The 55th Symposium on Ring Theory
and Representation Theory

ABSTRACT

Osaka Metropolitan University, Osaka

September 5 – 8, 2023

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10:35–11:05 Ayako Itaba (Tokyo University of Science)
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9:50–10:20 Yuta Kozakai (Tokyo University of Science), Ryotaro Koshio (Tokyo University of Science)
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10:35–11:05 Sota Asai (The University of Tokyo), Osamu Iyama (The University of Tokyo)
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11:20–11:50 Takuma Aihara (Tokyo Gakugei University)
On trivial tilting theory

On quiver Heisenberg algebras and the algebras $B(Q)$

Hiroyuki Minamoto

Let K be an algebraically closed field of arbitrary characteristic and Q a finite acyclic quiver. Recall that for an element $v = (v_i)_{i \in Q_0}$, which we call weight, the *quiver Heisenberg algebra* (QHA) ${}^v\Lambda(Q)$ is defined to be

$$ {}^v\Lambda(Q) := \frac{K[z]Q}{(\rho_i - v_i z \mid i \in Q_0)} $$

where ρ_i denotes the mesh relation at the vertex i. QHA is a special class of algebras previously introduced in [2, 3, 4]

A weight $v \in KQ_0$ is called *regular* if it is not orthogonal to the dimension vectors of indecomposable KQ-modules M (i.e., $\sum_{i \in Q_0} v_i \dim e_i M \neq 0$ inside K). In the case Q is Dynkin and $K = \mathbb{C}$, we may identify KQ_0 with the Cartan subalgebra $\mathfrak{h}$ of the Lie algebra $\mathfrak{g}$ of type Q. Then our terminology coincides with that of Lie theory.

Theorem 1. *Assume that Q is Dynkin. Then, QHA ${}^v\Lambda(Q)$ is finite dimensional if and only if the weight v is regular. If this is the case, ${}^v\Lambda(Q)$ is a symmetric algebra.*

In the case char$K = 0$, the "if" part of the first statement is proved by Etingof and Rains [4], and the second is proved for a generic weight by Etingof, Latour and Rains [5].

For the proof of the second statement, we introduce an algebra ${}^vB(Q)$ which is constructed from ${}^v\Lambda(Q)$. A point here is that the second quasi-Veronese algebra of ${}^v\Lambda(Q)$ is isomorphic to the 3-preprojective algebra of ${}^vB(Q)$. Another key ingredient is a description of the cohomology algebra of the derived $d+1$-preprojective algebra of a d-representation finite algebra.

Compare to the preprojective algebras $\Pi(Q)$ which are only Frobenius in general, QHA ${}^v\Lambda(Q)$ can be said to be well-behaved, since they are always symmetric. Making use of this, we investigate silting theory of QHA of Dynkin type. We obtain the following results which are analogous to the results for $\Pi(Q)$ by Aihara-Mizuno [1].

Theorem 2. *Assume that Q is Dynkin and v is regular.*

(1) *There is a bijection between the Weyl group W_Q of Q and the set $s\tau\mathrm{tilt}\,{}^v\Lambda(Q)$ of support τ-tilting modules of ${}^v\Lambda(Q)$. In particular, the algebra ${}^v\Lambda(Q)$ is a τ-tilting finite algebra.*

(2) *The algebra ${}^v\Lambda(Q)$ is silting discrete.*

(3) *An algebra which is derived equivalent to ${}^v\Lambda(Q)$ is isomorphic to ${}^{w(v)}\Lambda(Q)$ for some $w \in W_Q$.*

This talk is based on joint work with Martin Herschend.

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On Interval Global Dimension of Posets:
a Characterization of Case 0

Toshitaka Aoki, Emerson Gaw Escolar, and Shunsuke Tada

We study the relative homological algebra of posets with respect to the intervals. We introduce our recent research on the properties of the supports of interval approximations and interval resolution global dimension. We also provide necessary and sufficient conditions on a poset to ensure that any representation is interval decomposable (i.e. a characterization of the case where interval resolution global dimension is equal to 0).

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A classification of t-structures by a lattice of torsion classes

Arashi Sakai

In representation theory of a finite dimensional algebra, there are two important classes of subcategories, *torsion classes* and *wide subcategories*. Let Λ be a finite dimensional algebra. Torsion pairs in $\mathsf{mod}\Lambda$ bijectively correspond to *intermediate t-structures* on $D^b(\mathsf{mod}\Lambda)$ [4]. Besides wide subcategories of $\mathsf{mod}\Lambda$ bijectively correspond to H^0*-stable thick subcategories* of $D^b(\mathsf{mod}\Lambda)$ [7]. Recently, in [3], torsion classes and wide subcategories are unified by the notion of *ICE-closed subcategories*, subcategories closed under taking images, cokernels and extensions. It is natural to ask what objects in $D^b(\mathsf{mod}\Lambda)$ correspond to ICE-closed subcategories of $\mathsf{mod}\Lambda$. To give an answer, we consider a sequence of ICE-closed subcategories instead of ICE-closed subcategories.

We introduce the notion of *ICE sequence*, a sequence of ICE-closed subcategories satisfying some conditions. This is equivalent to the notion of *narrow sequence* defined in [6]. Combining this and a result in [6], we obtain the following bijection.

Theorem 1. [5] *There exists a bijective correspondence between*
1. *the set of ICE sequences in* $\mathsf{mod}\Lambda$*,*
2. *the set of homology-determined preaisles in* $D^b(\mathsf{mod}\Lambda)$*.*

We say that an ICE sequence *has a length* n provided that it corresponds to n-intermediate t-structure in the above. We give a description of ICE sequences of length $n+1$ from the viewpoint of the lattice $\mathsf{tors}\Lambda$ consisting of torsion classes in $\mathsf{mod}\Lambda$. In [1], Asai and Pfeifer introduced the notion of *wide intervals* and *meet intervals* in $\mathsf{tors}\Lambda$. We introduce a *decreasing sequence of maximal meet intervals* in $\mathsf{tors}\Lambda$, which is defined by only a lattice-theoretical property, and we obtain the following classification. We assume that Λ is τ*-tilting finite* [2], that is, every torsion class in $\mathsf{mod}\Lambda$ is functorially finite.

Theorem 2. [5] *Let* Λ *be a* τ*-tilting finite algebra and* n *a positive integer. Then there are one-to-one corresponcenses between*
1. *the set of* $(n+1)$*-intermediate* t*-structures on* $D^b(\mathsf{mod}\Lambda)$ *whose aisles are homology-determined,*
2. *the set of ICE sequence in* $\mathsf{mod}\Lambda$ *of length* $n+1$*,*
3. *the set of decreasing sequences of maximal meet intervals in* $\mathsf{tors}\Lambda$ *of length* n*.*

By the above, we obtain t-structures from the lattice-theoretical information of $\mathsf{tors}\Lambda$. Moreover if Λ is hereditary and of finite representation type, then every bounded t-structure is obtained from the above correspondence up to shifts in $D^b(\mathsf{mod}\Lambda)$.

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Computation of the structure of module categories using FD Applet

Haruhisa Enomoto

I will give an overview of *FD Applet* [1], an easy-to-use application that I am developing. It can compute various properties related to modules over (mainly representation-finite special biserial) algebras. For example, FD Applet can of course compute the Auslander-Reiten quiver of a given (representation-finite special biserial) algebra. In addition, this can compute the complete lists (and the numbers) of

- (Miyashita / Wakamatsu / τ- / classical / cluster-)tilting modules,
- 2-term simple-minded collections,
- τ-tilting-theoretical objects such as support τ-tilting modules, τ-rigid modules, semibricks, torsion classes, wide subcategories,
- cotorsion pairs,
- resolving subcategories,
- subcategories closed under images and extensions / images, (co)kernels, and extensions,

and so on. Moreover, this can compute various modules / subcategories from a given module / subcategory:

- from a module to the smallest torsion class containing it,
- from a support τ-tilting module to the corresponding semibrick,
- from a semibrick to the corresponding 2-term simple-minded collection,
- from a torsion class to the corresponding wide subcategory,
- from a given subcategory to the Ext-projective objects in it,

and so on. This can also compute some quivers like the quiver of support τ-tilting modules with the brick labeling and the Hasse quiver of various subcategories.

