INFINITE SEQUENCES OF FROBENIUS EXTENSIONS

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Abstract. In this talk, to each Frobenius extension of first kind A/R we associate a sequence of ring extensions $A_0 = R \subset A_1 = A \subset \cdots \subset A_n \subset \cdots$ such that each A_{i+1}/A_i is a Frobenius exteinsion of first kind.

Introduction

In this talk, to each Frobenius extension of first kind A/R we associate a sequence of ring extensions

$$A_0 = R \subset A_1 = A \subset \cdots \subset A_n \subset \cdots$$

such that each A_{i+1}/A_i is a Frobenius exteinsion of first kind.

An important thing on Frobenius extensions is that Frobenius extensions of Auslander-Gorenstein rings are Auslander-Gorenstein. It should be noted that Auslander-Gorenstein rings appear in various fields of current research in mathematics.

1. Preliminaries

Throughout the rest of this talk, R stands for an arbitrary ring.

We recall the notion of Frobenius extensions of rings due to Nakayama and Tsuzuku [4, 5], which we modify as follows (cf. [1, 3]).

Definition 1 ([3])**.** A ring A is said to be an extension of R if A contains R as a subring, and the notation A/R is used to denote that A is an extension ring of R. A ring extension A/R is said to be Frobenius if the following conditions are satisfied:
(F1) A is finitely generated as a left R-module;
(F2) A is finitely generated projective as a right R-module;
(F3) $A \cong \mathrm{Hom}_R(A, R)$ as right A-modules.

We refer to [2] for the definition of Auslander-Gorenstein rings.

Proposition 2 ([3, Proposition 1.9])**.** *For any Frobenius extension A/R, if R is an Auslander-Gorenstein ring, then so is A with* $\mathrm{inj\ dim}\ A \leq \mathrm{inj\ dim}\ R$.

2. Main results

We deal with Frobenius extensions of first kind and state the main results.

Definition 3 (cf. [4, 5])**.** A Frobenius extension A/R is said to be of first kind if $A \cong \mathrm{Hom}_R(A, R)$ as R-A-bimodules.

The detailed version of this paper will be submitted for publication elsewhere.

Theorem 4. *Let P be a finitely generated projective right R-module and $Q = \mathrm{Hom}_R(P, R)$. Put $\Lambda = \mathrm{End}_R(P)$. Assume that there exists a subring $A \subset \Lambda$ satisfying the following conditions:*

(1) *Q is finitely generated projective as a right A-module;*

(2) *$P \cong \mathrm{Hom}_A(Q, A)$ as A-R-bimodules.*

Then Λ/A is a Frobenius extension of first kind.

Theorem 5. *For any Frobenius extension of first kind A/R there exists a sequence of ring extensions*

$$A_0 = R \subset A_1 = A \subset \cdots \subset A_{n+1} = \mathrm{End}_{A_{n-1}}(A_n) \subset \cdots$$

such that A_{i+1}/A_i is a Frobenius extension of first kind for all $i \geq 0$.

References

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