with multiplicity m in the center (or equivalently is a symmetric Nakayama algebra with e simple modules and nilpotency degree of the radical being $m+1$). In [2], Rickard showed that a Brauer tree algebra is determined by the number of the edges and the multiplicity of the exceptional vertex up to derived equivalence. This fact was shown by proving that A is derived equivalent to the algebra B by constructing a one-sided tilting complex T over A with endomorphism ring B which is called Rickard tree-to-star complex. Moreover, in [4], Rickard and Schaps constructed one-sided tilting complexes over A with endomorphism ring B by applying operations called foldings to the Rickard tree-to-star complex T. The complexes are called Rickard-Schaps tree-to-star complexes.

On the other hand, in [3] and [1] it is shown that for a one-sided tilting complex over an algebra there exists a two-sided tilting complex which is isomorphic to the one-sided tilting complex when restricted to the one-sided action. Hence there should exist two-sided tilting complexes of A-B-bimodules corresponding to the Rickard tree-to-star complex and the Rickard-Schaps tree-to-star complexes. In this talk, we construct the two-sided tilting complex of A-B-bimodules which is isomorphic to Rickard tree-to-star complex T in the bounded derived category of A-modules by using concrete A-B-bimodules. Moreover, we realize operations on the two-sided tilting complex corresponding to the foldings, hence we give the construction of the two-sided tilting complexes which correspond to Rickard-Schaps tree-to-star complexes.

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Singularity categories and silting objects

Takuma Aihara

Tilting theory is now an essential tool in the study of finite dimensional algebras, and it influences many branches of mathematics. In the theory, silting objects play a central and important role. Then, we would think how many such objects there exist. By silting mutation [1], once we find a silting object, then one can get infinitely many silting ones. On the other hand, we know that the singularity category of a non-semisimple selfinjective algebra, which is triangule equivalent to the stable module category [2], has no silting object [1].

In this talk, we will discuss the existence of silting objects, and particularly consider about silting objects of a singularity category. A main result is the following:

Theorem 1. *Let Λ be a finite dimensional algebra over an algebraically closed field and denote by $\mathsf{D_{sg}}(\Lambda)$ the singularity category of Λ. If the right selfinjective dimension* $\mathrm{inj.dim}(\Lambda_\Lambda)$ *of Λ is finite, then $\mathsf{D_{sg}}(\Lambda)$ and $\mathsf{D_{sg}}(\Lambda^{\mathrm{op}})$ admit no silting object. Here, Λ^{op} stands for the opposite algebra of Λ.*

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Two-term silting complexes over radical square zero algebras

Toshitaka Aoki

Let k be an algebraically closed field. In the representation theory of finite dimensional k-algebras, the notion of silting complexes is introduced as a generalization of tilting complexes. In particular, we are interested in *two-term silting complexes* for a given finite dimensional k-algebra Λ. It is known that they correspond to several important objects in representation theory such as support τ-tilting modules, and the classification of them has been established for certain classes of algebras, preprojective algebras of Dynkin type or Brauer graph algebras for example. We denote by $\mathsf{2\text{-}silt}\Lambda$ (respectively, $\mathsf{tilt}\Lambda$) the set of isomorphism classes of basic two-term silting complexes for Λ (respectively, finitely generated basic tilting Λ-modules). Note that they have the natural structure of partially ordered set.

In this talk, we give a classification of two-term silting complexes for *algebras with radical square zero*, that is, any such algebra Λ satisfying $J_\Lambda^2 = 0$ where J_Λ is the Jacobson radical of Λ. They provide one of the most fundamental classes of algebras. Let Λ be a finite dimensional k-algebra with radical square zero, and let $Q = (Q_0, Q_1)$ be the ordinary quiver of Λ. For the representation theory of Λ, a path algebra kQ^s of Q^s is important, where $Q^s := (Q_0^s, Q_1^s)$ is a *separated quiver* of Q defined by $Q_0^s := \{i^+ | i \in Q_0\} \coprod \{i^- | i \in Q_0\}$ and $Q_1^s := \{i^+ \to j^- | i \to j \text{ in } Q_1\}$. Indeed, it was used to characterize the radical square zero algebras of finite representation type, that is, having only finitely many indecomposable modules up to isomorphisms (see, for example, [2]). Now, we consider a map $\epsilon\colon Q_0 \to \{+,-\}$ called a signature on Q and define a full subquiver Q_ϵ of Q^s for ϵ whose vertices are given by a set $\{i^{\epsilon(i)} \in Q_0^s | i \in Q\}$. Then we have the following theorem.

Theorem 1. *Let Λ be a finite dimensional k-algebra with radical square zero, and let Q be the ordinary quiver of Λ. Then there is a bijection between*

$$\mathsf{2\text{-}silt}\Lambda \quad \overset{1-1}{\longleftrightarrow} \quad \coprod_{\substack{\epsilon\colon \text{a signature} \\ \text{on } Q}} \mathsf{tilt}kQ_\epsilon^{\mathrm{op}}$$

where Q_ϵ^{op} is the opposite quiver of Q_ϵ. Moreover, if we fix a signature ϵ on Q, then the restriction of this correspondence induces an isomorphism of partially ordered sets.

This result is a refinement of a work of Adachi [1], in which he gives a characterization of radical square zero algebras having only finitely many (support) τ-tilting modules up to isomorphisms. As an application, we calculate the number of isomorphism classes of basic two-term tilting complexes over *Brauer line algebras*, which are a special class of Brauer tree algebras.

Theorem 2. *Let Γ_n be a Brauer line algebra corresponding to the multiplicity-free Brauer line with n vertices. Then the number of isomorphism classes of basic two-term tilting complexes for Γ_n is $\binom{2n}{n}$.*

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Central elements of the Jennings basis and certain Morita invariants

Taro Sakurai

From Morita theoretic viewpoint, computing Morita invariants is important. We prove that the intersection of the center and the nth (right) socle $ZS^n(A) := Z(A) \cap \mathrm{Soc}^n(A)$ of a finite-dimensional algebra A is a Morita invariant; This is a generalization of important Morita invariants — the center $Z(A)$ and the Reynolds ideal $ZS^1(A)$. (See Table 1.)

As an example, we also studied $ZS^n(FG)$ for the group algebra FG of a finite p-group G over a field F of positive characteristic p. Such an algebra has a basis along the socle filtration, known as the Jennings basis. We prove certain elements of the Jennings basis are central and hence form a linearly independent set of $ZS^n(FG)$. In fact, such elements form a basis of $ZS^n(FG)$ for every integer $1 \leq n \leq p$ if G is powerful. As a corollary we have $\mathrm{Soc}^p(FG) \subseteq Z(FG)$ if G is powerful.

TABLE 1. What is known about $ZS^n(FG) = Z(FG) \cap \mathrm{Soc}^n(FG)$ for a finite group G.

	dimension (representation-theoretic)	basis (group-theoretic)
$Z(FG)$	$k(G)$	conjugacy class sums
$ZS^n(FG)$	unknown	unknown
$ZS^2(FG)$	$\ell(G) + \sum \dim \mathrm{Ext}^1(S,S)$	unknown[1]
$ZS^1(FG)$	$\ell(G)$	p-regular section sums

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[1] Except finite p-groups.

Simple modules in the Auslander-Reiten quivers for finite group algebras

Shigeo Koshitani

We will be looking at the positions of simple modules in the stable Auslander-Reiten quivers for group algebras of finite groups over a field. This is joint work with Caroline Lassueur.

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Functor categories on derived categories of hereditary algebras

Yuta Kimura

For a given triangulated category $\mathcal{T}$, it is nature to ask whether $\mathcal{T}$ is triangle equivalent to the (bounded) derived category of an abelian category. Since there are many studies on the derived category of an abelian category, describing a triangulated category as a derived category is very useful.

