

# Characterizations of standard derived equivalences of diagrams of dg categories and their gluing

Hideto Asashiba (Shizuoka, KUIAS, OCAMI)

2022-09-09, Symposium on ring theory and representation theory

# 1. Motivation

$k$ : alg. closed field

Representation-finite selfinjective algebras

standard

non-standard

only  $\text{char } k = 2$

der. eq. type:  $(D_{3m}, \frac{1}{3}, 1)$

$(m \geq 2)$

$$A \cong \hat{B}/G$$

- $B$ : a tilted alg of Dynkin
- $\hat{B}$ : the repetitive cat of  $B$ ,  
having  $G$ -action ( $G$ : infin cyclic gp)

⊙  $\exists$  a  $G$ -covering  $\hat{B} \xrightarrow{P} A$

$A'$ : another such alg with the  $G$ -covering  $\hat{B}' \xrightarrow{P'} A'$

Main tools

(1)  $B \stackrel{der}{\sim} B' \implies \hat{B} \stackrel{der}{\sim} \hat{B}'$

(2)  $\hat{B} \stackrel{der}{\sim} \hat{B}' + \text{some compatibility with } P, P' \implies A \stackrel{der}{\sim} A'$

(2) is generalized (2013):

$k$ : any commutative ring

the 2-cat of small  $k$ -cats

$G$ : a cyclic gp

$\mathcal{C} = \hat{B}$ : locally bdd  $k$ -cat

$G$ -action  $X: G \rightarrow \text{Aut}(\mathcal{C})$

$$\begin{aligned} &\Downarrow \\ X: G &\rightarrow k\text{-Cat} \\ * &\mapsto \mathcal{C} \end{aligned}$$

$\mathcal{C}/G$

$\text{Mod } \mathcal{C}$

$\mathcal{D}(\text{Mod } \mathcal{C})$

$\mathcal{C} \stackrel{der}{\sim} \mathcal{C}' \iff \mathcal{D}(\text{Mod } \mathcal{C}) \simeq \mathcal{D}(\text{Mod } \mathcal{C}')$   
as tri cats

$I$ : a small cat

$\mathcal{C}$ : a small or light  $k$ -cat

(colax) functor  $X: I \rightarrow k\text{-Cat}$

eg.  $I = \text{free}(1 \xrightarrow{a} 2 \xrightarrow{b} 3)$

$X(a): X(1) \xrightarrow{X(a)} X(2) \xrightarrow{X(b)} X(3)$  in  $k\text{-Cat}$

$\int X$  := the Grothendieck construction of  $X$

$$\begin{array}{ccc} \text{Mod } X & \xrightarrow{\text{Mod } X} & \text{Mod } X \\ \mathcal{D}(\text{Mod } X) & : I \xrightarrow{X} k\text{-Cat} \xrightarrow{\text{Mod}} k\text{-ModCat} & \\ & & \downarrow \mathcal{D} \\ & & k\text{-TRI}^2 \end{array}$$

$X \stackrel{der}{\sim} X' \iff \mathcal{D}(\text{Mod } X) \simeq \mathcal{D}(\text{Mod } X')$   
as colax funs

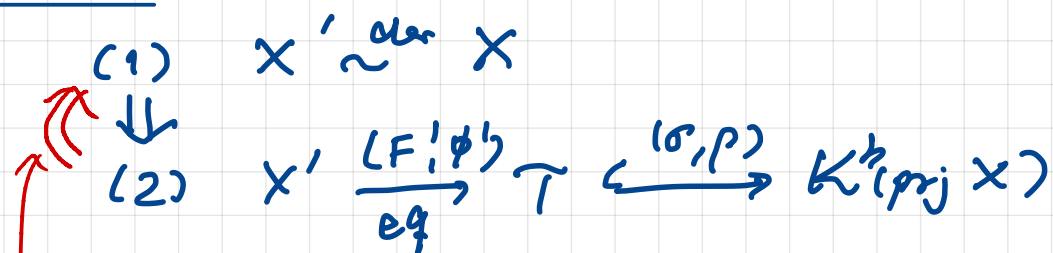
Q1. Define and characterize derived eq. for  $X, X'$

Q2. When  $\int X' \stackrel{\text{der}}{\sim} \int X$  ?

A1  $X' \stackrel{\text{der}}{\sim} X \iff \exists (F, \phi) : \mathcal{D}(\text{Mod } X') \rightarrow \mathcal{D}(\text{Mod } X)$   
 an eq in  $\text{Colax}(I, \mathbb{k}\text{-Cat})$

Prop  $\iff \begin{cases} \forall i \in I_0, F(i) : \text{triangle eq} \\ \forall a \in I_1, \phi_a : \text{nat iso} \end{cases}$

Thm 1



$X : \mathbb{k}\text{-flat}$

$\forall i \in I_0, \mathcal{T}(i) : \text{tilting subcat of } K^b(\text{proj } X(i)).$

