FUNCTOR CATEGORIES ON DERIVED CATEGORIES OF HEREDITARY ALGEBRAS

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ABSTRACT. For a triangulated category $\mathcal{T}$, it is known that the category of finitely presented functors $\mathsf{mod}\,\mathcal{T}$ on $\mathcal{T}$ is a Frobenius category. Let A be a representation finite hereditary algebra. Iyama and Oppermann [IO] showed that the category $\underline{\mathsf{mod}}\,\mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,A)$ is triangle equivalent to the bounded derived category of the stable Auslander algebra of A. In this paper, we extend this triangle equivalence to the case when A is a representation infinite hereditary algebra.

1. Introduction

Let k be a field and A be a finite dimensional k-algebra. In [IO], it was shown that if A is a representation finite hereditary algebra, then there exists a triangle equivalence

$$\underline{\mathsf{mod}}\mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,A) \simeq \mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,\Gamma_A), \tag{1.1}$$

where Γ_A is the stable Auslander algebra of A, that is, the Auslander algebra of $\underline{\mathsf{mod}}\,A$.

In this paper, we extend a triangle equivalence (1.1) to the case when A is a representation infinite hereditary algebra, that is, our main theorem of this paper is Theorem 14. In this case, the role of the stable Auslander algebra is played by the category $\mathsf{mod}(\underline{\mathsf{mod}}\,A)$ of finitely presented functors on $\underline{\mathsf{mod}}A$.

To prove Theorem 14, we need to give general preliminary results on functor categories and repetitive categories. The functor category $\mathsf{mod}(\underline{\mathsf{mod}}\,A)$ is an abelian category with enough projectives and enough injectives since the category $\underline{\mathsf{mod}}\,A$ forms a dualizing k-variety, which is a distinguished class of k-linear categories introduced by Auslander and Reiten [AR]. A key role is played by the repetitive category $\mathsf{R}(\underline{\mathsf{mod}}\,A)$ of $\underline{\mathsf{mod}}\,A$. We see that for a dualizing k-variety $\mathcal{A}$, its repetitive category $\mathsf{R}\mathcal{A}$ is also a dualizing k-variety (Theorem 12). Moreover, we see that $\mathsf{mod}\mathsf{R}\mathcal{A}$ is a Frobenius abelian category.

In the case where A is a representation finite hereditary algebra, the Happel's theorem (Theorem 5) played an important role in the proof of a triangle equivalence (1.1). We see that a categorical analog of this triangle equivalence for dualizing k-varieties holds. In fact, we deal with the following more general class of categories including dualizing k-varieties. For a k-linear additive category $\mathcal{A}$, we denote by $\mathsf{proj}\mathcal{A}$ the category of finitely generated projective $\mathcal{A}$-modules and by $\mathsf{mod}_\infty\mathcal{A}$ the category of $\mathcal{A}$-modules having resolutions by $\mathsf{proj}\mathcal{A}$. We consider the following conditions:

(IFP) $\mathrm{D}\mathcal{A}(X,-)$ is in $\mathsf{mod}_\infty\mathcal{A}$ for each $X \in \mathcal{A}$, where $\mathrm{D} = \mathrm{Hom}_k(-,k)$.

(G) $\mathrm{D}\mathcal{A}(X,-)$ has finite projective dimension over $\mathcal{A}$ for each $X \in \mathcal{A}$.

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For example, if $\mathcal{A}$ is a dualizing k-variety, then $\mathcal{A}$ satisfies the condition (IFP). On the other hand, the condition (G) is a categorical version of Gorensteinness. Gorenstein-projective modules (also known as Cohen-Macaulay modules, totally reflexive modules) are important class of modules. We denote by $\mathsf{GP}(\mathsf{R}\mathcal{A}, \mathcal{A})$ the category of Gorenstein-projective $\mathsf{R}\mathcal{A}$-modules of finite projective dimension as $\mathcal{A}$-modules. Under the conditions (IFP) and (G), we can show Theorem 9, which induces a categorical analog of Happel's triangle equivalence.