Let $\mathcal{C}$ be an additive category. A $\mathcal{C}$-module is a contravariant functor from $\mathcal{C}$ to $\mathcal{A}b$, where $\mathcal{A}b$ is the category of abelian groups. This is an analog of modules over rings when we regard $\mathcal{C}$ as a ring with several objects. A finitely presented $\mathcal{C}$-module is also defined in the same way as defining a finitely presented module over a ring. We denote by $\mathsf{mod}\,\mathcal{C}$ the category of finitely presented $\mathcal{C}$-modules. If $\mathcal{C}$ is triangulated, then it is known that $\mathsf{mod}\,\mathcal{C}$ is Frobenius and abelian, and its stable category $\underline{\mathsf{mod}}\,\mathcal{C}$ is also triangulated.

In this talk, we focus on the triangulated category $\underline{\mathsf{mod}}\,\mathsf{D}^{\mathrm{b}}(\mathsf{mod}H)$, where H is a finite dimensional hereditary algebra. We construct a triangle equivalence between this category and the bounded derived category of some abelian category. An important step is to establish a functor category analog of Happel's triangle equivalence [2, Chapter II, 4.9 Theorem] for repetitive algebras.

Let H be a finite dimensional hereditary algebra of finite representation type. Namely, there exists a basic H-module M satisfying $\mathsf{add}\,M = \mathsf{mod}\,H$. We call the algebra $\Gamma_H := \mathsf{End}_{\underline{\mathsf{mod}}\,H}(M)$ the stable Aunslander algebra of H. Iyama and Oppermann [3] showed that $\underline{\mathsf{mod}}\,\mathsf{D}^{\mathrm{b}}(\mathsf{mod}H)$ is triangle equivalent to the bounded derived category of Γ_H.

We extend the triangle equivalence of Iyama and Oppermann to the case when H is a representation infinite hereditary algebra. If H is representation finite, then $\mathsf{mod}(\underline{\mathsf{mod}}\,H) \simeq \mathsf{mod}\,\Gamma_H$ holds. Therefore the role of the stable Auslander algebra Γ_H is played by the stable category of H. It is known that $\mathsf{mod}(\underline{\mathsf{mod}}\,A)$ is abelian for any finite dimensional algebra A. We have the following theorem.

Theorem 1. [3, 4] *Let H be a finite dimensional hereditary algebra. Then we have a triangle equivalence*

$$\underline{\mathsf{mod}}\,\mathsf{D}^{\mathrm{b}}(\mathsf{mod}H) \simeq \mathsf{D}^{\mathrm{b}}(\mathsf{mod}(\underline{\mathsf{mod}}\,H)).$$

The category $\underline{\mathsf{mod}}\,H$ is a so-called dualizing variety, which was introduced and studied by Auslander and Reiten [1]. To show Theorem 1, we need the following theorem. For a category $\mathcal{C}$, we denote by $\mathsf{R}\mathcal{C}$ the repetitive category of $\mathcal{C}$.

Theorem 2. [4] *Let $\mathcal{A}$ be a dualizing variety. Then the following holds.*

(a) $\mathsf{R}\mathcal{A}$ *is a dualizing variety.*
(b) $\mathsf{mod}\,\mathsf{R}\mathcal{A}$ *is a Frobenius abelian category.*
(c) *If any modules in* $\mathsf{mod}\mathcal{A}$ *and* $\mathsf{mod}\mathcal{A}^{\mathrm{op}}$ *have finite projective dimension, then we have a triangle equivalence* $\underline{\mathsf{mod}}\,\mathsf{R}\mathcal{A} \simeq \mathsf{D}^{\mathrm{b}}(\mathsf{mod}\mathcal{A})$.

The category of finitely generated projective modules $\mathsf{proj}A$ over a finite dimensional algebra A is a typical example of a dualizing variety. If $\mathcal{A} = \mathsf{proj}A$ and A is of finite global dimension, then Theorem 2 gives a Happel's triangle equivalence for repetitive algebras.

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d-representation-finite self-injective algebras

Erik Darpö

In this talk, I shall present a systematic method to construct self-injective algebras which are d-representation-finite in the sense of higher-dimensional Auslander–Reiten theory. Such algebras are given as orbit algebras of the repetitive categories of algebras of finite global dimension satisfying a certain finiteness condition for the Serre functor. The condition holds, in particular, for all fractionally Calabi-Yau algebras of global dimension at most d. This generalizes Riedtmann's classical construction of representation-finite self-injective algebras.

Applications include n-fold trivial extensions and higher preprojective algebras, which can be shown to be d-representation-finite in many cases.

BRICKS OVER PREPROJECTIVE ALGEBRAS

Sota Asai

In representation theory of finite-dimensional algebras over a field K, preprojective algebras of Dynkin type are an important class of algebras. For each Dynkin diagram Δ, the preprojective algebra Π of type Δ is defined as a quotient of the path algebra of the double quiver Q of type Δ.

One of the characteristic properties of the preprojective algebras is that they have many connections with the Coxeter groups W corresponding to type Δ. There is an ideal $I(w)$ of Π associated to each element $w \in W$, which is defined in [5] and [2] by using a reduced expression of w. Actually, Mizuno [6] proved that the ideal $I(w)$ is a support τ-tilting Π-module, and that the quotient $\Pi/I(w)$ is a support τ^{-1}-tilting Π-module, and that this correspondence $w \mapsto \Pi/I(w)$ gives a bijection from W to the set $\mathsf{s}\tau^{-1}\text{-}\mathsf{tilt}\,\Pi$ of basic support τ^{-1}-tilting Π-modules, compatible with the poset structures.

Mizuno [6] also proved that every torsion-free class in $\mathsf{mod}\,\Pi$ is functorially finite. Thus, from my previous paper [1], we get a bijection from the set $\mathsf{s}\tau^{-1}\text{-}\mathsf{tilt}\,\Pi$ to the set $\mathsf{sbrick}\,\Pi$ of semibricks in $\mathsf{mod}\,\Pi$ given by $M \mapsto \mathsf{soc}_{\mathsf{End}_\Pi(M)} M$. Here, the term "semibricks" means Π-modules which are direct sums of some bricks, and bricks are Π-modules whose endomorphism rings are division K-algebras.

Therefore, there exists a bijection $S\colon W \to \mathsf{sbrick}\,\Pi$. I am studying the explicit behaviors of this bijection.

It follows from [4] and [3] that the bijection $S\colon W \to \mathsf{sbrick}\,\Pi$ is restricted to a bijection from the set $\mathsf{j\text{-}irr}\,W$ of join-irreducible elements in W to the set $\mathsf{brick}\,\Pi$ of bricks in $\mathsf{mod}\,\Pi$. Thanks to the combinatorial works on indecomposable τ^{-1}-tilting Π^{-1}-modules developed in [4], we can explicitly describe the restricted bijection $S\colon \mathsf{j\text{-}irr}\,W \to \mathsf{brick}\,\Pi$.

In order to extend the explicit description of $S\colon \mathsf{j\text{-}irr}\,W \to \mathsf{brick}\,\Pi$ to the original bijection $S\colon W \to \mathsf{sbrick}\,\Pi$, we consider the following problem:

> let $w \in W$ and $S(w)$ be decomposed into bricks as a direct sum $S(w) = S_1 \oplus \cdots \oplus S_m$, and take the unique join-irreducible element $w_i \in \mathsf{j\text{-}irr}\,W$ such that $S(w_i) = S_i$ for each i, then what is the relationship between w and $w_1, \ldots, w_m$?

The answer is the following. This is one of the main results in my talk.