$X : \mathbb{k}\text{-flat}$   $\iff X(i)(x, y) : \mathbb{k}\text{-flat } (i \in I_0, x, y \in X(i)_0)$



$$\begin{array}{ccccc}
 x'(i) & \xrightarrow[\sim]{F(i)} & \overset{\text{tilting}}{\mathcal{T}(i)} & \xrightarrow{\sigma(i)} & K^b(\text{proj } X(i)) \\
 (2) \quad x'(a) & \xrightarrow[\sim]{L} & \downarrow & \xrightarrow[\sim]{P_a} & \downarrow K^b(\text{proj } X(a)) \\
 & \searrow \phi_a & & & \\
 x'(j) & \xrightarrow[\sim]{F(j)} & \mathcal{T}(j) & \xrightarrow{\sigma(j)} & K^b(\text{proj } X(j))
 \end{array}$$

A2

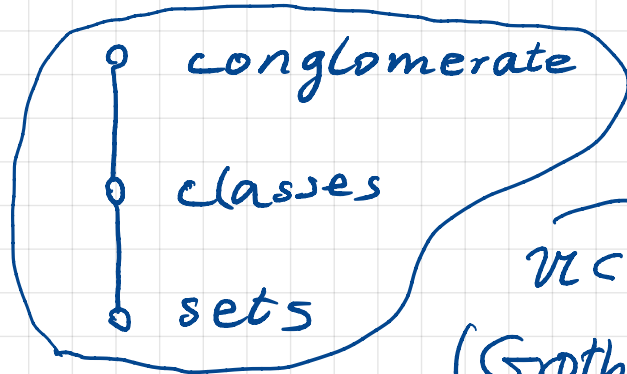
Thm 2 (2)  $\Rightarrow \int x' \overset{\text{der}}{\sim} \int x.$

Problem Generalize these to dg cat.<sup>s</sup>

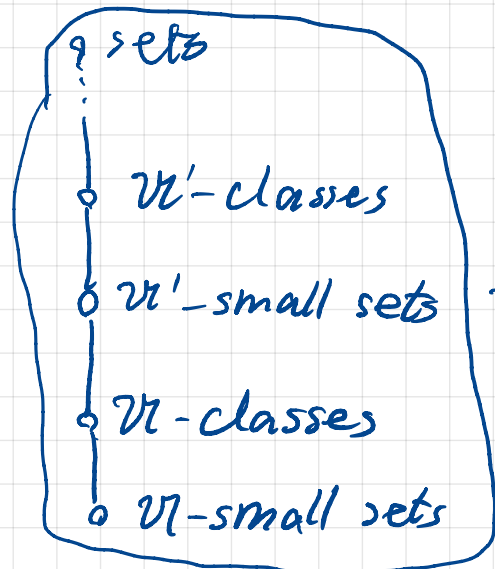
Characterization of der eq is well controlled in the setting of dg cat.<sup>s</sup>

[Keller: Deriving DG categories]

## 2. Collection of categories



$\mathcal{U} \subset \mathcal{U}' \subset \dots$   
(Grothendieck) universes



← refine

$\forall S: \text{a set}, S: \underline{\mathcal{U}\text{-small}} \Leftrightarrow S \in \mathcal{U}$   
 $S: \underline{\mathcal{U}\text{-class}} \Leftrightarrow S \subseteq \mathcal{U}$

Fix only one universe  $\mathcal{U} (\ni \mathbb{N})$

### Def. $\mathcal{C}$ : a cat

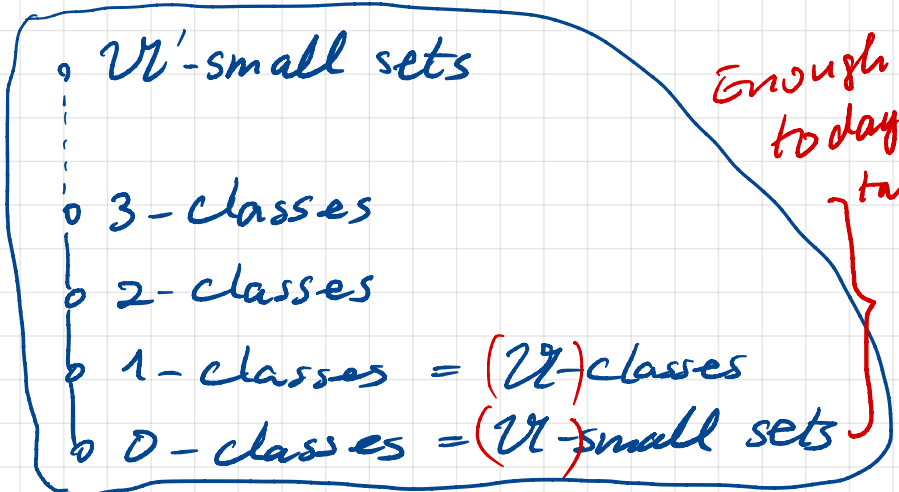
$\mathcal{C}: \underline{\text{small}} \Leftrightarrow \begin{cases} \mathcal{C}_0: \text{small} \\ \mathcal{C}(x, y): \text{small } \forall x, y \in \mathcal{C}_0 \end{cases}$

$\mathcal{C}: \underline{\text{light}} \Leftrightarrow \begin{cases} \mathcal{C}_0: \text{1-class} \\ \mathcal{C}(x, y): \text{small } \forall x, y \in \mathcal{C}_0 \end{cases}$