Notation. In this paper, we denote by k a field. All categories are k-linear additive categories. All subcategories are full and closed under isomorphisms. Let $\mathcal{C}$ be a k-linear additive category and $\mathcal{S}$ be a subclass of objects of $\mathcal{C}$ or a subcategory of $\mathcal{C}$. We denote by $\mathsf{add}\,\mathcal{S}$ the subcategory of $\mathcal{C}$ whose objects are direct summands of finite direct sums of objects in $\mathcal{S}$. For objects $X, Y \in \mathcal{C}$, we denote by $\mathcal{C}(X, Y)$ the set of morphisms from X to Y in $\mathcal{C}$. We call $\mathcal{C}$ *Hom-finite* if $\mathcal{C}(X, Y)$ is finitely generated over k for any $X, Y \in \mathcal{C}$. We call a category *skeletally small* if the class of isomorphism class of objects is a set. We assume that all categories in this paper are skeletally small.

All algebras are k-algebras. For an algebra A, we denote by $\mathsf{mod}\,A$ the category of finitely generated left A-modules and by $\underline{\mathsf{mod}}\,A$ the projectively stable category of A.

2. Functor categories

In this section, we recall the definition of modules over categories. Let $\mathcal{A}$ be a k-linear additive category. An *$\mathcal{A}$-module* is a contravariant additive k-linear functor from $\mathcal{A}$ to $\mathsf{mod}\,k$. We denote by $\mathsf{Mod}\mathcal{A}$ the category of $\mathcal{A}$-modules, where morphisms of $\mathsf{Mod}\mathcal{A}$ are morphisms of functors. Since $\mathcal{A}$ is skeletally small, $\mathsf{Mod}\mathcal{A}$ is a category. It is well known that $\mathsf{Mod}\mathcal{A}$ is abelian.

Example 1. For each $X \in \mathcal{A}$, we have an $\mathcal{A}$-module $\mathcal{A}(-, X)$. By Yoneda's lemma, $\mathcal{A}(-, X)$ is projective in $\mathsf{Mod}\mathcal{A}$.

Let M be an $\mathcal{A}$-module. We call M a *finitely presented* module if there exists an exact sequence $\mathcal{A}(-, X) \to \mathcal{A}(-, X) \to M \to 0$ in $\mathsf{Mod}\mathcal{A}$ for some $X, Y \in \mathcal{A}$. We denote by $\mathsf{mod}\mathcal{A}$ the subcategory of $\mathsf{Mod}\mathcal{A}$ consisting of finitely presented functors. Note that $\mathsf{mod}\mathcal{A}$ is not necessarily an abelian category.

Let A be a finite dimensional algebra. We denote by $\mathsf{proj}\,A$ the category of finitely generated projective A-modules. Then we have an equivalence $\mathsf{mod}(\mathsf{proj}\,A) \simeq \mathsf{mod}\,A$. Assume that A is a representation finite algebra, that is, there exists a basic A-module M satisfying $\mathsf{add}\,M = \mathsf{mod}A$. We call the algebra $\Gamma_A := \mathrm{End}_{\underline{\mathsf{mod}}\,A}(M)$ *the stable Auslander algebra of* A. We have an equivalence $\mathsf{proj}\,\Gamma_A \simeq \underline{\mathsf{mod}}\,A$.

A finite dimensional algebra is said to be *hereditary* if the global dimension is at most one. We recall the result of Iyama and Oppermann.

Theorem 2. [IO, Corollary 4.11] *Let A be a finite dimensional representation finite hereditary algebra and Γ_A the stable Auslander algebra of A. Then we have a triangle equivalence*

$$\underline{\mathsf{mod}}\,\mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,A) \simeq \mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,\Gamma_A).$$

In this paper, we extend the triangle equivalence of Theorem 2 to the case when A is a representation infinite hereditary algebra. We deal with the derived category of $\mathsf{mod}(\underline{\mathsf{mod}}\,A)$ in stead of that of $\mathsf{mod}\,\Gamma_A$, since $\mathsf{mod}\,\Gamma_A \simeq \mathsf{mod}(\underline{\mathsf{mod}}\,A)$ holds if A is a representation finite algebra.