In this talk, I will demonstrate the use of FD Applet, and then explain the algorithm used in FD Applet. More precisely, we review the structure of modules over special biserial algebras and see that we only need *string combinatorics* to calculate the above things (we do not even need linear algebras). Then I will explain how specific objects like torsion classes and resolving subcategories can be computed.

In addition, I would like to talk about possible applications or impacts of FD Applet. One such application is my preprint about orthogonal modules [2], and possibly many enumerative research and experiments can be done using FD Applet. I would like to appreciate any feature requests from the audience, and also any contributions or developers to FD Applet.

If time permits, I will also talk about the combination of computer and mathematics in another direction: the proof assistant system *Lean*: we can formalize (that is, teach to computer) various theories of modern mathematics including algebras and category theory.

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K_0 of weak Waldhausen extriangulated categories

Yasuaki Ogawa and Amit Shah

The higher algebraic K-theory for an exact category $\mathcal{C}$ was introduced by Quillen, which is now called Quillen's Q-construction. It was used to produce a long exact sequence of K-groups associated to the Serre quotient of abelian categories. After Quillen, a new class of categories was introduced to define the K-theory of a more general type of category than exact categories. Such a structure that generalizes well to K-theory is that of a category $\mathcal{C}$ equipped with cofibrations co and weak equivalences w, which is called a Waldhausen category $(\mathcal{C}, co, w)$. As an application of Waldhausen's K-theory, an abstract localization theorem due to Thomason and Waldhausen was established, which says that "a short exact sequence of triangulated category gives rise to a long exact sequence of K-groups". Thus, it seems quite natural to generalize Waldhausen's K-theory to extriangaulated categories due to Nakaoka-Palu which is a generalization of exact categories and triangulated categories.

In this talk, focusing only on the Grothendieck groups, we generalize a part of the Waldhausen theory on exact categories to the extriangulated case, more specifically, we define the weak Waldhausen extriangulated category $(\mathcal{C}, co, w)$, and define its Grothendieck group $K_0(w\mathcal{C})$. As one might expect, it behaves nicely for extriangulated versions of Quillen's localization and resolution theorem.

Theorem A (Localization)**.** Let $\mathcal{N}^w \to (\mathcal{C}, co, v) \to (\mathcal{C}, co, w)$ be a localization situation of weak Waldhausen additive categories satisfying certain conditions. Then, we have an exact sequence

$$K_0(\mathcal{N}^w) \longrightarrow K_0(v\mathcal{C}) \longrightarrow K_0(w\mathcal{C}) \longrightarrow 0$$

of abelian groups. Moreover, if the weak Waldhausen homotopy category $\mathsf{Ho}(w\mathcal{C})$ exists, we have a group isomorphism $K_0(w\mathcal{C}) \cong K_0(\mathsf{Ho}(w\mathcal{C}))$.

Theorem B (Resolution)**.** Assume that $\mathcal{X}$ is a full subcategory closed under taking extensions and cocones of $\mathfrak{s}$-deflations in an extriangulated category $\mathcal{C}$. If any object $C \in \mathcal{C}$ admits a finite $\mathcal{X}$-resolution, then we have a group isomorphism $K_0(\mathcal{C}) \cong K_0(\mathcal{X})$.

The above theorems are closely related to Palu's index isomorphism which provides an interpretation of $K_0(\mathcal{C})$ of a triangulated category $\mathcal{C}$ via its cluster tilting subcategory $\mathcal{X}$.

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Resolving subcategories of derived categories

Ryo Takahashi

The notion of a *t-structure* in a triangulated category has been introduced by Beĭlinson, Bernstein and Deligne [2] in the early 1980s. Understanding t-structures in a given triangulated category $\mathcal{T}$, which is equivalent to understanding *aisles* or *coaisles* of $\mathcal{T}$, has been a basic problem in representation theory.

Let R be a commutative noetherian ring. Denote by $\operatorname{mod} R$ the category of finitely generated R-modules, and by $\mathsf{D}^{\mathsf{b}}(R)$ the bounded derived category of $\operatorname{mod} R$. Under the assumption that R admits a dualizing complex (this is equivalent to saying that R is a homomorphic image of a Gorenstein ring of finite Krull dimension [6]), Alonso Tarrío, Jeremías López and Saorín [1] completely classified the aisles of $\mathsf{D}^{\mathsf{b}}(R)$ in terms of the filtrations by supports that satisfy the *weak Cousin condition*, which had been used implicitly by Deligne, Bezrukavnikov and Kashiwara [3, 5]. Recently, the speaker [8] has generalized this classification theorem to the case where R has finite Krull dimension such that $\operatorname{Spec} R$ is a *CM-excellent* scheme in the sense of Česnavičius [4].

Now that the aisles of $\mathsf{D}^{\mathsf{b}}(R)$ (hence the coaisles of $\mathsf{D}^{\mathsf{b}}(R)$ as well) have been classified under the mild assumption mentioned above, it would be reasonable to move on to the investigation of the structure of *pre(co)aisles* of $\mathsf{D}^{\mathsf{b}}(R)$. Here we recall their definitions.

Definition 1. A full subcategory $\mathcal{X}$ of $\mathsf{D}^{\mathsf{b}}(R)$ is called a *preaisle* (resp. *precoaisle*) if the following two conditions are satisfied.

(1) For each exact triangle $A \to B \to C \to A[1]$, if A and C are in $\mathcal{X}$, then so is B.
(2) If X is an object in $\mathcal{X}$, then so is $X[n]$ for all positive (resp. negative) integers n.

An aisle (resp. coaisle) is none other than a preaisle (resp. precoaisle) whose inclusion functor has a right (resp. left) adjoint [7], and in general there is a big difference between being a (co)aisle and a pre(co)aisle.

In this talk, mimicking the definition of a resolving subcategory of $\operatorname{mod} R$, we shall introduce the notion of a *resolving subcategory* of $\mathsf{D}^{\mathsf{b}}(R)$.

Definition 2. Let $\mathcal{X}$ be a full subcategory of $\mathsf{D}^{\mathsf{b}}(R)$. We say that $\mathcal{X}$ is *resolving* provided that it satisfies the following three conditions.

(1) Every finitely generated projective R-module belongs to $\mathcal{X}$.
(2) Any direct summand of any object in $\mathcal{X}$ belongs to $\mathcal{X}$.
(3) For each exact triangle $A \to B \to C \to A[1]$ with $C \in \mathcal{X}$, one has $A \in \mathcal{X}$ if and only if $B \in \mathcal{X}$.

Note that a full subcategory of $\mathsf{D}^{\mathsf{b}}(R)$ is resolving if and only if it is a precoaisle of $\mathsf{D}^{\mathsf{b}}(R)$ containing R and closed under direct summands. In this talk, we will study resolving subcategories of $\mathsf{D}^{\mathsf{b}}(R)$ by relating them with certain subsets of $\operatorname{Spec} R$.

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Embeddings into modules of finite projective dimensions and the n-torsionfreeness of syzygies

Yuya Otake

Throughout this talk, let R be a commutative noetherian ring. We assume that all modules are finitely generated ones. It is a natural and classical question to ask when a given module can be embedded in a module of finite projective dimension. By Auslander–Buchweitz approximation theory [2], any module over a Gorenstein local ring has a *finite projective hull*. Conversely, Foxby [4] and Takahashi, Yassemi and Yoshino [6] figured out that if every R-module can be embedded in an R-module of finite projective dimension, then R is Gorenstein. In this talk, we consider the following question to give a refinement of these results.