$\mathcal{C}: \underline{\text{1-moderate}} \Leftrightarrow \begin{cases} \mathcal{C}_0: \text{1-class} \\ \mathcal{C}(x, y): \text{1-class } \forall x, y \in \mathcal{C}_0 \end{cases}$

$k \geq 1$   
 $\mathcal{C}: \underline{\text{k-moderate}} \Leftrightarrow \begin{cases} \mathcal{C}_0 \\ \mathcal{C}(x, y) \end{cases} \text{ k-class}$

2-sets  
Cat  
CAT  
CAT  
CAT<sup>k</sup>



Enough for today's talk

remove "U" from names

## 2. Preparation

Dfn A 2-category is a sequence of data

(1)  $\mathbb{C}_0 \neq \emptyset$  : a set

(2)  $(\mathbb{C}(x, y))_{x, y \in \mathbb{C}_0}$  : a family of categories

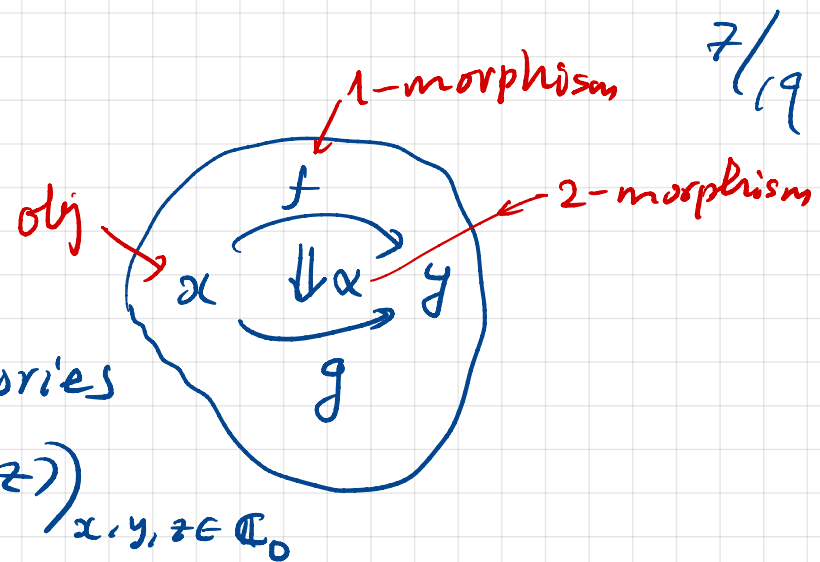
(3)  $\circ = (\circ_{x, y, z} : \mathbb{C}(y, z) \times \mathbb{C}(x, y) \rightarrow \mathbb{C}(x, z))_{x, y, z \in \mathbb{C}_0}$

a family of functors

(4)  $\mathbb{1} = (u_x : \mathbb{1} \rightarrow \mathbb{C}(x, x))_{x \in \mathbb{C}_0}$  : a family of functors

$* \ni \mathbb{1}_*$

$\mathbb{1}_x := u_x(*), \mathbb{1}_{\mathbb{1}_x} := u_x(\mathbb{1}_*)$



that satisfies associativity and unitality.

Exm (1)  $k\text{-Cat}$  :  $\left\{ \begin{array}{l} \bullet \text{ small } k\text{-cats} \\ \bullet k\text{-functors} \\ \bullet \text{ natural transformations} \end{array} \right.$

(2)  $\mathcal{C}$  : a cat.  $\mathcal{C}$  is regarded as a 2-cat  $\left\{ \begin{array}{l} \bullet \text{ objects of } \mathcal{C} \\ \bullet \text{ morphisms of } \mathcal{C} \\ \bullet \mathbb{1}_f \left( \begin{array}{l} f \in \mathcal{C}(x, y) \\ x, y \in \mathcal{C}_0 \end{array} \right)$  \end{array} \right.

Dfn  $A, B : 2\text{-cat}^s$  A 2-functor  $X : A \rightarrow B$  is a pair of data

(1)  $X_0 : A_0 \rightarrow B_0 : \text{a map}$   $X(x) := X_0(x) \ (x \in A_0)$

(2)  $\left( X_{(x,y)} : A(x,y) \rightarrow B(x,y) \right)_{x,y \in A_0} : \text{a family of functors}$   
 $X(gf) = X(g) \circ X(f)$   
 $X(1_x) = 1_{X(x)}$   
 $X(f) := X_{(x,y)}(f) \ (f \in A(x,y))$

that preserves compositions and identities.