3. Repetitive categories

In this section, we recall the definition of the repetitive category of an additive category. As we see in Proposition 4, the repetitive category of the stable category of A is equivalent to the derived category of A. Let $\mathrm{D} := \mathrm{Hom}_k(-, k)$ be the standard k-dual.

Definition 3. Let $\mathcal{A}$ be a k-linear additive category. The *repetitive category* $\mathsf{R}\mathcal{A}$ is the k-linear additive category generated by the following category: the class of objects is $\{(X, i) \mid X \in \mathcal{A}, i \in \mathbb{Z}\}$ and the morphism space is given by

$$\mathsf{R}\mathcal{A}\big((X,i),(Y,j)\big) = \begin{cases} \mathcal{A}(X,Y) & i = j, \\ \mathrm{D}\mathcal{A}(Y,X) & j = i+1, \\ 0 & \text{else.} \end{cases}$$

For $f \in \mathsf{R}\mathcal{A}\big((X,i),(Y,j)\big)$ and $g \in \mathsf{R}\mathcal{A}\big((Y,j),(Z,k)\big)$, the composition is given by

$$g \circ f = \begin{cases} g \circ f & i = j = k, \\ \big(\mathrm{D}\mathcal{A}(Z,f)\big)(g) & i = j = k-1, \\ \big(\mathrm{D}\mathcal{A}(g,X)\big)(f) & i+1 = j = k, \\ 0 & \text{else.} \end{cases}$$

Since the global dimension of a hereditary algebra A is at most one, $\mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,A)$ has a Serre functor $\mathbb{S} = \mathrm{D}(A) \otimes^{\mathbb{L}}_A -$. If A is hereditary, then there exists a fully faithful functor $\underline{\mathsf{mod}}\,A \to \mathsf{mod}\,A$. By this functor, we regard $\underline{\mathsf{mod}}\,A$ as a subcategory of $\mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,A)$. The following proposition was shown by [IO] for a representation finite hereditary algebra and by [K] for a representation infinite hereditary algebra.

Proposition 4. [IO, K] *Let A be a hereditary algebra. Then we have an additive equivalence $\mathsf{R}(\underline{\mathsf{mod}}\,A) \simeq \mathsf{D}^{\mathrm{b}}(\mathsf{mod} A)$ given by $(X, i) \mapsto \mathbb{S}^{-i}(X)$.*

Therefore to show Theorem 14, it is enough to show that there exists an equivalence between the category $\underline{\mathsf{mod}}(\mathsf{R}(\underline{\mathsf{mod}}\,A))$ and the bounded derived category $\mathsf{D}^{\mathrm{b}}(\mathsf{mod}(\underline{\mathsf{mod}}\,A))$. As we see in the following, if A is a representation finite hereditary algebra, then this equivalence is nothing but Happel's triangulated equivalence.

Let A be a representation finite hereditary algebra. It is easy to see that $\mathsf{mod}\,\mathsf{R}(\mathsf{proj}\Gamma_A) \simeq \mathsf{mod}\,\widehat{\Gamma_A}$, where $\widehat{\Gamma_A}$ is the repetitive algebra of Γ_A (see [H]). About the repetitive algebras, Happel showed the following theorem.

Theorem 5. [H] *Let A be a finite dimensional algebra and $\widehat{A}$ be the repetitive algebra of A. Then $\mathsf{mod}\,\widehat{A}$ is a Frobenius abelian category. If the global dimension of A is finite, then we have a triangle equivalence $\underline{\mathsf{mod}}\,\widehat{A} \simeq \mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,A)$.*

By applying Proposition 4 and Theorem 5, we have Theorem 2.

Proof of Theorem 2. Let A be a finite dimensional representation finite hereditary algebra. Then we have

$$\underline{\mathsf{mod}}\,\mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,A) \simeq \underline{\mathsf{mod}}\,\mathsf{R}(\underline{\mathsf{mod}}\,A) \simeq \underline{\mathsf{mod}}\,\widehat{\Gamma_A} \simeq \mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,\Gamma_A).$$

□

In the next section, we see that the categorical analog of Theorem 5 holds. And by using it, in Section 5, we show Theorem 14 holds.