Theorem 1. *In above, $w = w_1 \vee \cdots \vee w_m$ holds, and it is the canonical join decomposition of w.*

This means that w coincides with the join of $w_1, \ldots, w_m$, and that $w_1, \ldots, w_m$ satisfy some kinds of minimality, see [4] for the precise definition. I emphasize that, though canonical join decompositions are defined in a purely combinatorial way, they give the indecomposable direct summands of semibricks, which are in the representation-theoritic side. In other words, the bricks as the indecomposable direct summands of a semibrick also have some combinatorial aspect.

In this talk, I would like to explain the bijection $S\colon W \to \mathsf{sbrick}\,\Pi$ from these viewpoints. I will also give some examples of getting semibricks from elements of the Coxeter group W.

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Wide subcategories are semistable

Toshiya Yurikusa

In this talk, we provide a complement of Ingalls-Thomas-type bijections for finite dimensional algebras.

For the path algebra kQ of a finite connected acyclic quiver Q over a field k, Ingalls and Thomas [1] gave bijections between the following objects:

(1) Isomorphism classes of basic support tilting modules in $\mathsf{mod}(kQ)$.
(2) Functorially finite torsion classes in $\mathsf{mod}(kQ)$.
(3) Functorially finite wide subcategories of $\mathsf{mod}(kQ)$.
(4) Functorially finite semistable subcategories of $\mathsf{mod}(kQ)$.

Note that they also proved that (1)-(4) above correspond bijectively with the clusters in the cluster algebra of Q and the isomorphism classes of basic cluster tilting objects in the cluster category of kQ.

By τ-tilting theory, for an arbitrary finite dimensional algebra Λ, there are Ingalls-Thomas-type bijections between the following objects:

(1) Isomorphism classes of basic support τ-tilting modules in $\mathsf{mod}\Lambda$.
(2) Functorially finite torsion classes in $\mathsf{mod}\Lambda$.
(3) Left finite wide subcategories of $\mathsf{mod}\Lambda$.

Notice that the statement for semistable subcategories of $\mathsf{mod}\Lambda$ is missing. The aim of this talk is to give the following complement of Ingalls-Thomas-type bijections.

Theorem 1. [2] The following objects are the same.

(3) Left finite wide subcategories of $\mathsf{mod}\Lambda$.
(4) Left finite semistable subcategories of $\mathsf{mod}\Lambda$.

More generally, we show that wide subcategories of $\mathsf{mod}\Lambda$ associated with two-term presilting complexes of Λ are semistable.

Theorem 2. [2] For a two-term presilting complex U of Λ, the wide subcategory ${}^{\perp}\mathrm{H}^{-1}(\nu U) \cap \mathrm{H}^0(U)^{\perp}$ of $\mathsf{mod}\Lambda$ is semistable.

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Tilting theory for Gorenstein rings in dimension one

Osamu Iyama

The stable category of Cohen-Macaulay modules is a basic object in Cohen-Macaulay representation theory. It has a structure of a triangulated category when the ring is Gorenstein. It is known that the stable category is often triangle equivalent to the derived category of a (non-commutative) ring. I will discuss such triangle equivalences in dimension one. This talk is based on joint works with R. Buchweitz, M. Herschend and K. Yamaura.

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A categorical approach to algebras and coalgebras

Robert Wisbauer

As mentioned in my first talk, categorical techniques turned out to be very effective in algebra and representation theory. Hereby, it was a key observation that module theory of an algebra A over a field K is the theory of the functor

$$A \otimes_K - : \mathbb{M}_K \to \mathbb{M}_K,$$

an endofunctor of the category of K-vektor spaces. The algebra A is defined by K-linear maps multiplication $A \otimes_K A \to A$ and unit $e : K \to A$, subject to associativity and unitality conditions. Left A-modules are given by a K-vector space V with K-linear maps $\varrho : A \otimes_K V \to V$, also subject to associativity and unitality conditions. Together with A-linear maps, this yields the category ${}_A\mathbb{M}$ of left A-modules, The algebra (A, m) and $(A \otimes_K V, m \otimes V)$, are left A-modules and this leads to the free and forgetful functors

$$\phi_A : \mathbb{M}_K \to {}_A\mathbb{M},\ V \mapsto (A \otimes V, m \otimes V), \quad U_A : {}_A\mathbb{M} \to \mathbb{M}_K,\ (M, \rho) \mapsto M$$

and the bijection

$$\mathrm{Hom}_A(A \otimes_K V, M) \to \mathrm{Hom}_K(V, U_A(M)),$$

that is, the functor U_A is right adjoint to $A \otimes_K -$. This all is very well known and now it is no problem to replace $\mathbb{M}_K$ by an arbitrary category $\mathbb{A}$ and the functor $A \otimes_K -$ by any endofunctor $F : \mathbb{A} \to \mathbb{A}$.

Then we need natural transformations, $m : FF \to F$ and $e : 1 \to F$, which should satisfy the respective associativity and unitality conditions. This gives F a *monad* structure. An F-module is an object $V \in \mathbb{A}$ with a morphism $\varrho : F(V) \to V$ and (F, m) and $(F(V), m_V)$ are first examples of this. Morphisms of F-modules are morphisms from $\mathbb{A}$ respecting the module structures and they yield the category $\mathbb{A}_F$ of F-modules with free and forgetful functors

$$\phi_F : \mathbb{A} \to \mathbb{A}_F,\ V \mapsto (F(V), m_V), \quad U_F : \mathbb{A}_F \to \mathbb{A},\ (M, \varrho) \mapsto M,$$

and the bijection

$$\mathrm{Mor}_F(F(V), M) \to \mathrm{Mor}_{\mathbb{A}}(V, U_F(M)),\ F(V) \xrightarrow{f} M \ \mapsto\ V \xrightarrow{ev} F(V) \xrightarrow{f} M,$$

showing that the functor U_F is right adjoint to ϕ_F.

This shows at which elementary level structures from module theory can be trasnferred to arbitrary categories and one is no longer restricted to tensor functors. If the functor F has a right adjoint G, then the monad structure on F provides G with the structure of a comonad. Thus this approach leads naturally to comonads (coalgebras, bocses) and comodules.

Düsseldorf

Hisao Tominaga, one of founders of this Symposium

Shuichi Ikehata

The Symposium on Ring Theory and Representation Theory has been held annually in Japan and the Proceedings have been published by the organizing committee. The first Symposium was organized in 1968 by Hisao Tominaga, Hiroyuki Tachikawa, Manabu Harada and Shizuo Endo. In particular, Tominaga made a great contribution to the development of this symposium. He was a professor at Okayama University since 1966. His research covers a wide area of the theory of noncommutative rings which contains Galois theory of rings, the structure theory of rings, and etc. He wrote many excellent papers and raised many disciples. He also edited Mathematical Journal of Okayama University for a long time. His contributions to ring theory have been acclaimed by many mathematicians. He died in Okayama, February 23, 1994. In this talk I would like to talk about his mathematics and his personality.

Okayama University

Infinite sequences of Frobenius extensions

Mitsuo Hoshino, Noritsugu Kameyama and Hirotaka Koga

We use the notation A/R to denote that a ring A contains a ring R as a subring. Recall that a ring extension A/R is said to be a Frobenius extension of first kind if A is finitely generated projective as a right R-module and if $A \cong \mathrm{Hom}_R(A, R)$ as (R, A)-bimodules. In this talk, to each Frobenius extension of first kind A/R we associate a sequence of ring extensions

$$A_0 = R \subset A_1 = A \subset \cdots \subset A_n \subset \cdots$$

such that each A_{i+1}/A_i is a Frobenius exteinsion of first kind.