Dfn  $A, B : 2\text{-cat}^s$ . A colax functor  $X : A \rightarrow B$  is a sequence of data

(1), (2) as above

(3)  $(X_x : X(1_x) \Rightarrow 1_{X(x)})_{x \in A_0}$  a family of 2-mor<sup>s</sup>

(4)  $(X_{b,a} : X(ba) \Rightarrow X(b)X(a))_{\leftarrow \begin{smallmatrix} b \\ a \end{smallmatrix} \leftarrow \text{in } A}$  a fam of 2-mor<sup>s</sup>, natural in  $a, b$

that satisfies the axioms

(a) counitality

$$\begin{array}{ccc} X(a1_x) \Rightarrow X(a)X(1_x) & & X(1_y a) \Rightarrow X(1_y)X(a) \\ \parallel \circlearrowleft & \downarrow & \parallel \circlearrowleft \downarrow \\ & X(a)1_{X(x)} & 1_{X(y)}X(a) \end{array} \quad \left( \begin{array}{c} x \xrightarrow{a} y \\ \text{in } A \end{array} \right)$$

(b) associativity

$$\begin{array}{ccc} X(cba) \Rightarrow X(c)X(ba) & & \\ \downarrow \circlearrowleft & \downarrow & \\ X(cb)a \Rightarrow X(c)X(b)X(a) & & \end{array} \quad \left( \begin{array}{c} c \\ w \leftarrow z \leftarrow y \leftarrow x \text{ in } A \end{array} \right)$$

- A lax functor  $X: A \rightarrow B$  is a colax fun  $X: A \rightarrow B^{co}$
- A pseudofunctor is a colax fun with ↑ reverse the 2-mor of B  
all  $X_x, X_{b,a}: 2\text{-iso}^s$
- A 2-functor is nothing but a colax fun with all  $X_i, X_{b,a}$  identities

Defn (1)  $A: a \text{ cat}$

$A: a \text{ dg cat} \iff \begin{cases} A(x, y): a \text{ (cochain) complex of } k\text{-modules } (x, y \in A_0). \\ A(y, z) \otimes_k A(x, y) \rightarrow A(x, z): a \text{ chain map } (x, y, z \in A_0). \end{cases}$

(2)  $A, B: \text{dg cat}^s$ . A dg functor  $F: A \rightarrow B$  is a sequence of data

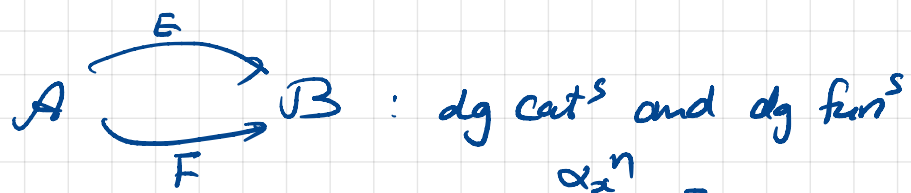
(i)  $F_0: A_0 \rightarrow B_0: a \text{ map } F(x) := F_0(x) \ (x \in A_0)$

(ii)  $F_{(x, y)}: A(x, y) \rightarrow B(F(x), F(y)): a \text{ chain map } (x, y \in A_0)$

$F(f) := F_{(x, y)}(f) \ (f \in A(x, y))$

that preserves compositions and identities

Dfn (dg nat tr, derived tr)  
 $n \in \mathbb{Z}$ .



$$\bullet \text{Hom}(E, F)^n := \left\{ (\alpha_x^n)_{x \in A_0} \in \prod_{x \in A_0} \mathcal{B}(E_x, F_x)^n \mid \begin{array}{ccc} E_x & \xrightarrow{\alpha_x^n} & F_x \\ E_f \downarrow & \swarrow (-1)^{mn} & \downarrow F_f \\ E_y & \xrightarrow{\alpha_y^n} & F_y \end{array} \forall f \in A(x, y)^m \right\}$$

$\forall m \in \mathbb{Z}$   
 $\forall x, y \in A_0$

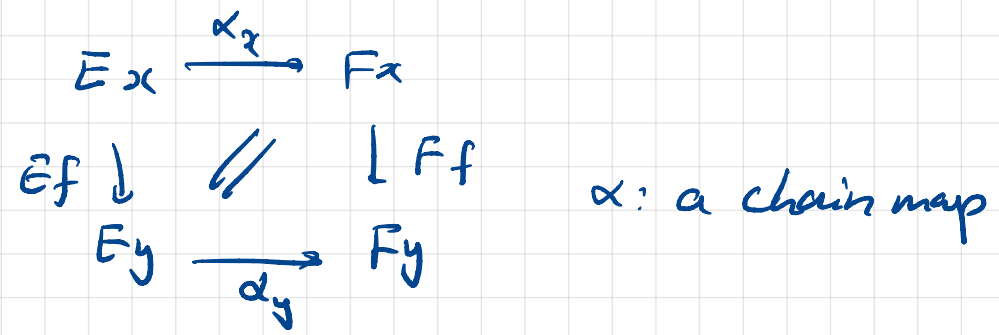
$$\bullet \text{Hom}(E, F) := \bigoplus_{n \in \mathbb{Z}} \text{Hom}(E, F)^n$$

$$\bullet \text{Hom}(E, F)^n \longrightarrow \text{Hom}(E, F)^{n+1}$$

$$(\alpha_x^n) \longmapsto (d_B(\alpha_x^n))_x$$

$\alpha^n = (\alpha_x^n)_{x \in A_0}$  : a derived transformation of degree n  
 $\alpha = (\alpha^n)_{n \in \mathbb{Z}}$  : a derived transformation  
 $\mathcal{Z}^*(\text{Hom}(E, F)) \ni \alpha$  is called a dg natural transformation