4. Gorenstein-projective modules and Happel's theorem

We define Gorenstein-projective modules. Let $\mathcal{A}$ be an additive category. We first define a contravariant functor

$$(-)^* : \mathsf{Mod}\mathcal{A} \to \mathsf{Mod}\mathcal{A}^{\mathrm{op}}$$

as follows: for $M \in \mathsf{Mod}\mathcal{A}$ and $X \in \mathcal{A}$, let $(M)^*(X) := (\mathsf{Mod}\mathcal{A})(M, \mathcal{A}(-, X))$. By the same way, we define a contravariant functor $(-)^* : \mathsf{Mod}\mathcal{A}^{\mathrm{op}} \to \mathsf{Mod}\mathcal{A}$. Let $P_\bullet := (P_i, d_i : P_i \to P_{i+1})_{i\in\mathbb{Z}}$ be a complex of finitely generated projective $\mathcal{A}$-modules. We say that $P_\bullet$ is *totally acyclic* if complexes $P_\bullet$ and $\cdots \to (P_{i+1})^* \to (P_i)^* \to (P_{i-1})^* \to \cdots$ are acyclic.

Definition 6. Let $\mathcal{A}$ be an additive category. An $\mathcal{A}$-module M is said to be *Gorenstein-projective* if there exists a totally acyclic complex $P_\bullet$ such that the image of d_0 is isomorphic to M. We denote by $\mathsf{GP}\mathcal{A}$ the full subcategory of $\mathsf{Mod}\mathcal{A}$ consisting of all Gorenstein-projective $\mathcal{A}$-modules.

For instance, a finitely generated projective $\mathcal{A}$-module is Gorenstein-projective. We denote by $\mathsf{mod}_\infty\mathcal{A}$ the category of $\mathcal{A}$-modules having resolutions by $\mathsf{proj}\mathcal{A}$. In general, $\mathsf{GP}\mathcal{A} \subset \mathsf{mod}_\infty\mathcal{A}$ holds. The following proposition is well-known.

Proposition 7. *Let $\mathcal{A}$ be an additive category. Then $\mathsf{GP}\mathcal{A}$ is a Frobenius category, where the relative-projective objects are precisely finitely generated $\mathcal{A}$-modules.*

We consider the Gorenstein-projective modules over repetitive categories and certain subcategories of it. Let $\mathcal{A}$ be a k-linear additive category and $i \in \mathbb{Z}$. Put the following full subcategory of $\mathsf{R}\mathcal{A}$:

$$\mathcal{A}_i := \mathsf{add}\{\, (X, i) \in \mathsf{R}\mathcal{A} \mid X \in \mathcal{A} \,\}.$$

An inclusion functor $\mathcal{A}_i \to \mathsf{R}\mathcal{A}$ induces an exact functor

$$\rho_i : \mathsf{Mod}\mathsf{R}\mathcal{A} \to \mathsf{Mod}\mathcal{A}_i.$$

Moreover, we denote by

$$\rho : \mathsf{Mod}\mathsf{R}\mathcal{A} \to \mathsf{Mod}\mathcal{A}$$

the forgetful functor, that is, $\rho(M) := \bigoplus_{i\in\mathbb{Z}} \rho_i(M)$ for any $M \in \mathsf{Mod}\mathsf{R}\mathcal{A}$, where we regard an $\mathcal{A}_i$-module $\rho_i(M)$ as an $\mathcal{A}$-module by the equivalence $\mathsf{Mod}\mathcal{A}_i \simeq \mathsf{Mod}\mathcal{A}$. We denote by $\mathsf{GP}(\mathsf{R}\mathcal{A}, \mathcal{A})$ the full subcategory of $\mathsf{GP}(\mathsf{R}\mathcal{A})$ consisting of all objects M such that the projective dimension of $\rho(M)$ over $\mathcal{A}$ is finite, that is,

$$\mathsf{GP}(\mathsf{R}\mathcal{A}, \mathcal{A}) := \{\, M \in \mathsf{GP}(\mathsf{R}\mathcal{A}) \mid \mathrm{projdim}_{\mathcal{A}}\, \rho(M) < \infty \,\}.$$

We consider the following condition on $\mathcal{A}$:

(G) : the projective dimension of $\mathrm{D}\mathcal{A}(X,-)$ over $\mathcal{A}$ is finite for any $X \in \mathcal{A}$.