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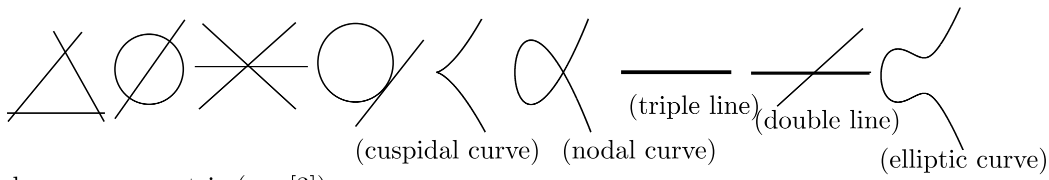
The defining relations and the Calabi-Yau property of
3-dimensional quadratic AS-regular algebras

Ayako Itaba, Ryo Onozuka, James Eccles

Let k be an algebraically closed field of characteristic 0, A a graded k-algebra finitely generated in degree 1 and V a k-vector space. An AS-regular algebra introduced by Artin-Schelter [1] is one of the main interests in noncommutative algebraic geometry. Using a geometric pair (E, σ), 3-dimensional quadratic AS-regular algebras are classified by Artin-Tate-Van den Bergh [2], where E is the projective space $\mathbb{P}^2$ or a cubic curve of $\mathbb{P}^2$ as follows and σ is an automorphism of E.



These algebras are geometric (see [3]).

In this talk, we consider 3-dimensional quadratic AS-regular algebras. By using the normalization of a variety, we determine the defining relations of Type CC and Type NC 3-dimensional quadratic AS-regular algebras (these algebras correspond to cuspidal and nodal cubic curve in the projective space). Also, we consider the following conjecture: for a 3-dimensional quadratic AS-regular algebra A, there exists a Calabi-Yau AS-regular algebra C such that A and C are graded Morita equivalent. Using the twist of a superpotential in the sense of Mori-Smith [4] and the defining relations determined above, we will show that this conjecture holds in most cases.

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Strongly quasi-hereditary algebras and rejective subcategories

Mayu Tsukamoto

Quasi-hereditary algebras were introduced by Scott [6] to study highest weight categories in the representation theory of semisimple complex Lie algebras and algebraic groups. Ringel [5] introduced a special class of quasi-hereditary algebras called *right-strongly quasi-hereditary* algebras, motivated by Iyama's finiteness theorem of representation dimensions of artin algebras [3]. One of the advantages of right-strongly quasi-hereditary algebras is that they have better upper bound of global dimension than that of general quasi-hereditary algebras [5, §4].

For an artin algebra A and its factor algebra B, we naturally regard $\mathsf{mod}B$ as a full subcategory of $\mathsf{mod}A$. In this case, each $X \in \mathsf{mod}A$ has a right (resp. left) ($\mathsf{mod}B$)-approximation of X which is monic (resp. epic) in $\mathsf{mod}A$. More generally, Iyama [3, 4] called subcategories of an additive category with these properties a *right (resp. left) rejective subcategories*. They are a special class of *coreflective (resp. reflective) subcategories* appearing in the classical theory of localizations of abelian categories. Using the notion of right rejective (resp. left rejective, coreflective, reflective) subcategories, we introduce the notion of *total right rejective (resp. total left rejective, coreflective, reflective) chains* of an additive category. In this talk, we characterize right-strongly (resp. left-strongly) quasi-hereditary algebras in terms of these chains.

Theorem 1 ([7, Theorem 3.33])**.** *Let A be an artin algebra and*

$$A = Ae_0A > Ae_1A > \cdots > Ae_iA > Ae_{i+1}A > \cdots > Ae_{n_1}A > 0 \qquad (1)$$

a chain of idempotent ideals of A, where e_i is an idempotent of A. Then the following conditions are equivalent:

(i) (1) *is a right-strongly (resp. left-strongly) heredity chain.*

(ii) *The following chain is a total right (resp. left) rejective chain of* $\mathsf{proj}A$.

$$0 \subset \mathsf{add}e_{n-1}A \subset \mathsf{add}e_{n-1}A \subset \cdots \subset \mathsf{add}e_iA \subset \cdots \subset \mathsf{add}e_0A = \mathsf{proj}A.$$

(iii) (1) *is a heredity chain of A and the following chain is a coreflective (resp. reflective) chain of* $\mathsf{proj}A$.

$$0 \subset \mathsf{add}e_{n-1}A \subset \mathsf{add}e_{n-1}A \subset \cdots \subset \mathsf{add}e_iA \subset \cdots \subset \mathsf{add}e_0A = \mathsf{proj}A.$$

As application, we sharpen a well-known result of Dlab-Ringel [1, Theorem 2] stating that any artin algebra of global dimension at most two is quasi-hereditary. In fact, we prove that such an algebra is always right-strongly (resp. left-strongly) quasi-hereditary by combining [4, Theorem 3.6] and Theorem 1. Moreover we show that the Auslander algebra of a representation-finite algebra A is strongly quasi-hereditary if and only if A is a Nakayama algebra. It also can be proved from a recent result [2, Theorem 3], which is shown by a different method.

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Degenerations of Cohen-Macaulay modules via matrix representations

Naoya Hiramatsu

The notion of degenerations of modules appears in geometric methods of representation theory of finite dimensional algebras. Yoshino [6] gives a scheme-theoretical definition of degenerations, so that it can be considered for modules over a noetherian algebra which is not necessary finite dimensional. Many authors [1, 3, 6, 7] have studied the degeneration problem of modules. The author and Yoshino [2] give the complete description of degenerations over a ring of even-dimensional simple hypersurface singularity of type (A_n).

Let $(R, \mathfrak{m})$ a commutative noetherian complete local k-algebra with a residue field k. It is known that there exists a regular local k-subalgebra S of R such that R is a module-finite S-algebra. (Cohen's structure theorem for a complete local rings). Let M be a Cohen-Macaulay R-module. Then M is free as S-module, so that we can obtain a k-algebra homomorphism $R \to \mathrm{End}_S(M)$. It is called a matrix-representation of M over S.

Let R be a hypersurface ring which is of the form

$$k[[x_0, x_1, \dots, x_d]]/(x_0^2 + x_2^2 + \cdots x_d^2).$$

It is known that R has countable representation type and all objects in $\mathrm{CM}(R)$ are classified (e.g. [4]).

The purpose of the talk is to give the necessary condition for the degenerations of Cohen-Macaulay modules by considering it via matrix representations. As an application, we will give the description of degenerations of indecomposable Cohen-Macaulay R-modules in the case where R is of dimension 1 and 2. Note that the matrices in the theorem below are matrix representations of indecomposable Cohen-Macaulay R-modules.

Theorem 1. *Let k be an algebraically closed field of characteristic zero.*

(1) *Let $R = k[[x, y]]/(x^2)$. Then* $\begin{pmatrix} 0 & y^a \\ 0 & 0 \end{pmatrix}$ *degenerates to* $\begin{pmatrix} 0 & y^b \\ 0 & 0 \end{pmatrix}$ *if and only if $a \le b$ and $a \equiv b$* mod 2.

(2) *Let $R = k[[x, y, z]]/(x^2 - yx)$. Then* $\begin{pmatrix} 0 & z^a \\ 0 & y \end{pmatrix}$ *(resp.* $\begin{pmatrix} y & z^a \\ 0 & 0 \end{pmatrix}$*)* <u>*never*</u> *degenerates to* $\begin{pmatrix} 0 & z^b \\ 0 & y \end{pmatrix}$ *(resp.* $\begin{pmatrix} y & z^b \\ 0 & 0 \end{pmatrix}$*) for all $a < b$.*

This talk is based on a joint work with Yuji Yoshino.