i.e  $\alpha_x \in \mathcal{B}(E_x, F_x)^0$ ,  $d(\alpha) = 0$ ,



Def  $\mathcal{C}_{dg}(k) :=$  the cat of (co)chain complexes of  $k$ -modules

$$\mathcal{C}_{dg}(k)(M, N) = \prod_{p \in \mathbb{Z}} \text{Hom}_k(M^p, N^{p+n})$$

$$d(f) := \left( d_N^{p+n} f^p - (-1)^n f^{p+1} d_M^p \right)_{p \in \mathbb{Z}} \quad \forall f = (f^p)_{p \in \mathbb{Z}} \in \mathcal{C}_{dg}(M, N)^n.$$

$$\begin{array}{ccc} M^p & \xrightarrow{d_M^p} & M^{p+1} \\ f^p \downarrow & & \downarrow f^{p+1} \\ N^{p+n} & \xrightarrow{d_N^{p+n}} & N^{p+n+1} \end{array}$$

$\mathcal{C}_{dg}(k)$  is a light dg cat.

$k$ -dgCat := the 2-cat of small dg cats, dg fun<sup>s</sup>, dg nat tr<sup>s</sup>.

	dg nat tr	derived
small	$k$ -dgCat	$k$ -DGCat
light	$k$ -dgCAT	$k$ -DGCAT

Let  $\mathcal{A} \in k\text{-dgCat}_0 = k\text{-DGCat}_0 \Rightarrow \mathcal{C}_{dg}(k)$

$\mathcal{C}_{dg}(\mathcal{A}) := k\text{-DGCat}(\mathcal{A}, \mathcal{C}_{dg}(k))$  • obj: right dg  $\mathcal{A}$ -module  $A \xrightarrow{M} \mathcal{C}_{dg}(k)$   
 $\in k\text{-dgCAT}_0 = k\text{-DGCAT}_0$  • mor: derived tr.

$\mathcal{E}(\mathcal{A}) := \mathcal{Z}^0(\mathcal{C}_{dg}(\mathcal{A}))$ : a Frobenius cat

$\mathcal{F}\mathcal{E}(\mathcal{A}) := H^0(\mathcal{C}_{dg}(\mathcal{A})) = \underline{\mathcal{E}(\mathcal{A})}$ : stable cat of  $\mathcal{E}(\mathcal{A})$

$$\mathcal{D}(A) := \mathcal{H}(A)[qis^{-1}]$$

$$\mathcal{C}_{dg}(A)_0 = \mathcal{C}(A)_0 = \mathcal{H}(A)_0 = \mathcal{D}(A)_0$$

Dfn. Let  $M \in \mathcal{C}_{dg}(A)_0$ .

$f: M \rightarrow N$  in  $\mathcal{C}(A)$  is quasi-iso (qis for short)  $\iff H^n(f): H^n(M) \rightarrow H^n(N)$   
iso  $\forall n \in \mathbb{Z}$

$M$  : acyclic  $\iff H^n(M) = 0, \forall n \in \mathbb{Z}$ .

$M$  : homotopically projective  $\iff \mathcal{H}(A)(M, A) = 0, \forall A : \text{acyclic}$   
 (htp proj) (or cofibrant)

$$\mathcal{H}_p(A) := \text{full } \{M \in \mathcal{H}(A) \mid M : \text{htp proj}\}$$

Dfn.  $\text{Colax}(I, k\text{-dgCat}) : 2\text{-cat}$

obj : colax functors  $I \rightarrow k\text{-dgCat}$  with suitable 1-mor<sup>s</sup> and 2-mor<sup>s</sup>