This condition induces that the category $\mathsf{GP}(\mathsf{R}\mathcal{A},\mathcal{A})$ is Frobenius. In fact, we can show the following proposition.

Proposition 8. *Let $\mathcal{A}$ be a k-linear, Hom-finite additive category. Then $\mathcal{A}$ satisfies (G) if and only if $\mathsf{projR}\mathcal{A} \subset \mathsf{GP}(\mathsf{R}\mathcal{A},\mathcal{A})$ holds. In this case, the following statements fold.*

(a) *$\mathsf{GP}(\mathsf{R}\mathcal{A},\mathcal{A})$ is a Frobenius category such that the projective objects is the objects of $\mathsf{projR}\mathcal{A}$.*

(b) *The inclusion functor $\mathsf{GP}(\mathsf{R}\mathcal{A},\mathcal{A}) \to \mathsf{GP}(\mathsf{R}\mathcal{A})$ induces a fully faithful triangle functor $\underline{\mathsf{GP}}(\mathsf{R}\mathcal{A},\mathcal{A}) \to \underline{\mathsf{GP}}(\mathsf{R}\mathcal{A})$.*

We also consider the following condition on $\mathcal{A}$:

$$\text{(IFP)} : \mathrm{D}\mathcal{A}(X,-) \in \mathsf{mod}_\infty\mathcal{A} \text{ holds for any } X \in \mathcal{A}.$$

Note that if $\mathcal{A}$ is a dualizing k-variety (see Definition 10), then $\mathsf{mod}_\infty\mathcal{A} = \mathsf{mod}\,\mathcal{A}$ holds and $\mathcal{A}$ satisfies (IFP). Then we can show the following theorem, which induces a categorical analog of Haapel's theorem, see also Theorem 12 (c).

Theorem 9. *Let $\mathcal{A}$ be a k-linear, Hom-finite additive category and assume that $\mathcal{A}$ and $\mathcal{A}^{\mathrm{op}}$ satisfy (IFP).*

(a) *If $\mathcal{A}$ and $\mathcal{A}^{\mathrm{op}}$ satisfy (G), then we have a triangle equivalence*

$$\mathsf{K}^{\mathrm{b}}(\mathsf{proj}\mathcal{A}) \simeq \underline{\mathsf{GP}}(\mathsf{R}\mathcal{A},\mathcal{A}).$$

(b) *If each object of $\mathsf{mod}_\infty\mathcal{A}$ and $\mathsf{mod}_\infty\mathcal{A}^{\mathrm{op}}$ has finite projective dimension, then $\mathsf{GP}(\mathsf{R}\mathcal{A}) = \mathsf{GP}(\mathsf{R}\mathcal{A},\mathcal{A})$ holds. In particular, we have a triangle equivalence*

$$\mathsf{K}^{\mathrm{b}}(\mathsf{proj}\mathcal{A}) \simeq \underline{\mathsf{GP}}(\mathsf{R}\mathcal{A}).$$

5. Dualizing varieties and the main theorem

We first recall the definition of dualizing k-varieties. Let $\mathcal{A}$ be a k-linear additive category. A morphism $e : X \to X$ in $\mathcal{A}$ is called an *idempotent* if $e^2 = e$. We call $\mathcal{A}$ *idempotent complete* if each idempotent of $\mathcal{A}$ has a kernel. The standard k-dual $\mathrm{D} = \mathrm{Hom}_k(-,k)$ induces a contravariant functors, we also denote them by D, $\mathrm{D} : \mathsf{Mod}\mathcal{A} \to \mathsf{Mod}\mathcal{A}^{\mathrm{op}}$ and $\mathrm{D} : \mathsf{Mod}\mathcal{A}^{\mathrm{op}} \to \mathsf{Mod}\mathcal{A}$ given by $(\mathrm{D}M)(X) := \mathrm{D}(M(X))$.