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Symmetric Hochschild extensions and the normalized 2-cocycles

Tomohiro Itagaki

In this talk, we give a sufficient condition related 2-cocycles for Hochschild extension algebras to be symmetric. Let K be an algebraically closed field. For a finite dimensional algebra A over a K, the trivial extension algebra $T(A) := A \ltimes \mathrm{Hom}_K(A, K)$ of A by the standard duality module $\mathrm{Hom}_K(A, K)$ is very important in the representation theory of self-injective algebras. This is also one of the Hochschild extension algebras of A. In particular, the equivalent class of the trivial extension corresponds to the cohomology class of zero map in the second Hochschild cohomology groups $\mathrm{H}^2(A, \mathrm{Hom}_K(A, K))$. It is well known that the trivial extension algebra $T(A)$ of K-algebra A is symmetric and the symmetric regular K-linear map $\mu : T(A) \to K$ is given by $\mu(a, f) = f(1)$, where $a \in A$ and $f \in \mathrm{Hom}_K(A, K)$. However, it is known that Hochschild extension algebras are self-injective [2] and they are not necessarily symmetric in general [1].

In [1], Ohnuki, Takeda and Yamagata gave a sufficient condition related to 2-cocycles for Hochschild extension algebras to be symmetric by giving a symmetric regular linear map. Moreover, they also shew that the condition is not a necessary condition for Hochschild extension algebras to be symmetric by giving an example of an algebra and a 2-cocycle.

In this talk, we give another sufficient condition related 2-cocycles for Hochschild extension algebras to be symmetric. The 2-cocycle in the above example satisfies our sufficient condition. Our idea is that we change $1 \in A$ which appeared at the image of μ into the same or another element in the center $\mathrm{Z}(A)$ of A depending on 2-cocycles.

Consequently, we can show that for any 2-cocycle α satisfied the sufficient condition in [1] there exists 2-cocycle β such that $[\beta] = [\alpha] \in \mathrm{H}^2(A, \mathrm{Hom}_K(A, K))$ and β satisfies the sufficient condition in our result. In order to prove that, we define E-normalized 2-cocycles, where E is a complete set of primitive orthogonal idempotents, and we also show that for any 2-cocycle α there exists a 2-cocycle $\overline{\alpha}$ such that the cohomology classes of α and $\overline{\alpha}$ coincide.

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On S-Noetherian rings

Jung Wook Lim

In commutative algebra, Noetherian rings are very important tools. Due to their importance, there have been several attempts to generalize the concept of Noetherian rings in order to extend the well-known results for Noetherian rings. One of them is the notion of S-Noetherian rings. Let R denote a commutative ring with identity, S a (not necessarily saturated) multiplicative subset of R, and M a unitary R-module. Anderson and Dumitrescu defined R an *S-Noetherian ring* if each ideal of R is S-finite, *i.e.*, for each ideal I of R, there exist an $s \in S$ and a finitely generated ideal J of R such that $sI \subseteq J \subseteq I$. They defined M to be *S-finite* if there exist an $s \in S$ and a finitely generated R-submodule F of M such that $sM \subseteq F$. Also, M is called *S-Noetherian* if each submodule of M is S-finite.

In this talk, we summarize some results on S-Noetherian rings.

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RELATIVE NON-COMMUTING GRAPH OF A FINITE RING

Jutirekha Dutta and Dhiren K. Basnet[*]

Let S be a subring of a finite ring R and $C_R(S) = \{r \in R : rs = sr \ \forall \ s \in S\}$. The relative non-commuting graph of the subring S in R, denoted by $\Gamma_{S,R}$, is a simple undirected graph whose vertex set is $R \setminus C_R(S)$ and two distinct vertices a, b are adjacent if and only if a or $b \in S$ and $ab \neq ba$. In this paper, we discuss some properties of $\Gamma_{S,R}$, determine diameter, girth, some dominating sets and chromatic index for $\Gamma_{S,R}$. Also, we derive some connections between $\Gamma_{S,R}$ and the relative commuting probability of S in R. Finally, we show that the relative non-commuting graphs of two relative $\mathbb{Z}$-isoclinic pairs of rings are isomorphic under some conditions. Some of the significant results are as given below.

Theorem 1. *Let S be a non-commutative subring of a ring R. Then $\Gamma_{S,R}$ is not a star graph or a bipartite graph or an n-regular graph for any square free odd positive integer n.*

Theorem 2. *Let R be a ring with unity and S a subring of R. Then $\Gamma_{S,R}$ is not complete.*

Theorem 3. *Let S be a non-commutative subring of a ring R. If $Z(S) = \{0\}$ then* $\text{diam}(\Gamma_{S,R}) = 2$ *and* $\text{girth}(\Gamma_{S,R}) = 3$.

Theorem 4. *Let S be a subring of a ring R and $A \subseteq V(\Gamma_{S,R})$. Then A is a dominating set for $\Gamma_{S,R}$ if and only if $C_R(A) \subseteq A \cup C_R(S)$.*

Theorem 5. *Let R be a ring. Then the non-commuting graph $\Gamma_{R,R}$ is of class* 2.

Theorem 6. *Let S be a subring of a ring R. Then the relation between $\Gamma_{S,R}$ and the relative commuting probability* $\Pr(S, R))$ *is*

$$|E(\Gamma_{S,R})| = |S||R|(1 - \Pr(S, R)) - \frac{|S|^2}{2}(1 - \Pr(S)).$$

Theorem 7. *Let S_1 and S_2 be two subrings of the finite rings R_1 and R_2 respectively. Let the pairs (S_1, R_1) and (S_2, R_2) are relative $\mathbb{Z}$-isoclinic. Then $\Gamma_{S_1,R_1} \cong \Gamma_{S_2,R_2}$ if $|Z(R_1) \cap S_1| = |Z(R_2) \cap S_2|$ and $|Z(R_1)| = |Z(R_2)|$.*

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The paper is in a final form and no version of it will be submitted for publication elsewhere.

On Nakayama Conjecture and related conjectures - Review

Masahisa Sato

Nakayama Conjecture is the old and world wide famous conjecture proposed by Japanese mathematician "Tadashi Nakayama" in 1958 who was the professor in Nagoya University. This conjecture was his challenge to homological algebra, in fact, he suspected the power of homological algebra which had been created in this era.
In fact, Professor Tachikawa taught us in his last lecture in 1994 that Professor Nakayama had proposed;

"**If homological algebra is enough powerful to mathematics,**
then solve this problem."

Since then, many many ring theorists have attempted to solve Nakayama Conjecture.
I believe that the greatest contributor of studying of Nakayama Conjecture is Professor Tachikawa.
Let's consider one aspect of Nakayama Conjecture, like the injective envelope $E(R)$ of an artinian ring R is projective. A ring with this property is called *QF-3 ring.*
Every one knows that theory of QF-3 rings are very important in Ring Theory.
The theory of QF-3 rings is developed and summarized in Tachikawa's lecture note LNM 351; "Quasi-Frobenius Rings and Generalizayuions, QF-3 and QF-1 Rings, " published in 1973.

In this talk, we shed light on Nakayama Conjecture again as turning point "**50th**" of our symposium hoping some Japanese young ring theorists are interested in and solve Nakayama Conjecture to progress ring theory more actively.

Nakayama Conjecture is as following.

Nakayama Conjecture *Let A be a finite dimensional algebra over a field K and $0 \to A \to E_1 \to E_2 \to \dots$ a minimal injective resolution of A. If all E_i's are projective, then A is self-injective.*

Also we give relating problems so called "Generalized Nakayama Conjecture", "Strong Nakayama Conjecture", "Tachikawa Conjecture", "Finitistic dimension Conjecture" and so on. We discuss relations between their conjectures.

This lecture is the same one as the Invited lecture I did in English for International Conference on Algebra and its Applications held in Aligarh Muslim University, India . So this time, I will give my lecture in Japanese in order that I can explain more correctly and young Japanese ring theorists can understand much more.

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When is an abelian category a quantum projective space?