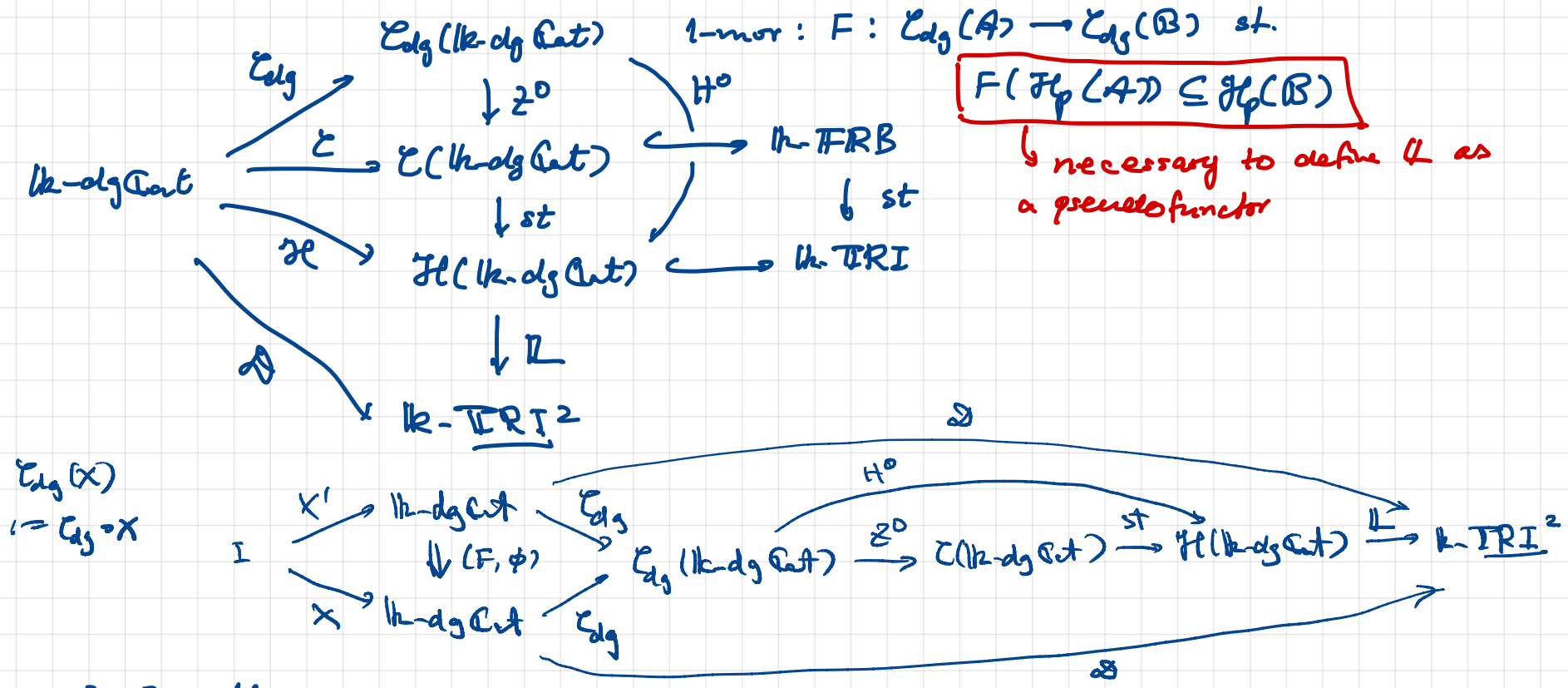
1-mor :  $(F, \phi) : X' \rightarrow X$ , where  $F = (F(i) : X'(i) \rightarrow X(i))_{i \in I_0}$  and  
 $\uparrow$  dg functor

$\phi = (\phi_a)_{a \in I_1}$   
 $\uparrow$   
 dg nat  $*$

$$\begin{array}{ccc} X'(i) & \xrightarrow{F(i)} & X(i) \\ X'(a) \downarrow & \phi_a \swarrow & \downarrow X(a) \\ X'(j) & \xrightarrow{F(j)} & X(j) \end{array}$$

satisfying suitable axioms.  
 $(a : i \rightarrow j \text{ in } I)$



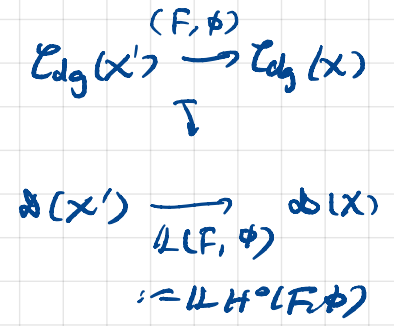


3. Results

Thm 1 Let  $X, X' \in \text{Colax}(I, k\text{-dgCat})$ . Then  $(\mathcal{Q})$ :

(1)  $\exists (F, \phi) : \mathcal{C}_{dg}(X') \rightarrow \mathcal{C}_{dg}(X) : 1\text{-mor in Colax}(I, k\text{-dgCat})$   
 st.  $\forall i \in I_0, F(i)$  preserves ltp projectives, and  
 $U(F, \phi) : \mathcal{D}(X') \rightarrow \mathcal{D}(X)$  an eq in  $\text{Colax}(I, k\text{-TRI}^2)$ .

(2)  $\exists$  a quasi-eg.  $X \xrightarrow{(F, \phi)} \mathcal{T} \xrightarrow{(\sigma, \rho)} \mathcal{C}_{dg}(X)$  in  $\text{Colax}(I, k\text{-dgCat})$   
 st.  $\mathcal{T}$ : a tilting colax functor for  $X$



①  $F(i) : X'(i) \rightarrow X(i)$  preserves htp proj  $\Leftrightarrow F(i)(\mathcal{H}_p(X'(i))_0) \subseteq \mathcal{H}_p(X(i))_0$  14  
19

①  $T$  is a tilting colax subfunctor for  $X$  if  $\exists$  1-mor  $(\sigma, \rho) : T \hookrightarrow \mathcal{L}_{dg}(X)$   
with  $\sigma(i) : T(i) \hookrightarrow \mathcal{L}_{dg}(X(i))$  the inclusion,  $T(i)$  is a tilting dg subcat  
for  $X(i)$ ,  $\forall i \in I_0$  ①'

①' A dg subcat  $T$  of  $\mathcal{L}_{dg}(A)$  is called a tilting dg subcat for  $A$   
if  $\forall M \in T$  is compact and  $\text{Loc}(T_0) = \mathcal{D}(A)$ .  
the smallest  $\swarrow$  localizing subcat of  $\mathcal{D}(A)$  containing  $T_0$

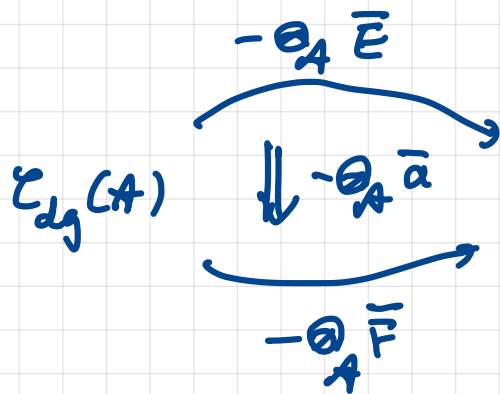
Note: no extension vanishing conditions. ( $\Rightarrow$  tilting = s-tilting.)