Question 1. *Let M be an R-module and $n \geq 0$ an integer. When can M be embedded in an R-module of projective dimension at most n?*

Auslander and Bridger [1] introduced the notion of *n-torsionfree modules* as a generalization of the notion of torsionfree modules over integral domains. We will see that the above question is closely related to the $(n+1)$-torsionfreeness of n-syzygies.

As an application, we consider the n-torsionfreeness of syzygies of the residue field k over a local ring R. Let $t = \operatorname{depth} R$ and $\Omega^t k$ be the tth syzygy of the residue field k. Dey and Takahashi [3] proved that $\Omega^t k$ is $(t+1)$-torsionfree, and it is a $(t+2)$nd syzygy if and only if the local ring R has type one. Motivated by their results, we study higher torsionfreeness of the syzygy $\Omega^t k$.

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The Auslander–Reiten conjecture for normal rings

Kaito Kimura

Auslander and Reiten [3] proposed the *generalized Nakayama conjecture*, which is rooted in the *Nakayama conjecture* [9] and asserts that for any artin algebra Λ, any indecomposable injective Λ-module appears as a direct summand in the minimal injective resolution of Λ. In addition, they proposed another conjecture, characterizing the projectivity of a module in terms of vanishing of Ext modules, which is called the *Auslander-Reiten conjecture*, and proved that this conjecture is true if and only if the generalized Nakayama conjecture is true. This long-standing conjecture is known to hold over several classes of algebras. For example, Auslander and Reiten [3] proved the conjecture for algebras of finite representation type. Hoshino [5] showed that it holds for symmetric artin algebras with radical cube zero. The Auslander-Reiten conjecture is closely related to other important conjectures such as the *Tachikawa conjecture* [10].

The Auslander-Reiten conjecture remains meaningful for arbitrary commutative noetherian rings for formalization by Auslander, Ding, and Solberg [2]. The conjecture is known as follows: for a commutative noetherian ring R, every finitely generated R-module M such that $\mathrm{Ext}_R^i(M, M \oplus R) = 0$ for all $i \geq 1$ is projective. This conjecture is known to hold true if R is a complete intersection [2], or if R is a locally excellent Cohen-Macaulay normal ring containing the field of rational numbers $\mathbb{Q}$ [6], or if R is a Gorenstein normal ring [1]. Recently, Kimura, Otake, and Takahashi [8] proved the conjecture for every Cohen-Macaulay normal ring. Even if R is not Cohen-Macaulay, it is known that R satisfies the conjecture if it is a quotient of a regular local ring and is a normal ring containing $\mathbb{Q}$ [4].

In this talk, we consider the above conjecture over normal rings. This talk is based on a preprint [7].

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The first Euler characteristic and the depth of associated graded rings

Kazuho Ozeki

The homological property of the associated graded ring of an ideal is an important problem in commutative algebra. In this talk, we explore the structure of the associated graded ring of $\mathfrak{m}$-primary ideals in the case where the first Euler characteristic attains almost minimal value in a Cohen-Macaulay local ring.

Throughout this talk, let A be a Cohen-Macaulay local ring with maximal ideal $\mathfrak{m}$ and $d = \dim A > 0$. For simplicity, we may assume the residue class field $A/\mathfrak{m}$ is infinite. Let I be an $\mathfrak{m}$-primary ideal in A and let

$$R = \mathrm{R}(I) := A[It] \quad \subseteq A[t] \text{ and } R' = \mathrm{R}'(I) := A[It, t^{-1}] \quad \subseteq A[t, t^{-1}]$$

denote, respectively, the Rees algebra and the extended Rees algebra of I. Let

$$G = \mathrm{G}(I) := R'/t^{-1}R' \cong \bigoplus_{n \geq 0} I^n/I^{n+1}$$

denotes the associated graded ring of I. Let $M = \mathfrak{m}G + G_+$ be the graded maximal ideal in G. Let $Q = (a_1, a_2, \cdots, a_d) \subseteq I$ be a parameter ideal in A which forms a reduction of I. Then, we set

$$\chi_1(a_1t, a_2t, \ldots, a_dt; G) := \ell(G/(a_1t, a_2t, \ldots, a_dt)G) - \mathrm{e}(a_1t, a_2t, \ldots, a_dt; G_M)$$

and call it the *first Euler characteristic* of G relative to $a_1t, a_2t, \ldots, a_dt$ (c.f. [1, 2, 3]), where $\ell(*)$ and $\mathrm{e}(*)$ denote the length and the multiplicity symbol, respectively.

It is well-known that $\chi_1(a_1t, a_2t, \ldots, a_dt; G) \geq 0$ holds true, and the equality $\chi_1(a_1t, a_2t, \ldots, a_dt : G) = 0$ holds true if and only if the associated graded ring G is Cohen-Macaulay. The aim of this talk is to explore the structure of the associated graded ring G with $\chi_1(a_1t, a_2t, \ldots, a_dt; G) = 1$ and, in particular, we prove that $\operatorname{depth} G = d - 1$.

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Classifying Serre subcategories of the category of maximal Cohen-Macaulay modules

Shunya Saito

Classifying subcategories is an active subject in the representation theory of algebras. Especially, several subcategories of the module category $\mathsf{mod}\,R$ of a commutative noetherian ring R have been classified so far. See [1, 2, 5, 6] for example. In this talk, we will give classification results of several subcategories of the category $\mathsf{cm}\,R$ of maximal Cohen-Macaulay modules over a one-dimensional Cohen-Macaulay ring R. The main result is the following:

Theorem 1. *Let R be a one-dimensional Cohen-Macaulay ring.*

(1) *There is a bijection between the following:*
- *The Serre subcategories of $\mathsf{cm}\,R$.*
- *The subsets of the set $\operatorname{Min} R$ of minimal prime ideals.*

(2) *The following are equivalent for a subcategory $\mathcal{X}$ of $\mathsf{cm}\,R$:*
- *$\mathcal{X}$ is a torsion class of $\mathsf{cm}\,R$.*
- *$\mathcal{X}$ is a Serre subcategory of $\mathsf{cm}\,R$.*

Moreover, if R admits a canonical module, then the following is also equivalent to the above:
- *$\mathcal{X}$ is a torsionfree class of $\mathsf{cm}\,R$.*

The major difference from previous studies is that $\mathsf{mod}\,R$ is an abelian category, while $\mathsf{cm}\,R$ is an exact category which is not abelian.

I have a plan to talk about the following: First, I will review previous works on the classification of subcategories of $\mathsf{mod}\,R$. Then I will introduce various subcategories of an exact category and give a method for studying them. Finally, I will show Theorem 1 using this method.

This talk is based on preprints [3, 4]

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Govorov–Lazard type theorems, big Cohen–Macaulay modules, and Cohen–Macaulay hearts

Tsutomu Nakamura

This talk is based on [2]. Let A be a ring and denote by $\mathsf{Mod}\,A$ (resp. $\mathsf{mod}\,A$) the category of (right) A-modules (resp. finitely presented A-modules). Let $\mathcal{C}$ be an additive subcategory of $\mathsf{Mod}\,A$ closed under direct limits. In general, it is a delicate problem whether every module in $\mathcal{C}$ can be presented as a direct limit of modules in $\mathcal{C} \cap \mathsf{mod}\,A$. If this is possible, we say that a *Govorov–Lazard type theorem* holds for $\mathcal{C}$, and write $\varinjlim(\mathcal{C} \cap \mathsf{mod}\,A) = \mathcal{C}$. For example, we have $\varinjlim \mathsf{mod}\,A = \mathsf{Mod}\,A$ and $\varinjlim \mathsf{proj}\,A = \mathsf{Flat}\,A$, where $\mathsf{Flat}\,A$ (resp. $\mathsf{proj}\,A$) denotes the category of flat (resp. finitely generated projective) A-modules. The second equality is due to Govorov (1965) and Lazard (1969). Holm–Jørgensen (2011) proved that a Govorov–Lazard type theorem may not hold for the category of Gorenstein-flat modules.