Izuru Mori and Kenta Ueyama

Fix a field k. For a locally finite graded right coherent algebra A over k, we define the quotient category $\operatorname{tails} A := \operatorname{grmod} A/\operatorname{fdim} A$ where $\operatorname{grmod} A$ is the category of finitely presented graded right A-modules, and $\operatorname{fdim} A$ is the full subcategory of $\operatorname{grmod} A$ consisting of modules finite dimensional over k. The k-linear abelian category $\operatorname{tails} A$ is called the noncommutative projective scheme associated to A by Artin-Zhang [1] since if A is commutative and finitely generated in degree 1 over k, then $\operatorname{tails} A$ is equivalent to the category of coherent sheaves over the projective scheme associated to A by Serre [4]. Moreover, if A is an AS-regular algebra introduced by Artin-Schelter [1], then $\operatorname{tails} A$ is regarded as a quantum projective space since an AS-regular algebra is a noncommutative generalization of a polynomial algebra, and a projective space is exactly the projective scheme associated to a polynomial algebra. Since a projective space is the most basic example of a projective scheme in commutative algebraic geometry, a quantum projective space is the most basic example of a noncommutative projective scheme in noncommutative algebraic geometry.

In this talk, we will characterize a k-linear abelian category $\mathscr{C}$ such that $\mathscr{C} \cong \operatorname{tails} A$ for some graded right coherent AS-regular algebra A over R introduced in [3]. If time permits, we will prove that if $\mathscr{C}$ is a smooth quadric surface in a quantum $\mathbb{P}^3$ in the sense of Smith-Van den Bergh [5], then there exists a right noetherian AS-regular algebra A over kK_2 of dimension 3 and of Gorenstein parameter 2 such that $\mathscr{C} \cong \operatorname{tails} A$ where kK_2 is the path algebra of the 2-Kronecker quiver.

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Two constructions of Iwanaga-Gorenstein algebras

Aaron Chan

We consider two series $\{A^{(m)}\}_{m\geq 1}, \{A^{[m]}\}_{m\geq 1}$ of algebras constructed from a given algebra A via two different methods. As far as we know, the algebra $A^{(m)}$ was first studied by Assem and Iwanaga [1], whereas $A^{[m]}$ was first studied by Yamagata [2]. Both of these works show that when A is hereditary, then the global dimension increases as m increases. In the case of $A^{[m]}$, the same holds for the series of dominant dimensions of $A^{[m]}$.

In this talk, we show how one can calculate the self-injective dimension and dominant dimension explicitly of $A^{(m)}$ for a hereditary algebra A. We use this to show that A being representation-finite is equivalent to having one (or equivalently, infinitely many) of the $A^{(m)}$'s being a higher Auslander algebra, i.e. the global dimension and dominant dimension coincides. Moreover, it turns out that the same result holds after replacing $A^{(m)}$ by $A^{[m]}$. If time allows, we will also explain how this can be generalised to a much wider class of Iwanaga-Gorenstein algebras.

This is a joint work with Osamu Iyama and René Marczinzik.

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Classifications of exact structures and Cohen-Macaulay-finite algebras

Haruhisa Enomoto

In this talk, I will discuss a classification of exact structures on a given additive category and its application, based on [1]. Exact categories, in the sense of Quillen, have been playing an important role in the representation theory of algebras. In general, an additive category has many exact structures. Recently, Rump [3] showed that every additive category has the largest exact structures, but no general description of exact structures was known. We give an explicit description of all exact structures on a given additive category $\mathcal{E}$ by using particular modules over $\mathcal{E}$ (equivalently, modules over the Auslander algebra of $\mathcal{E}$).

Let k be a field. For simplicity, all algebras are assumed to be finite dimensional over k. To this end, the following condition for simple modules plays an indispensable role.

Definition 1. Let Γ be an algebra and S a simple Γ-module. We say that *S satisfies the* 2*-regular condition* if the following conditions are satisfied.

(1) The projective dimension of S is equal to 2.
(2) $\mathrm{Ext}^i_\Gamma(S,\Gamma)=0$ for $i=0,1$.
(3) $\mathrm{Ext}^2_\Gamma(S,\Gamma)$ is simple Γ^{op}-module.

Surprisingly, the following shows that categorical notion (exact structures) is deeply related to homological condition (2-regular conditions). Also this can be seen as a classification of exact categories with finitely many indecomposables.

Theorem 2. *Let $\mathcal{E}$ be an idempotent complete Hom-finite additive k-category with finitely many indecomposables, and let Γ be its Auslander algebra. Then there exists a bijection between the following two classes.*

(1) *Exact structures on $\mathcal{E}$.*
(2) *Sets of simple Γ-modules satisfying the* 2*-regular condition.*
(3) *Sets of dotted arrows in the translation quiver $Q(\Gamma)$ associated with Γ.*

As an application, we give the Auslander-type correspondence for Cohen-Macaulay-finite Iwanaga-Gorenstein algebras. We say that an algebra Λ is *Iwanaga-Gorenstein* if the left and right injective dimension of Γ itself is finite. For such an algebra Λ, a finitely generated Γ-module X is called *Cohen-Macaulay* if $\mathrm{Ext}^i_\Lambda(X,\Lambda)=0$ for all $i>0$. We say that an Iwanaga-Gorenstein algebra is *Cohen-Macaulay-finite* (CM-finite) if there exist finitely many Cohen-Macaulay modules up to isomorphism. By using the previous theorem and the results in [2], we proved the following.

Theorem 3. *There exists a bijection between the following two classes.*

(1) *Morita-equivalence classes of CM-finite Iwanaga-Gorenstein algebras.*
(2) *Equivalence classes of pairs (Γ,X), where Γ is an algebra with finite global dimension and X is a union of stable τ-orbits in the translation quiver $Q(\Gamma)$.*

Moreover, we give an explicit method to construct a CM-finite algebra from the pair (Γ,X). This gives a systematic method to construct CM-finite Iwanaga-Gorenstein algebras, and all such algebras are obtained in this way. Thus our result reduces the classification problem of CM-finite Iwanaga-Gorenstein algebras to that of algebras with finite global dimension.

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Sigma-pure-injective modules for string algebras

William Crawley-Boevey

This is joint work with Raphael Bennett-Tennenhaus. I will discuss the classification of modules for string algebras, and especially the classification of Σ-pure-injective modules. It turns out that they are direct sums of string and band modules.

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The Feit-Thompson conjecture is true

Kaoru Motose

We set $f := \frac{q^p-1}{q-1}$ and $t := \frac{p^q-1}{p-1}$ for primes $2 < p < q$. Feit and Thompson conjectured f never divides t. It is trivial for $p = 2$. The utility and the cause of this conjecture are stated in [1], [2] and [3, B25].

If d is a common divisor of f and t then $d \equiv 1 \bmod 2pq$ (see for example [5]). Using this fact and a computer, Stephans found the unique example : for $p = 17$ and $q = 3313$, the prime $r = 2pq+1 = 112643$ is the greatest common divisor of f and t (see [6]). This example was the conductor to the proof of our theorem. In this talk, we shall give a proof of the next theorem.

Theorem. If f is a divisor of t, then f is prime and $f \le q^2 - q + 1$. Hence f never divides t.

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The number of partial matrix rings

Gangyong Lee*, Mauricio Medina-Bárcenas, and Khanh Tung Nguyen
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Recently, Lee, Roman, and Zhang defined a partial matrix ring. We call an $n \times n$ *partial matrix ring over a ring* A, denoted by $\mathsf{PM}_n(A)$, a subring of a full $n \times n$ matrix ring over A, with elements matrices whose entries are either elements of A or 0, such that nonzero entries are independent of each other. I.e., $\mathsf{PM}_n(A) = \sum_{(i,j)\in\mathcal{U}} e_{ij}A$ where e_{ij} are matrix units and $\mathcal{U}$ is a subset of the index set $\mathcal{I} \times \mathcal{I}$, $\mathcal{I} = \{1, 2, \ldots, n\}$. Note that in a partial matrix ring $R = \mathsf{PM}_n(A)$, $\sum_{i=1}^{n} e_{ii}A \subseteq R$ (because R has the unity) and not every choice of an index-pair set $\mathcal{U}$ will generate a structure closed under multiplication of matrices.