(2)  $(F, \phi') : X \rightarrow \mathcal{T}$  is quasi-ef  $\Leftrightarrow \begin{cases} \forall i \in I_0, F(i) : \text{quasi-ef} & \text{--- (2)'} \\ \forall \alpha \in I_1, \phi(\alpha) : \text{2-qis} & \text{--- (2)''} \end{cases}$

(2)' A dg fun  $F : \mathcal{A} \rightarrow \mathcal{B}$  is quasi-ef  $\Leftrightarrow \begin{cases} \forall n \in \mathbb{Z}, H^n F : H^n \mathcal{A} \rightarrow H^n \mathcal{B} \text{ fully faith.} \\ H^0 F : H^0 \mathcal{A} \rightarrow H^0 \mathcal{B} \text{ dense} \end{cases}$

(2)'' A dg nat. tr  $\mathcal{A} \begin{matrix} \xrightarrow{E} \\ \Downarrow \alpha \\ \xrightarrow{F} \end{matrix} \mathcal{B}$  is 2-qis  $\Leftrightarrow \mathcal{D}(\mathcal{A}) \begin{matrix} \xrightarrow{-\Theta_{\mathcal{A}} \bar{E}} \\ \Downarrow -\Theta_{\mathcal{A}} \bar{\alpha} \\ \xrightarrow{-\Theta_{\mathcal{A}} \bar{F}} \end{matrix} \mathcal{D}(\mathcal{B})$   
 $-\Theta_{\mathcal{A}} \bar{\alpha} : \text{iso}$

$\mathcal{L}_{dg}$



${}_{\mathcal{A}} \bar{E}_{\mathcal{B}} := \mathcal{B}(-, E(?)) : \text{bimodule}$   
 $\bar{\alpha} := \mathcal{B}(-, \alpha(?)) : \text{a mor of bimodules}$

Dfn •  $X'$  is standardly derived equivalent to  $X$  ( $X' \overset{sd}{\rightsquigarrow} X$ )

$\Leftrightarrow$  (1) and/or (2) above holds.

Prop •  $X \overset{sd}{\rightsquigarrow} X, X \overset{sd}{\rightsquigarrow} X', X' \overset{sd}{\rightsquigarrow} X'' \Rightarrow X \overset{sd}{\rightsquigarrow} X''$

• Not clear  $X \overset{sd}{\rightsquigarrow} X' \Rightarrow X' \overset{sd}{\rightsquigarrow} X$

Dfn •  $X'$  and  $X$  are standardly derived eq ( $X \overset{sd}{\rightsquigarrow} X'$ )

$\Leftrightarrow \exists X' = X_0, X_1, \dots, X_n = X$  st.  $X_0 \overset{sd}{\rightsquigarrow} X_1 \overset{sd}{\leftarrow} X_2 \overset{sd}{\rightsquigarrow} \dots \overset{sd}{\rightsquigarrow} X_n$

*preserving htp proj natural*

Prop  $(F, \phi) : \mathcal{C}_{dg}(X') \rightarrow \mathcal{C}_{dg}(X)$  1-mor st.  $\mathbb{L}(F, \phi) : \mathcal{D}(X') \rightarrow \mathcal{D}(X)$  eq.

$\Rightarrow \exists (F'(\iota) : \mathcal{C}_{dg}(X') \rightarrow \mathcal{C}_{dg}(X))_{\iota \in I_0}$  st.  $\mathbb{L}F'(\iota) \cong \mathbb{L}F(\iota)$ .  $\forall \iota \in I_0$   
 $F'(\iota)$  preserves  
htp proj

$\exists \psi$  st.  $(\mathbb{L}F', \psi) : \mathcal{D}(X') \rightarrow \mathcal{D}(X)$  is an eq in  $\text{Colax}(\mathbb{L}, \mathbb{k}\text{-TRT}^2)$

But not clear whether  $\exists \psi'$  st.  $(F', \psi') : \mathcal{C}_{dg}(X') \rightarrow \mathcal{C}_{dg}(X)$  1-mor

$\Rightarrow \text{Colax}(\mathbb{L}, \mathbb{k}\text{-dgCAT})$  st.  $(\mathbb{L}F', \mathbb{L}\psi') : \mathcal{D}(X') \rightarrow \mathcal{D}(X)$  is an eq.

Remark We do not need  $k$ -flatness condition on  $X$ .

17  
/19

It is possible to remove  $k$ -flatness assumption also from Keller's Thm for dg cat<sup>s</sup>.