Let R be a commutative noetherian local ring. An R-module M is called *(balanced) big Cohen–Macaulay* if every system of parameters of R is an M-regular sequence. We call an R-module M a *weak* big CM (=Cohen–Macaulay) if every system of parameters of R is a weak M-regular sequence. We denote by $\mathsf{WCM}\,R$ the category of weak big CM modules. Then $\mathsf{WCM}\,R \cap \mathsf{mod}\,R = \mathsf{CM}\,R$, where the right-hand side denotes the category of (maximal) CM modules. Holm (2018) showed that $\varinjlim \mathsf{CM}\,R = \mathsf{WCM}\,R$ holds for any CM local ring R with a canonical module. Our first result extends this to orders over a CM local ring with a canonical module.

Theorem 1. *Let R be a CM local ring with a canonical module and let A be an R-order. Then we have $\varinjlim \mathsf{CM}\,A = \mathsf{WCM}\,A$, where $\mathsf{CM}\,A$ (resp. $\mathsf{WCM}\,A$) denotes the category of A-modules being CM (resp. weak big CM) as R-modules.*

It is well known that every pure-injective module over an Artin algebra is a direct summand of a direct product of finitely presented modules. Using the above theorem, we can extend this fact to complete orders.

Corollary 2. *Let R and A be as above and assume R is complete. Then every pure-injective complete big CM module is a direct summand of a direct product of CM modules.*

If R is not CM, there would be little hope that we could have $\varinjlim \mathsf{CM}\,R = \mathsf{WCM}\,R$; this impression comes from difficulty of the *small CM conjecture* (which is still open). Instead, we would like to give another formulation. The only assumption we need is that R is a homomorphic image of a CM local ring.

We use the *Cohen–Macaulay heart* $\mathcal{H}_{\mathsf{CM}}$ of R introduced in [1]. This is the heart of some compactly generated generated t-structure in the derived category $\mathsf{D}(R)$. There are several remarkable facts: $\mathcal{H}_{\mathsf{CM}}$ is a locally coherent Grothendieck category and derived equivalent to $\mathsf{Mod}\,R$. Furthermore, we have $\mathcal{H}_{\mathsf{CM}} \cap \mathsf{Mod}\,R = \mathsf{WCM}\,R$.

Theorem 3. *Let R be a homomorphic image of a CM local ring. Then every weak big CM module is a direct limit of finitely presented objects in $\mathcal{H}_{\mathsf{CM}}$.*

If R admits a dualizing complex D (such that $\inf\{i \mid H^i(D) \neq 0\} = 0$), there is an equivalence

$$\mathrm{RHom}_R(-, D) : (\mathsf{mod}\,R)^{\mathrm{op}} \xrightarrow{\sim} \mathrm{fp}(\mathcal{H}_{\mathsf{CM}}),$$

where $\mathrm{fp}(\mathcal{H}_{\mathsf{CM}})$ denotes the subcategory of finitely presented objects in $\mathcal{H}_{\mathsf{CM}}$.

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The spectrum of the category of maximal Cohen-Macaulay modules

Naoya Hiramatsu

The Ziegler spectrum of an associative algebra is a topological space whose points are the isomorphism classes of indecomposable pure-injective modules, whose topology is defined in terms of positive primitive formulas over the algebra. Many studies of Ziegler spectrums are given in the context of representation theory of algebras [1, 2, 3]. In this talk we consider an analogue of the Ziegler spectrum for the (stable) category of maximal Cohen-Macaulay (abbr. MCM) modules over a complete Gorenstein local ring.

Let R be complete Cohen–Macaulay local ring with algebraic residue field k. We denote by $\mathcal{C}$ the category of MCM R-modules. We denote by $\mathrm{mod}(\mathcal{C})$ the category of finitely presented contravariant additive functors, that is

$$\mathrm{mod}(\mathcal{C}) := \{F : \mathcal{C} \to Ab \mid \text{finitely presented contravariant additive functors}\}$$

and also denote by $\underline{\mathrm{mod}}(\mathcal{C})$ the full subcategory of $\mathrm{mod}(\mathcal{C})$ consisting of functors with $F(R) = 0$. We put $\mathsf{Sp}(\mathcal{C})$ the set of isomorphism classes of the indecomposable MCM R-modules except R and 0.

For a subset $\mathcal{X}$ of $\mathsf{Sp}(\mathcal{C})$, we denote by $\Sigma(\mathcal{X})$ the subcategory of $\underline{\mathrm{mod}}(\mathcal{C})$ formed by the functors F such that $F(X) = 0$ for all $X \in \mathcal{X}$. For a subcategory $\mathcal{F}$ of $\underline{\mathrm{mod}}(\mathcal{C})$, we denote by $\gamma(\mathcal{F})$ the subset of $\mathsf{Sp}(\mathcal{C})$ satisfying $F(X) = 0$ for all $F \in \mathcal{F}$.

Theorem 1. *Let R be Gorenstein. Then the assignment $\mathcal{X} \mapsto \gamma \circ \Sigma(\mathcal{X})$ is a is a Kuratowski closure operator on $\mathsf{Sp}(\mathcal{C})$. In particular, it induces a topology on $\mathsf{Sp}(\mathcal{C})$.*

Here we put the definition of the Cantor-Bendixson rank. The Cantor-Bendixson rank measures the complexity of the topology. It measures how far the topology is from the discrete topology.

Definition 2. $\mathcal{T}$ is a topological space. If $x \in \mathcal{T}$ is an isolated point, then $\mathrm{CB}(x) = 0$. Put $\mathcal{T}' \subset \mathcal{T}$ is a set of the <u>non</u>-isolated point. Define the induced topology on $\mathcal{T}'$. Set $\mathcal{T}^{(0)} = \mathcal{T}, \mathcal{T}^{(1)} = \mathcal{T}^{(0)'}, \cdots, \mathcal{T}^{(n+1)} = \mathcal{T}^{(n)'}$. We define $\mathrm{CB}(x) = n$ if $x \in \mathcal{T}^{(n)} \backslash \mathcal{T}^{(n+1)}$ If there exists n such that $\mathcal{T}^{(n+1)} = \emptyset$ and $\mathcal{T}^{(n)} \neq \emptyset$, then $\mathrm{CB}(\mathcal{T}) = n$.

We say that a Cohen–Macaulay local ring is CM_+-finite if there exist only finitely many isomorphism classes of indecomposable MCM modules that are not locally free on the punctured spectrum [4].

Theorem 3. *If R is CM_+-finite then $\mathrm{CB}(\mathsf{Sp}(\mathcal{C})) \leq 1$.*

It is known that the hypersurface ring which is of countable CM representation type is CM_+-finite. Therefore we obtain $\mathrm{CB}(\mathsf{Sp}(\mathcal{C})) \leq 1$ over such the ring.

Remark 4. In this talk, we consider only finitely generated (pure-injective) modules. Previous studies [1, 2, 3] have also considered infinitely generated modules, which is different from our consideration.

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The classification of 3-dimensional cubic AS-regular algebras of Type P, S, T and WL

Masaki Matsuno and Yu Saito

In this talk, k is an algebraically closed field of characteristic 0 and an algebra A is a graded k-algebra finitely generated in degree 1, that is, $A = k\langle x_1, \cdots, x_n\rangle/I$ where $\deg x_i = 1$ and I is a homogeneous two sided ideal.

We introduce the notion of 3-geometric algebra as a generalization of geometric algebra introduced by Mori ([2]) to study 3-dimensional cubic AS-regular algebras.