Even if we do not know whether $\mathsf{PM}_n(A) = \sum_{(i,j)\in\mathcal{U}} e_{ij}A$ is a ring or not (it depends on a set $\mathcal{U}$ as above), in this talk, we will provide how to make partial matrix rings as subrings of $n \times n$ full matrix rings and the number of $n \times n$ partial matrix rings for some $n \in \mathbb{N}$. Using the concept of pre ordered sets and partial ordered sets, we will achieve our result. This talk is based on a joint work with Mauricio Medina-Bárcenas and Khanh Tung Nguyen.

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The ordinary quivers of Hochschild extension algebras for self-injective Nakayama algebras

Hideyuki Koie, Tomohiro Itagaki and Katsunori Sanada

This talk is based on [3]. Let K be an algebraically closed field, A a finite dimensional algebra over K and D the standard duality functor $\mathrm{Hom}_K(-, K)$. By a Hochschild extension over A by $D(A)$, we mean an exact sequence $0 \longrightarrow D(A) \xrightarrow{\kappa} T \xrightarrow{\rho} A \longrightarrow 0$ such that T is a K-algebra, ρ is an algebra epimorphism and κ is a T-bimodule monomorphism. The algebra T is called a Hochschild extension algebra. It is well known that T is a self-injective algebra (see [5]). Hochschild [2] proved that the set of equivalence classes of Hochschild extensions over A by $D(A)$ is in one-to-one correspondence with the second Hochschild cohomology group $H^2(A, D(A))$. We denote by $T_\alpha(A)$ the Hochschild extension algebra corresponding to a 2-cocycle α. Then, $T_0(A)$ is just the trivial extension algebra $A \ltimes D(A)$. In [1], Fernandez and Platzeck gave the ordinary quiver for the trivial extension algebra of A by $D(A)$. Moreover, they described the relations for the trivial extension algebra under the assumption that any oriented cycle in the ordinary quiver of A is zero in A. It seems that there is little information about the ordinary quivers for general Hochschild extension algebras.

We are interested in the ordinary quivers for Hochschild extension algebras of a self-injective Nakayama algebra. Let Δ be the following cyclic quiver with $s\,(\geq 1)$ vertices and s arrows. Suppose $n \geq 2$ and $A = K\Delta/R_\Delta^n$, where R_Δ^n is the two-sided ideal of $K\Delta$ generated by the paths of length n. We determine the ordinary quivers for Hochschild extension algebras of A by constructing the isomorphism $\Theta : \bigoplus_q D(HH_2(A)) \xrightarrow{\sim} H^2(A, D(A))$ and referring to the Sköldberg's results in [4], that is, the Hochschild homology group $HH_p(A)$ is $\mathbb{N}$-graded by the length of cycles in the quiver, and the degree q part $HH_{p,q}(A)$ is explicitly computed.

Theorem 1. *Suppose that $n \geq 2$, $A = K\Delta/R_\Delta^n$ and $n \leq q \leq 2n-1$. Let $\alpha : A \times A \to D(A)$ be a 2-cocycle such that the cohomology class $[\alpha]$ of α belongs to $\Theta(D(HH_{2,q}(A)))$ and $[\alpha] \neq 0$, and let $T_\alpha(A)$ be the Hochschild extension algebra of A defined by α. Then the ordinary quiver $\Delta_{T_\alpha(A)}$ is given by*

$$\Delta_{T_\alpha(A)} = \begin{cases} \Delta_{T_0(A)} & \text{if } n \leq q \leq 2n-2, \\ \Delta & \text{if } q = 2n-1. \end{cases}$$

Corollary 2. *Suppose that $n \geq 2$ and $A = K\Delta/R_\Delta^n$. Let $\alpha : A \times A \to D(A)$ be a 2-cocycle and $[\alpha] = \sum_{q=n}^{2n-1} [\beta_q]$, where $\beta_q : A \times A \to D(A)$ is a 2-cocycle such that the cohomology class $[\beta_q]$ of β_q belongs to $\Theta(D(HH_{2,q}(A)))$. Then the following equation holds*:

$$\Delta_{T_\alpha(A)} = \begin{cases} \Delta_{T_0(A)} & \text{if } [\beta_{2n-1}] = 0, \\ \Delta & \text{if } [\beta_{2n-1}] \neq 0. \end{cases}$$

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Complex rings, Quaternion rings and Octonion rings

Isao Kikumasa, Kazutoshi Koike and Kiyoichi Oshiro

In 1840s, Hamilton discovered quaternions and Kelly, Graves independently discovered octonions. These numbers are defined over real numbers and contain complex numbers. Through Frobenius, Wedderburn, Noether, many mathematician contineously studied these numbers. We may say that the roots of the ring theory begin with these numbers.

In order to define these numbers for any ring R, we consider free R-modules:

$\mathbb{C}(R) = e_0R \oplus e_1R$,
$\mathbb{H}(R) = e_0R \oplus e_1R \oplus e_2R \oplus e_3R$,
$\mathbb{O}(R) = e_0R \oplus e_1R \oplus \cdots \oplus e_7R$

We define $re_i = e_ir$ for any $r \in R$ and any e_i, and multiplications in $\{e_i\}$ are defined by using the so-called Kelly-Graves multiplication table. Then $\mathbb{C}(R)$, $\mathbb{H}(R)$ become rings and $\mathbb{O}(R)$ becomes a non-associative ring. We call that $\mathbb{C}(R)$ is a complex ring, $\mathbb{H}(R)]$ is a quaternion ring and $\mathbb{O}(R)$ is an octonion ring. For $\mathbb{C}(R)$, $\mathbb{H}(R)$, we put $e_0 = 1$, $e_1 = i$, $e_2 = j$, $e_3 = k$. Then multiplications for $\{i, j, k\}$ are usual forms as follows:

$$i^2 = j^2 = k^2 = -1,$$
$$ij = k,\ jk = i,\ ki = j,\ ji = -k,\ kj = -i,\ ik = -j$$

For the field $\mathbb{R}$ of real numbers and a commutative field F, $\mathbb{H}(\mathbb{R})$, $\mathbb{H}(F)$ are studied as number theory.

Recently, Lee-Oshiro showed the following result (Fronties Mathematics):

(A) If R is a Frobenius algebra, then $\mathbb{C}(R)$, $\mathbb{H}(R)$, $\mathbb{O}(R)$ are Frobenius algebras.

(B) If R is a quasi-Frobenius ring, then $\mathbb{C}(R)$、$\mathbb{H}(R)$ are quasi-Frobenius rings.

It follows from (B) that, for a division ring D, $\mathbb{C}(D)$, $\mathbb{H}(D)$ are quasi-Frobenius rings. In this talk, we report on the structure of these quasi-Frobenius rings. In particular, we show that if the characteristic of D is not 2, then these quasi-Frobenius rings are division rings or simple rings. We comment the conherence of this result with classical theory on $\mathbb{H}(F)$ over a field F.

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Source algebra version of Donovan's conjecture for finite group algebras

Shigeo Koshitani

In the modular representation theory of finite groups we have Donovan's conjecture, which says that for a given finite p-group D (where p is a prime number), there should be only FINITE number of Morita equivalence classes of block algebras B of certain finite group algebras such that D is a defect group of B. There is a stronger notion called a "Puig equivalence" than a Morita equivalence, and we have a similar conjecture. We will be discussing the finiteness of Puig equivalence classes of block algebras for finite group algebras. This is joint work with Caroline Lassueur as well.