Cor Let  $\mathcal{A}, \mathcal{A}'$  : small dg cat<sup>s</sup>. ( $\mathcal{Q}$ )

(1)  $\exists F: \mathcal{E}_{\text{dg}}(\mathcal{A}') \rightarrow \mathcal{E}_{\text{dg}}(\mathcal{A})$  a dg fun st.  $\mathbb{L}F: \mathcal{D}(\mathcal{A}') \rightarrow \mathcal{D}(\mathcal{A})$  a tri eq.

(2)  $\exists {}_{\mathcal{A}'}U_{\mathcal{A}}$  : a bimodule st.  $-\overset{\circ}{\otimes}_{\mathcal{A}'} U: \mathcal{D}(\mathcal{A}') \rightarrow \mathcal{D}(\mathcal{A})$  a tri eq.

(3)  $\exists$  a tri eq  $\mathcal{D}(\mathcal{A}') \rightarrow \mathcal{D}(\mathcal{A})$

(4)  $\mathcal{A}'$  is quasi-eg to a tilting dg subcat for  $\mathcal{A}$

(5)  $\exists F: \mathcal{E}_{\text{dg}}(\mathcal{A}') \rightarrow \mathcal{E}_{\text{dg}}(\mathcal{A})$  a dg fun preserving htp proj<sup>s</sup> st

$\mathbb{L}F: \mathcal{D}(\mathcal{A}') \rightarrow \mathcal{D}(\mathcal{A})$  is a tri eq.

Thm 2 Let  $X, X' \in \text{Colax}(\mathbb{I}, k\text{-dgCat})$ .

If  $X'$  is quasi-eg to a tilting colax functor for  $X$ , then  $\int X' \overset{\text{dar}}{\sim} \int X$ .

Exam In the case that  $I = G$  a group, we give an example, for which

Thm 2 is applied.  $G = \langle g \rangle \cong \mathbb{Z}(13)$ .

•  $(Q, w)$ :  $Q :=$

$\begin{matrix} & & 1 & & \\ & a_1 \swarrow & & \nwarrow a_6 & \\ & 2 & & 6 & \\ a_2 \swarrow & & & & \nwarrow a_5 \\ 3 & \xrightarrow{a_3} & 4 & \xrightarrow{a_4} & 5 \end{matrix}$

,  $w := a_5 a_4 a_3 a_2 a_1 a_6$

•  $(Q', w') := \mu_5 \circ \mu_3 \circ \mu_1 (Q, w)$

$Q' =$

$\begin{matrix} & & 1 & & \\ & a_1^* \swarrow & & \nwarrow a_6^* & \\ & 2 & & 6 & \\ a_2^* \swarrow & & & & \nwarrow a_5^* \\ 3 & \xrightarrow{[a_3 a_2]} & 4 & \xrightarrow{[a_5 a_4]} & 5 \\ & \xleftarrow{a_3^*} & & \xleftarrow{a_4^*} & \\ & & & & \end{matrix}$

,  $w' = [a_1 a_6] a_6^* a_1^* + [a_2 a_2] a_2^* a_3^* + [a_5 a_4] a_4^* a_5^* + [a_5 a_4] [a_3 a_2] [a_1 a_6]$

- Define an action of  $g$  on  $(Q, W)$  by  $\begin{cases} i \mapsto i-2 \\ a_i \mapsto a_{i-2} \end{cases}$

- $(Q_G, W_G) := (G, W)/G$

$$Q_G = \begin{array}{ccc} & \beta & \\ & \curvearrowright & \\ 1 & & 2 \\ & \curvearrowleft & \\ & \alpha & \end{array}, \quad W_G = (\beta\alpha)^3 \pmod{6} \quad 1 \leq i \leq 6$$

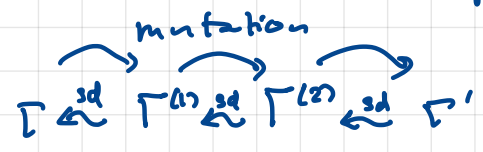
- Define an action of  $g$  on  $(Q', W')$  by  $\begin{cases} i \mapsto i-2 \\ a_i^* \mapsto a_{i-2}^* \end{cases}, [a_i a_{i+5}] \mapsto [a_{i-2} a_{i+3}]$

- $(Q'_G, W'_G) := (G', W')/G$

$$Q'_G = \begin{array}{ccc} & \beta' & \\ & \curvearrowright & \\ 1 & & 2 \curvearrowright \gamma \\ & \curvearrowleft & \\ & \alpha' & \end{array}, \quad W'_G = 3\gamma\beta\alpha + \gamma^3$$

- $\Gamma := \hat{\Gamma}(Q, W), \Gamma' := \hat{\Gamma}(Q', W')$  complete Ginzburg dg algebras (cat')

$$G \xrightarrow{X} \Gamma, \quad G' \xrightarrow{X'} \Gamma'$$



by Keller-Yang Thm 3.2 in

[Der eqs from muts of QP]

Thm 2  
der



mutation not possible to apply [K-Y, Thm 3.2]

$X' \xrightarrow{q\text{-eq}} \mathcal{T} \xrightarrow{\text{tilting}} \text{Cdg}(\Gamma)$