Definition 10. Let $\mathcal{A}$ be a k-linear, Hom-finite, idempotent complete additive category. We call $\mathcal{A}$ a *dualizing k-variety* if the functor $\mathrm{D} : \mathsf{Mod}\mathcal{A} \to \mathsf{Mod}\mathcal{A}^{\mathrm{op}}$ induces a duality between $\mathsf{mod}\mathcal{A}$ and $\mathsf{mod}\mathcal{A}^{\mathrm{op}}$.

The following is typical examples of dualizing k-varieties.

Example 11. [AR]

(a) If $\mathcal{A}$ is a dualizing k-variety, then $\mathcal{A}^{\mathrm{op}}$ is a dualizing k-variety and $\mathsf{mod}\mathcal{A}$ is an abelian dualizing k-variety.

(b) Let A be a finite dimensional algebra. Then $\mathsf{mod}\,A$ and $\mathsf{proj}\,A$ are dualizing k-varieties.

We can show that the repetitive category of a dualizing k-variety is a dualizing k-variety.

Theorem 12. [K] *Let $\mathcal{A}$ be a dualizing k-variety. Then the following holds.*

(a) $\mathsf{R}\mathcal{A}$ *is a dualizing k-variety.*
(b) $\mathsf{mod}\,\mathsf{R}\mathcal{A} = \mathsf{GP}(\mathsf{R}\mathcal{A})$ *holds, which is a Frobenius abelian category.*
(c) *If any modules in* $\mathsf{mod}\mathcal{A}$ *and* $\mathsf{mod}\mathcal{A}^{\mathrm{op}}$ *have finite projective dimension, then* $\mathcal{A}$ *and* $\mathcal{A}^{\mathrm{op}}$ *satisfy (G) and* $\mathsf{GP}(\mathsf{R}\mathcal{A}) = \mathsf{GP}(\mathsf{R}\mathcal{A}, \mathcal{A})$ *holds. In particular, we have a triangle equivalence* $\underline{\mathsf{mod}}\,\mathsf{R}\mathcal{A} \simeq \mathsf{D}^{\mathrm{b}}(\mathsf{mod}\mathcal{A})$*.*

Note that the statements (b) of the above theorem is a direct consequence of (a), and (c) follows from Theorem 9.

Let A be a finite dimensional algebra. By Example 11 (b), $\mathsf{proj}\,A$ is a dualizing k-variety. If the global dimension of A is finite, then by applying Theorem 12 to $\mathsf{proj}\,A$, we have Theorem 5.

We need the following result by Auslander and Reiten to show our main theorem.

Proposition 13. [AR, Propositions 6.2, 10.2] *Let $\mathcal{A}$ be a dualizing k-variety and $\mathcal{B} :=$ $\mathsf{mod}\mathcal{A}$. Let $\mathcal{P}$ be the full subcategory of $\mathcal{B}$ consisting of the projective modules. Then the following statements hold.*
(a) *$\mathcal{B}/[\mathcal{P}]$ is a dualizing k-variety.*
(b) *Assume that the global dimension of* $\mathsf{mod}\mathcal{A}$ *is at most n, then the global dimension of* $\mathsf{mod}(\mathcal{B}/[\mathcal{P}])$ *is at most $3n-1$.*

Now we are ready to show the main theorem of this paper.

Theorem 14. [IO, K] *Let A be a finite dimensional hereditary algebra. Then we have a triangulated equivalence*

$$\underline{\mathsf{mod}}\,\mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,A) \simeq \mathsf{D}^{\mathrm{b}}(\mathsf{mod}(\underline{\mathsf{mod}}\,A)).$$

Proof. By Proposition 4, we have $\underline{\mathsf{mod}}\,\mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,A) \simeq \underline{\mathsf{mod}}\,\mathsf{R}(\underline{\mathsf{mod}}\,A)$. By Proposition 13 and Theorem 12, we have $\underline{\mathsf{mod}}\,\mathsf{R}(\underline{\mathsf{mod}}\,A) \simeq \mathsf{D}^{\mathrm{b}}(\mathsf{mod}(\underline{\mathsf{mod}}\,A))$. □

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