Let $A = k\langle x_1, \cdots, x_n\rangle/(f_1, \cdots, f_l)$ be a 3-homogeneous algebra. We define Γ_A as

$$\Gamma_A := \{(p, q, r) \in (\mathbb{P}^{n-1})^{\times 3} \mid f_i(p, q, r) = 0,\ i = 1, \cdots, l\}.$$

A pair (E, σ) is called 3-*geometric* if $E \subset (\mathbb{P}^{n-1})^{\times 2}$ is a projective variety and $\sigma \in \operatorname{Aut}_k E$ satisfies $\pi_1\sigma = \pi_2$ where $\pi_i : \mathbb{P}^{n-1} \times \mathbb{P}^{n-1} \to \mathbb{P}^{n-1}$ are the i-th projections.

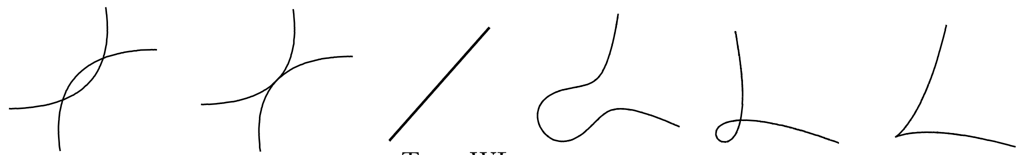
Definition 1. A 3-homogeneous algebra $A = k\langle x_1, \cdots, x_n\rangle/(f_1, \cdots, f_l)$ is 3-*geometric* if there exists a 3-geometric pair (E, σ) such that

(G1) $\Gamma_A = \{(p, q, \pi_2\sigma(p, q)) \mid (p, q) \in E\}$,

(G2) $(f_1, \cdots, f_l)_3 = \{f \in k\langle x_1, \cdots, x_n\rangle_3 \mid f(p, q, \pi_2\sigma(p, q)) = 0\ \forall (p, q) \in E\}$.

In this case, we denote $A = \mathcal{A}(E, \sigma)$.

Artin, Tate and Van den Bergh ([1]) showed that every 3-dimensional cubic AS-regular algebra is a 3-geometric algebra $\mathcal{A}(E, \sigma)$ where $E = \mathbb{P}^1 \times \mathbb{P}^1$ (Type P) or $E \subset \mathbb{P}^1 \times \mathbb{P}^1$ is a bidegree $(2, 2)$ curve.

$E =$ 

Type S Type T Type WL etc

In this talk, we study 3-dimensional cubic AS-regular algebras of Type P, S and T. One of our main results is that we give a complete list of defining relations of 3-dimensional cubic AS-regular algebras of those types.

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Quantum projective planes and Beilinson algebras of 3-dimensional quantum polynomial algebras for Type S'

Ayako Itaba

This talk is based on [6]. Throughout this talk, let k be an algebraically closed field of characteristic 0, and all algebras are defined over k. In noncommutative algebraic geometry, Artin and Schelter [1] gave the classifications of low dimensional quantum polynomial algebras. Moreover, Artin, Tate and Van den Bergh [2] found a nice correspondence between 3-dimensional quantum polynomial algebras and geometric pair (E, σ), where E is the projective plane $\mathbb{P}^2$ or a cubic divisor in $\mathbb{P}^2$, and $\sigma \in \mathrm{Aut}_k E$. So, this result allows us to write a 3-dimensional quantum polynomial algebra A as the form $A = \mathcal{A}(E, \sigma)$.

For a 3-dimensional quantum polynomial algebra $A = \mathcal{A}(E, \sigma)$, Artin, Tate and Van den Bergh [3] showed that A is finite over its center $Z(A)$ if and only if the order $|\sigma|$ of σ is finite. For a 3-dimensional quantum polynomial algebra $A = \mathcal{A}(E, \sigma)$ with the Nakayama automorphism ν of A, the author and Mori [7] proved that the order $|\nu^* \sigma^3|$ of $\nu^* \sigma^3$ is finite if and only if the norm $||\sigma||$ of σ introduced by Mori [8] is finite if and only if the noncommutative projective plane $\mathsf{Proj}_{\mathrm{nc}} A$ in the sense of Artin and Zhang [4] is finite over its center. This result is a categorical analogue of this ATV's result.

In this talk, we prove the following results for Type S' algebra $A = \mathcal{A}(E, \sigma)$, where $E \subset \mathbb{P}^2$ is a union of a line and a conic meeting at two points, and $\sigma \in \mathrm{Aut}_k E$.

Proposition 1 ([6, Proposition 3.2])**.** *Let $A = \mathcal{A}(E, \sigma) = k\langle x, y, z\rangle/(g_1, g_2, g_3)$ be a 3-dimansional Calabi-Yau quantum polynomial algebra of Type S', where*

$$\begin{cases} g_1 = yz - \alpha zy + x^2, \\ g_2 = zx - \alpha xz, \\ g_3 = xy - \alpha yx \quad (\alpha^3 \neq 0, 1). \end{cases}$$

Define $g := xyz + (1 - \alpha^3)^{-1} x^3 \in A_3$.

(1) *If A is finite over its center $Z(A)$ (that is, $|\alpha|$ is finite), then $Z(A) = k[x^{|\alpha|}, y^{|\alpha|}, z^{|\alpha|}, g]$.*
(2) *If A is not finite over its center $Z(A)$ (that is, $|\alpha|$ is infinite), then $Z(A) = k[g]$.*

Theorem 2 ([6, Theorem 4.4])**.** *For a 3-dimensional quantum polynomial algebra A of Type S', the following are equivalent*:

(1) *The noncommutative projective plane $\mathsf{Proj}_{\mathrm{nc}} A$ is finite over its center.*
(2) *The Beilinson algebra ∇A of A is 2-representation tame in the sense of Herschend, Iyama and Oppermann* [5].
(3) *The isomorphism classes of simple 2-regular modules over ∇A are parameterized by $\mathbb{P}^2$.*

Remark 3. Note that the same statement of Theorem 2 holds for Type S algebra $A = \mathcal{A}(E, \sigma)$ by Mori [8], where E is a triangle in $\mathbb{P}^2$ and $\sigma \in \mathrm{Aut}_k E$. Theorem 2 satisfies the expectation in [7, Remark 4.3].

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AS-regular $\mathbb{Z}$-algebras

Izuru Mori

This talk is a report based on a series of papers [2], [3] with Adam Nyman. In this talk, a $\mathbb{Z}$-algebra introduced in [1] is simply an infinite matrix algebra indexed by $\mathbb{Z} \times \mathbb{Z}$. AS-regular algebras are an important class of algebras in noncommutative algebraic geometry. In fact, every noncommutative deformation of $\mathbb{P}^2$ has a 3-dimensional quadratic AS-regular algebra as a homogeneous coordinate algebra. On the other hand, not every noncommutative deformation of $\mathbb{P}^1 \times \mathbb{P}^1$ has a 3-dimensional cubic AS-regular algebra as a homogeneous coordinate algebra, and a 3-dimensional cubic AS-regular $\mathbb{Z}$-algebra fills in this gap [5]. As such, AS-regular $\mathbb{Z}$-algebras are also an important class of algebras in noncommutative algebraic geometry, however, they have been rarely studied in literature except for the 3-dimensional case. In this talk, after reviewing basic definitions and properties for $\mathbb{Z}$-algebras, we will formally define the notions of AS-regular $\mathbb{Z}$-algebra and ASF-regular $\mathbb{Z}$-algebra, and study their properties. This project is motivated to understand noncommutative smooth quadric surfaces [3], [4], [5].

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Periodic dimensions of modules and algebras

Satoshi Usui

This talk is based on [5]. Let R be a Noetherian semiperfect ring. A finitely generated R-module whose sufficiently higher syzygies are all periodic is called *eventually periodic*. Each eventually periodic R-module M yields the degree n and the period p of its first periodic syzygy $\Omega_R^n(M)$. So far, several authors, such as Avramov [1], Bergh [2], Croll [3] and Küpper [4], have provided characterizations of eventually periodic modules and have studied the values of the associated degrees and periods.