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On finitely graded IG-algebras and the stable categories of their (graded) CM-modules

Hiroyuki Minamoto and Kota Yamaura

First we discuss homological dimensions of trivial extension algebras $A = \Lambda \oplus C$. The global dimension had been studied by Fossum-Griffith-Reiten [3], Reiten [8], Palmer-Roos [7]. Finally, Löfwall [6] gave a formula for the global dimension of A in terms of Λ and C by using "multiple Tor". We give formulas of injective and projective dimensions by using the iterated derived tensor product $C^a := C \otimes^{\mathbb{L}}_{\Lambda} C \otimes^{\mathbb{L}}_{\Lambda} \cdots \otimes^{\mathbb{L}}_{\Lambda} C$. We always equip A with the grading $\deg \Lambda = 0, \ \deg C = 1$.

Theorem 1. *Let M be a graded (right) A-module such that $M_i = 0$ for $i \neq 0, 1$. Then,*

$$\mathrm{id}_A M = \mathrm{gr.id}_A M = \sup\{\mathrm{id}_\Lambda M_1, \mathrm{id}_\Lambda(\Theta^a_M) + a + 1 \mid a \geq 0\}$$

where Θ^a_M denotes the cone of the morphism $\mathbb{R}\mathrm{Hom}_\Lambda(C^a, M_0) \to \mathbb{R}\mathrm{Hom}_\Lambda(C^{a+1}, M_1)$ induced from the A-module structure on M.

Next we discuss a finitely graded Iwanaga-Gorenstein(IG) algebra A. Representation theory of IG-algebra was initiated by Auslander-Reiten [1], Happel [4] and Buchweitz [2], has been studied by many researchers and is recently getting interest from other areas. We remark that every finitely graded algebra is graded Morita equivalent to a trivial extension algebra via quasi-Veronese algebra construction introduced by Mori [5]. So we concentrate on this case. Theorem 1 gives a condition that $A = \Lambda \oplus C$ to be IG, in terms of Λ and C. We study the condition in the case where Λ is IG. But for simplicity, in what follows, we assume that $\mathrm{gldim}\Lambda < \infty$.

Theorem 2. *Assume that ${}_\Lambda C$ and C_Λ are finitely generated module of* $\mathrm{pd} < \infty$. *Then (1) A is IG if and only if $\mathsf{D}^b(\mathrm{mod}\Lambda)$ has an admissible subcategory T such that the functor $F = - \otimes^{\mathbb{L}}_{\Lambda} C$ acts on it as an equivalence and that F nilpotently acts on $\mathsf{T}^{\perp}$.*

(2) If (1) is the case, then the restriction $\varpi|_{\mathsf{T}}$ of canonical functor $\varpi : \mathsf{D}^b(\mathrm{mod}\Lambda) \to \underline{\mathsf{CM}}^{\mathbb{Z}} A$ is an equivalence.

We remark that there is a characterization involving duality by C. The characterization (1) enables us to give a classification of C such that $\Lambda \oplus C$ is IG in the case where Λ is a path algebra of A_2 or A_3-quivers. It follows from (2) that if A is finite dimensional, then the Grothendick group $K_0(\underline{\mathsf{CM}}^{\mathbb{Z}} A)$ is finite free module. Other applications will be given in the talk or the poster.

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LATTICE OF TORSION CLASSES

Laurent DEMONET

This is a report on a joint work with Osamu Iyama, Nathan Reading, Idun Reiten and Hugh Thomas.

Let A be a finite dimensional algebra on a field k. A torsion class $\mathcal{T}$ in the category $\operatorname{mod} A$ of finite-dimensional A-modules is a full subcategory that is closed under extensions and quotients. Suppose that $\operatorname{mod} A$ contains only finitely many torsion classes. In this case, the set of torsion classes ordered by inclusion is a finite lattice $\operatorname{tors} A$. We study the lattice quotients of $\operatorname{tors} A$ from an algebraic point of view. In particular, we give an algebraic categorification of the *forcing* relation on arrows of the Hasse quiver of $\operatorname{tors} A$. We deduce some important combinatorial results about $\operatorname{tors} A$, in particular that it is *congruence uniform*. Suppose now that B is a quotient of A by an ideal. It is elementary that $\operatorname{tors} B$ is a lattice quotient of $\operatorname{tors} A$ via $\mathcal{T} \mapsto \mathcal{T} \cap \operatorname{mod} B$. Therefore, we are interested in characterizing lattice quotients L of $\operatorname{tors} A$ that are of the form $\operatorname{tors} B$. We give several necessary conditions on L which become sufficient when A is sufficiently easy. Il is the case in particular for preprojective algebras of type A_n and hereditary algebras of finite representation type. Finally, using preprojective algebras, we get new algebraic proofs of results on Cambrian lattices.

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How to capture t-structures by silting theory

Takahide Adachi and Mizuno Yuya
(This is a joint work with Dong Yang.)

The notion of t-structures, introduced by Beilinson-Bernstein-Deligne [2], has been playing an increasing important role, due to relationships to silting theory [6, 7], cluster tilting theory [1, 5], the theory of Bridgeland's stability conditions [3], and so on.

In this talk, we study bounded t-structures from the viewpoint of silting theory and cluster tilting theory. We introduce the notion of ST-pairs, which gives us a framework of our study. The following examples are motivating examples of an ST-pair.

Example 1. (1) Let Λ be a finite-dimensional algebra. Let $\mathcal{C} := \mathsf{K}^{\mathrm{b}}(\mathsf{proj}\Lambda)$ the bounded homotopy category of finitely generated projective Λ-modules, and $\mathcal{D} := \mathsf{D}^{\mathrm{b}}(\mathsf{mod}\Lambda)$ the bounded derived category of finitely generated Λ-modules. Then $(\mathcal{C}, \mathcal{D})$ is an ST-pair.

(2) Let Γ be a dg algebra satisfying the following conditions:
- $H^p(\Gamma) = 0$ for each integer $p > 0$.
- $H^0(\Gamma)$ is finite dimensional.
- $\mathsf{D}_{\mathrm{fd}}(\Gamma) \subseteq \mathsf{per}\Gamma$.

Here $\mathcal{C} := \mathsf{per}\Gamma$ is the perfect derived category of Γ and $\mathcal{D} := \mathsf{D}_{\mathrm{fd}}(\Gamma)$ the derived category of Γ with finite dimensional total cohomology. Then $(\mathcal{C}, \mathcal{D})$ is an ST-pair.

In both cases in Example 1, there is an injective map from the set of isoclasses of basic silting objects of $\mathcal{C}$ to the set of bounded t-structures on $\mathcal{D}$ (see [7] and [4]).

We generalize this result to the case of an ST-pair. Moreover, we give a characterization of the condition that the injective map is bijective. This characterization allows us to capture all bounded t-structures by silting theory. As an application, using the characterization and cluster tilting theory, we give a classification of silting-discrete perfect derived categories of derived preprojective algebras and Ginzburg dg algebras.

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Representations of equipped graphs

William Crawley-Boevey

An *equipped graph* can be thought of as a generalization of a quiver, in which one is allowed not just arrows $\bullet \longrightarrow \bullet$, but also edges with two heads $\bullet \longleftrightarrow \bullet$ or two tails $\bullet$ —— $\bullet$.

Gelfand and Ponomarev [2] introduced representations of equipped graphs by vector spaces and linear relations. Generalizing Gabriel's Theorem, they showed that the equipped graphs of finite representation type are exactly those whose underlying graph is a Dynkin diagram.

For example, by taking all edges to be two headed or two tailed, one can study the representation theory of a graph without needing to choose an orientation.

I will discuss an analogue of Kac's Theorem for equipped graphs, which I obtained in [1], and give illustrations of the Auslander-Reiten theory for equipped graphs.

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