In this talk, we focus on the degree of the first periodic syzygy of an eventually periodic module. For this, we introduce the notion of the periodic dimension of a module. By definition, if a given module is eventually periodic, then its periodic dimension equals the desired degree. After providing some basic properties of periodic dimensions, we explain that when R is a left artin ring and M is an eventually periodic R-module having finite Gorenstien dimension, the periodic dimension of M can be computed from the Gorenstein dimension of M, and vice versa. Moreover, as an application, we determine the periodic dimension of an eventually periodic Gorenstein algebra viewed as a regular bimodule.

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Classification of twisted algebras of 3-dimensional Sklyanin algebras

Masaki Matsuno

This talk is based on [2]. Throughout this talk, let k be an algebraically closed field of characteristic 0 and a graded algebra A means a connected graded algebra, that is, $A = \oplus_{i\in\mathbb{N}} A_i$ and $A_0 = k$. For a graded algebra, the notion of twisting system was introduced by Zhang [4] aiming to produce a deformation of the original algebra. A twisting system is called algebraic if it is given by some graded algebra automorphism. We can say that a twisting system is a generalization of a graded algebra automorphism. A twisting system has played a pivotal role when we study noncommutative graded algebras. In fact, many fundamental properties of graded algebras, such as domain, noetherian, Gelfand-Kirillov dimension, Artin-Schelter regularity and so on, are preserved under a twisting system. One of the main applications of a twisting system is that the graded module category of twisted algebra is equivalent to that of the original graded algebra. However, it is often difficult to construct non-algebraic twisting systems even if a graded algebra is given by explicit generators and defining relations. It is a fundamental problem to classify all possible twisting systems of a given graded algebra and to provide criteria about whether a twisting system is algebraic.

A geometric algebra $A = \mathcal{A}(E, \sigma)$ introduced by Mori [3] is a quadratic algebra which determines and is determined by the pair (E, σ) where $E \subset \mathbb{P}(A_1^*)$ is a projective scheme and $\sigma \in \operatorname{Aut} E$. Cooney-Grabowski [1] showed that a twisted algebra A^θ of a geometric algebra $A = \mathcal{A}(E, \sigma)$ is isomorphic to a geometric algebra $\mathcal{A}(E, \Phi(\theta)|_E\sigma)$ where $\Phi(\theta)$ is the element of $\mathrm{PGL}(A_1^*)$ corresponding to a twisting system θ on A. This means that there is a nice correspondence between a twisted algebra and a geometric algebra. We denote by $M(E, \sigma)$ (resp. $Z(E, \sigma)$) the set of $\Phi(\theta)$ where θ varies through all possible (resp. algebraic) twisting systems. The following is one of our main results.

Theorem 1 ([2, Theorem 3.5]). *Let $A = \mathcal{A}(E, \sigma)$ be a geometric algebra.*

(1) $\{A^\theta \mid \theta$ *is a twisting system on* $A\ \}/_{\cong} = \{\mathcal{A}(E, \tau|_E\sigma) \mid \tau \in M(E, \sigma)\}/_{\cong}$.

(2) $\{A^\theta \mid \theta$ *is a algebraic twisting system on* $A\ \}/_{\cong} = \{\mathcal{A}(E, \tau|_E\sigma) \mid \tau \in Z(E, \sigma)\}/_{\cong}$.

A 3-dimensional Sklyanin algebra is a geometric algebra $A = \mathcal{A}(E, \sigma)$ where E is an elliptic curve in the projective plane $\mathbb{P}^2$ and σ is a translation by some point of E. It is known that defining relations of a 3-dimensional Sklyanin algebra A are given as follows;

$$A = k\langle x, y, z\rangle/(ayz + bzy + cx^2, azx + bxz + cy^2, axy + byx + cz^2)$$

where $abc \neq 0$ and $(a^3 + b^3 + c^3)^3 \neq (3abc)^3$. It is difficult to even determine all graded algebra automorphisms of A by using defining relations, however, it is easy to calculate $Z(E, \sigma)$ and $M(E, \sigma)$ for each pair (E, σ) because we can use some properties of E and σ. In this talk, we give a complete list of the two subsets $Z(E, \sigma)$ and $M(E, \sigma)$ for all 3-dimensional Sklyanin algebras. As an application, we classify twisted algebras of 3-dimensional Sklyanin algebras up to graded algebra isomorphism.

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The moduli of 4-dimensional subalgebras of the full matrix ring of degree 3

Kazunori Nakamoto and Takeshi Torii

Let k be an algebraically closed field. We say that k-subalgebras A and B of $\mathrm{M}_3(k)$ are equivalent if $P^{-1}AP = B$ for some $P \in \mathrm{GL}_3(k)$. There are 26 equivalence classes of k-subalgebras of $\mathrm{M}_3(k)$.

Definition 1. We say that a subsheaf $\mathcal{A}$ of $\mathcal{O}_X$-algebras of $\mathrm{M}_n(\mathcal{O}_X)$ is a *mold* of degree n on a scheme X if $\mathrm{M}_n(\mathcal{O}_X)/\mathcal{A}$ is a locally free sheaf. We denote by $\mathrm{rank}\mathcal{A}$ the rank of $\mathcal{A}$ as a locally free sheaf.

Proposition 2 ([1, Definition and Proposition 1.1], [2, Definition and Proposition 3.5])**.** *The following contravariant functor is representable by a closed subscheme of the Grassmann scheme* $\mathrm{Grass}(d, n^2)$*:*

$$\begin{array}{rccl} \mathrm{Mold}_{n,d} & : & (\mathbf{Sch})^{op} & \to & (\mathbf{Sets}) \\ & & X & \mapsto & \left\{ \mathcal{A} \;\middle|\; \mathcal{A} \textit{ is a rank } d \textit{ mold of degree } n \textit{ on } X \right\}. \end{array}$$

We consider the moduli $\mathrm{Mold}_{3,d}$ of rank d molds of degree 3 over $\mathbb{Z}$. For $d = 1, 2, 3, 6, 7, 8, 9$, we have:

Example 3 ([3])**.** Let $n = 3$. If $d \leq 3$ or $d \geq 6$, then

$$\begin{array}{rcl} \mathrm{Mold}_{3,1} & = & \mathrm{Spec}\mathbb{Z}, \\ \mathrm{Mold}_{3,2} & \cong & \mathbb{P}^2_{\mathbb{Z}} \times \mathbb{P}^2_{\mathbb{Z}}, \\ \mathrm{Mold}_{3,3} & = & \overline{\mathrm{Mold}^{\mathrm{reg}}_{3,3}} \cup \overline{\mathrm{Mold}^{\mathrm{S}_2}_{3,3}} \cup \overline{\mathrm{Mold}^{\mathrm{S}_3}_{3,3}}, \text{ where the relative dimensions of} \\ & & \overline{\mathrm{Mold}^{\mathrm{reg}}_{3,3}}, \overline{\mathrm{Mold}^{\mathrm{S}_2}_{3,3}}, \text{ and } \overline{\mathrm{Mold}^{\mathrm{S}_3}_{3,3}} \text{ over } \mathbb{Z} \text{ are } 6, 4, \text{ and } 4, \text{ respectively}, \\ \mathrm{Mold}_{3,6} & \cong & \mathrm{Flag} := \mathrm{GL}_3/\{(a_{ij}) \in \mathrm{GL}_3 \mid a_{ij} = 0 \text{ for } i > j\}, \\ \mathrm{Mold}_{3,7} & \cong & \mathbb{P}^2_{\mathbb{Z}} \coprod \mathbb{P}^2_{\mathbb{Z}}, \\ \mathrm{Mold}_{3,8} & = & \emptyset, \\ \mathrm{Mold}_{3,9} & = & \mathrm{Spec}\mathbb{Z}. \end{array}$$

In this talk, we describe the moduli $\mathrm{Mold}_{3,4}$ of rank 4 molds of degree 3.

Theorem 4 ([3])**.** *When $d = 4$, we have an irreducible decomposition*

$$\mathrm{Mold}_{3,4} = \overline{\mathrm{Mold}^{\mathrm{B}_2\times\mathrm{D}_1}_{3,4}} \coprod \mathrm{Mold}^{\mathrm{S}_7}_{3,4} \coprod \mathrm{Mold}^{\mathrm{S}_8}_{3,4},$$

whose irreducible components are all connected components. The relative dimensions of $\overline{\mathrm{Mold}^{\mathrm{B}_2\times\mathrm{D}_1}_{3,4}}$*,* $\mathrm{Mold}^{\mathrm{S}_7}_{3,4}$*, and* $\mathrm{Mold}^{\mathrm{S}_8}_{3,4}$ *over $\mathbb{Z}$ are* 5*,* 2*, and* 2*, respectively. Moreover, both* $\mathrm{Mold}^{\mathrm{S}_7}_{3,4}$ *and* $\mathrm{Mold}^{\mathrm{S}_8}_{3,4}$ *are isomorphic to* $\mathbb{P}^2_{\mathbb{Z}}$*, and* $\overline{\mathrm{Mold}^{\mathrm{B}_2\times\mathrm{D}_1}_{3,4}}$ *contains* $\mathrm{Mold}^{\mathrm{S}_6}_{3,4} \cup \mathrm{Mold}^{\mathrm{S}_9}_{3,4} \cup \mathrm{Mold}^{\mathrm{N}_3}_{3,4}$.

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Wall-and-chamber structures of stability parameters for some dimer quivers

Yusuke Nakajima

For a quiver (with relations), we can consider the *stability condition* in the sense of A. King [3]. In this talk, we consider the stability condition for quivers associated to dimer models. A *dimer model* Γ is a bipartite graph described on the real two-torus, and we can obtain the quiver $Q = Q_\Gamma$ as the dual graph of Γ. For the quiver Q, we consider the weight space

$$\Theta(Q) := \Big\{\theta = (\theta_v)_{v\in Q_0} \in \mathbb{R}^{Q_0} \mid \sum_{v\in Q_0} \theta_v = 0\Big\},$$

where Q_0 is the set of vertices of Q. For $\theta \in \Theta(Q)$, which is called a *stability parameter*, we define θ-(semi)stable representations of Q.

Definition 1. Let M be a representation of Q of dimension vector $\mathbf{1} = (1, \dots, 1)$. For a subrepresentation N of M, we define $\theta(N) := \sum_{v\in Q_0} \theta_v(\dim N_v)$, and hence $\theta(M) = 0$ in particular.

We say that a representation M is *θ-semistable* if $\theta(N) \geq 0$ for any subrepresentation N of M, and M is *θ-stable* if $\theta(N) > 0$ for any non-zero proper subrepresentation N of M.

The space $\Theta(Q)$ of stability parameters has the *wall-and-chamber structure*, that is, it is decomposed into chambers (which are open cones in $\Theta(Q)$) separated by walls (which are codimension one faces of the closures of chambers). By the result in [3], for a certain stability parameter θ, we can construct a moduli space $\mathcal{M}_\theta(Q, \mathbf{1})$ which parametrizes θ-stable representations of Q of dimension vector $\mathbf{1}$. If a dimer model Γ satisfies the "consistency" condition and θ is contained in a chamber of $\Theta(Q)$, then the moduli space $\mathcal{M}_\theta(Q, \mathbf{1})$ is a projective crepant resolution of a three-dimensional Gorenstein toric singularity [1], and any projective crepant resolution of this singularity can be obtained in this way [2].

In this talk, we focus on dimer models giving projective crepant resolutions of the toric singularity of the form $\mathbb{C}[x, y, z, w]/(xy - z^a w^b)$, which is a *toric cDV (compound Du Val) singularity* of type cA_{a+b-1}, where a, b are integers with $a \geq 1$ and $a \geq b \geq 0$. In particular, we show that a special class of paths on a dimer model called *zigzag paths* determine the wall-and-chamber structure of $\Theta(Q)$, and the variations of stable representations under wall-crossings in $\Theta(Q)$ can be described by using *perfect matchings* of a dimer model. Note that the homological minimal model program [5] also detects the wall-and-chamber structure of $\Theta(Q)$, whereas our method provides a more combinatorial way to observe it.

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On inductions and restrictions of support τ-tilting modules over group algebras

Ryotaro Koshio, Yuta Kozakai

Since the τ-tilting theory was introduced by Adachi-Iyama-Reiten in [1], classifications and features of the support τ-tilting modules have been given for many kinds of algebras. In particular, for group algebras and their block algebras, the considerations of the support τ-tilting modules are equivalent to those of two-term tilting complexes, hence they are expected to solve the Broué's Abelian Defect Group Conjecture.

Let k be an algebraically closed field of characteristic $p > 0$, G a finite group with a cyclic Sylow p-subgroup, $\tilde{G}$ a finite group with the normal subgroup G, and M a support τ-tilting kG-module. In [2, 4, 3], the authors showed that if M is $\tilde{G}$-invariant, then the induced module $\mathrm{Ind}_G^{\tilde{G}}M$ is a support τ-tilting $k\tilde{G}$-module, where $\mathrm{Ind}_G^{\tilde{G}} := k\tilde{G}\otimes_{kG} -$ means the induction functor. Naturally, we consider when $\mathrm{Res}_G^{\tilde{G}}\tilde{M}$ is a support τ-tilting kG-module for support τ-tilting $k\tilde{G}$-module $\tilde{M}$, where $\mathrm{Res}_G^{\tilde{G}}$ means the restriction functor. As a positive answer to this question, we have the following result.

Theorem 1. *Let k be an algebraically closed field of characteristic $p > 0$, G a finite group with a cyclic Sylow p-subgroup, $\tilde{G}$ a finite group with the normal subgroup G, and $\tilde{M}$ a support τ-tilting $k\tilde{G}$-module. If $\tilde{M}$ is relatively G-projective and it holds that $\mathrm{Ind}_G^{\tilde{G}}\mathrm{Res}_G^{\tilde{G}}\tilde{M} \in \mathrm{add}\,\tilde{M}$, then the restricted module $\mathrm{Res}_G^{\tilde{G}}\tilde{M}$ is a support τ-tilting kG-module.*

Moreover we have the equivalent conditions to the one of the above theorem, that is the condition that the support τ-tilting $k\tilde{G}$-module $\tilde{M}$ is relatively G-projective and $\mathrm{Ind}_G^{\tilde{G}}\mathrm{Res}_G^{\tilde{G}}\tilde{M} \in \mathrm{add}\,\tilde{M}$.

Theorem 2. *Let k be an algebraically closed field of characteristic $p > 0$, G a finite group with a cyclic Sylow p-subgroup, $\tilde{G}$ a finite group with the normal subgroup G, and $\tilde{M}$ a support τ-tilting $k\tilde{G}$-module. The following conditions are equivalent:*

- *The support τ-tilting $k\tilde{G}$-module $\tilde{M}$ is relatively G-projective and it holds that $\mathrm{Ind}_G^{\tilde{G}}\mathrm{Res}_G^{\tilde{G}}\tilde{M} \in \mathrm{add}\,\tilde{M}$.*
- *$\mathrm{add}\,\tilde{M} = \mathrm{add}\,\mathrm{Ind}_G^{\tilde{G}}M$ for some $\tilde{G}$-invariant support τ-tilting kG-module M.*
- *For each simple $k(\tilde{G}/G)$-module S, it holds that $S \otimes_k \tilde{M} \in \mathrm{add}\,\tilde{M}$.*

As an application of this theorem, we determine the image of the set of $\tilde{G}$-invariant support τ-tilting kG-module under the induction functor $\mathrm{Ind}_G^{\tilde{G}}$.

Theorem 3. *Let $(\mathrm{s}\tau\text{-tilt}kG)^{\tilde{G}}$ be the set of $\tilde{G}$-invariant support τ-tilting kG-modules, and $(\mathrm{s}\tau\text{-tilt}k\tilde{G})^{\star}$ be the one of support τ-tilting $k\tilde{G}$-modules satisfying one of the equivalent conditions of Theorem 2. Then we have the following isomorphism of the partially ordered sets by the induction functor $\mathrm{Ind}_G^{\tilde{G}}$:*

$$\mathrm{Ind}_G^{\tilde{G}} : (\mathrm{s}\tau\text{-tilt}kG)^{\tilde{G}} \xrightarrow{\sim} (\mathrm{s}\tau\text{-tilt}k\tilde{G})^{\star} \ \ (M \mapsto \mathrm{Ind}_G^{\tilde{G}}M).$$

Here we consider the case that the quotient group $\tilde{G}/G$ is a p-group. Then the only simple $k(\tilde{G}/G)$-module is the trivial $k(\tilde{G}/G)$-module up to isomorphism. Hence we have the following result by using Theorems 2 and 3.

Corollary 4. *Let $\tilde{G}/G$ be a p-group. Then the following isomorphism of the partially ordered sets by the induction functor $\mathrm{Ind}_G^{\tilde{G}}$:*

$$\mathrm{Ind}_G^{\tilde{G}} : (\mathrm{s}\tau\text{-tilt}kG)^{\tilde{G}} \xrightarrow{\sim} (\mathrm{s}\tau\text{-tilt}k\tilde{G}) \ \ (M \mapsto \mathrm{Ind}_G^{\tilde{G}}M).$$

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Faces of certain neighborhoods of presilting cones

Sota Asai and Osamu Iyama

Let A be a finite-dimensional algebra over a field K, and $\mathsf{proj}\,A$ be the category of finitely generated projective A-modules. We consider the homotopy category $\mathsf{K}^{\mathrm{b}}(\mathsf{proj}\,A)$ of the bounded complexes on the category $\mathsf{proj}\,A$ and the set $\mathsf{2\text{-}presilt}\,A$ of basic 2-term presilting complexes in $\mathsf{K}^{\mathrm{b}}(\mathsf{proj}\,A)$.

To study 2-term presilting complexes, it is important to use the Grothendieck group $K_0(\mathsf{proj}\,A)$ and the real Grothendieck group $K_0(\mathsf{proj}\,A)_{\mathbb{R}}$. The Grothendieck group $K_0(\mathsf{proj}\,A)$ has the indecomposable projective modules $P(1), \ldots, P(n)$ as a canonical basis, so $K_0(\mathsf{proj}\,A) = \bigoplus_{i=1}^{n} \mathbb{Z}[P(i)]$ and $K_0(\mathsf{proj}\,A)_{\mathbb{R}} = \bigoplus_{i=1}^{n} \mathbb{R}[P(i)]$ hold. By [1, 2], if $U = \bigoplus_{i=1}^{m} U_i \in \mathsf{2\text{-}presilt}\,A$ with U_i indecomposable, then the elements $[U_1], \ldots, [U_m]$ in $K_0(\mathsf{proj}\,A)_{\mathbb{R}}$ are linearly independent. Therefore, it is natural to consider the *presilting cones* $C(U)$ and $C^\circ(U)$ in $K_0(\mathsf{proj}\,A)_{\mathbb{R}}$ defined by

$$C(U) := \sum_{i=1}^{m} \mathbb{R}_{\geq 0}[U_i], \quad C^\circ(U) := \sum_{i=1}^{m} \mathbb{R}_{>0}[U_i].$$

The presilting cone $C^\circ(U)$ is strongly related to the *numerical torsion pairs* introduced by [4]. They associated two torsion pairs $(\overline{\mathcal{T}}_\theta, \mathcal{F}_\theta)$ and $(\mathcal{T}_\theta, \overline{\mathcal{F}}_\theta)$ in $\mathsf{mod}\,A$ to each $\theta \in K_0(\mathsf{proj}\,A)_{\mathbb{R}}$ by using the linear inequalities given by θ. Thanks to [5, 7], the first speaker [3] showed that the presilting cone $C^\circ(U)$ is given by numerical torsion pairs as

$$C^\circ(U) = \{\theta \in K_0(\mathsf{proj}\,A)_{\mathbb{R}} \mid \mathcal{T}_\theta = \mathsf{Fac}\,H^0(U),\ \mathcal{F}_\theta = \mathsf{Sub}\,H^{-1}(\nu U)\}.$$

It is sometimes efficient to focus on the set $\mathsf{2\text{-}presilt}_U\,A$ of 2-term presilting complexes which contain a fixed $U \in \mathsf{2\text{-}presilt}\,A$ as direct summands. Jasso [6] established the method called *τ-tilting reduction* by constructing a finite-dimensional algebra B and a bijection $\mathsf{2\text{-}presilt}_U\,A \to \mathsf{2\text{-}presilt}\,B$. Numerical torsion pairs are useful also in this context. For any $V \in \mathsf{2\text{-}presilt}\,A$, the condition $V \in \mathsf{2\text{-}presilt}_U\,A$ holds if and only if $C^\circ(V) \subset N_U$, where N_U is an open neighborhood of $C^\circ(U)$ defined by

$$N_U := \{\theta \in K_0(\mathsf{proj}\,A)_{\mathbb{R}} \mid H^0(U) \in \mathcal{T}_\theta,\ H^{-1}(\nu U) \in \mathcal{F}_\theta\}.$$

The first speaker [3] gave a linear surjection $\pi\colon K_0(\mathsf{proj}\,A)_{\mathbb{R}} \to K_0(\mathsf{proj}\,B)_{\mathbb{R}}$ with $\mathsf{Ker}\,\pi = \sum_{i=1}^{m} \mathbb{R}[U_i]$ so that π is compatible with numerical torsion pairs for elements in N_U and $K_0(\mathsf{proj}\,B)_{\mathbb{R}}$. Moreover, the closure $\overline{N_U}$ is a rational polyhedral cone in $K_0(\mathsf{proj}\,A)_{\mathbb{R}}$, because it is given by finitely many inequalities for θ. Currently, we are actively studying the properties $\overline{N_U}$ as a rational polyhedral cone.

In this talk, we will explain that the set $\mathsf{Face}\,\overline{N_U}$ of faces of $\overline{N_U}$ are described in terms of the linear projection π. The key property is that, for any $F \in \mathsf{Face}\,\overline{N_U}$, the intersection $F \cap C(U)$ is a face of $C(U)$, so we can write $F \cap C(U) = C(U/U_I)$ for some subset $I \subset \{1, 2, \ldots, m\}$. We have proved the following properties for each $I \subset \{1, 2, \ldots, m\}$.

(1) π sends the set $\{F \in \mathsf{Face}\,\overline{N_U} \mid F \cap C(U) = C(U/U_I)\}$ to a rational polyhedral fan Σ_I.
(2) There exists an explicitly given $M_I \in \mathsf{mod}\,B$ which recovers Σ_I.

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On trivial tilting theory

Takuma Aihara

In mathematics, trivial cases are regularly trivial (insipid and uninteresting). For instance, we study that in group theory, a group with only trivial subgroup is a cyclic group of prime order, and in ring theory, a commutative ring with only trivial ideal is a field. These are the first exercises for beginners. Neverthless, we cannot turn away from them.

In this talk, we discuss trivial tilting thoery for a finite dimensional algebra Λ over an algebraically closed field. Tilting theory deals with (classical) tilting modules, one-sided tilting complexes and two-sided tilting complexes (derived Picard groups). For each object, the trivial ones are Λ_Λ, the one-sided stalk complexes $\Lambda_\Lambda[m]$ and the two-sided stalk complexes ${}_1\Lambda_\varphi[m]$ (φ is an algebra automorphism of Λ), respectively. We will give answers to the question "when does Λ have only trivial tiltings (in each case)".

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